Group Walk Random Graphs II

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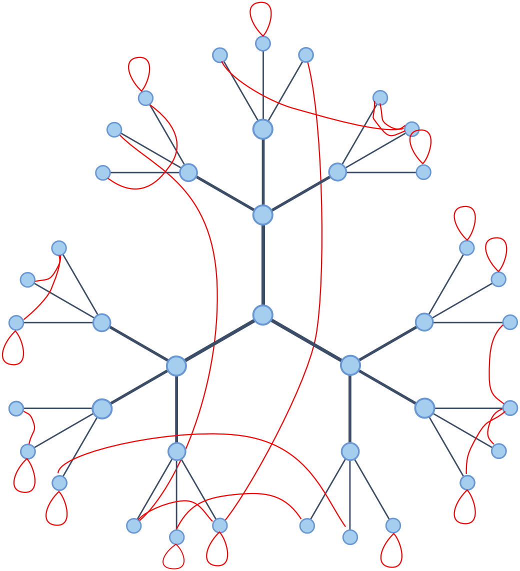
Oberwolfach, 1/16

Random Graphs from trees



$R_3^1(T)$

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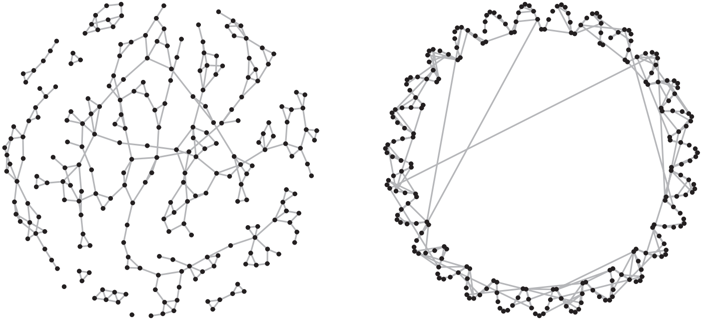
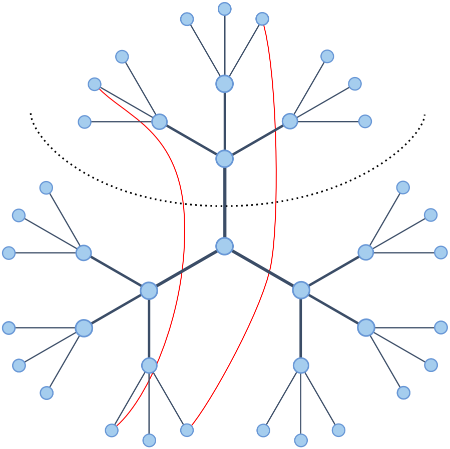


Simulation by A. Janse van Rensburg.
(Both figures depict the same graph.)

A nice property



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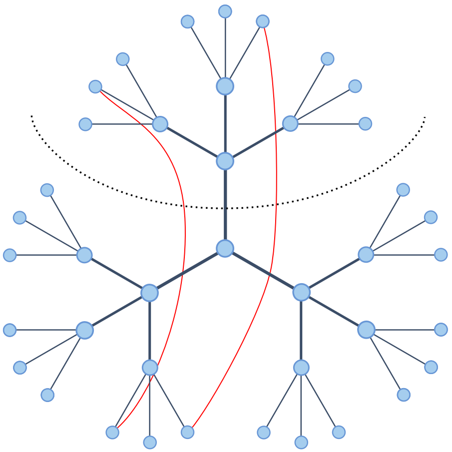

Proposition

$\mathbb{E}(\sharp$ *edges xy in* R_n *with x in X and y in Y*$)$

converges.

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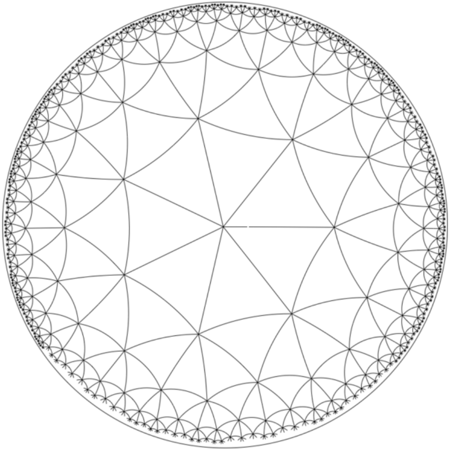

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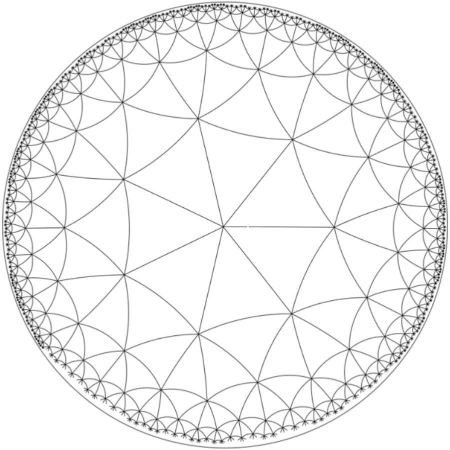

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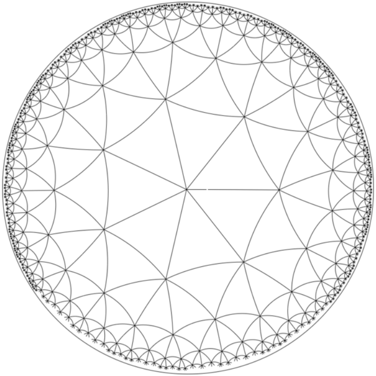

Proposition

For every two measurable subsets X, Y of the Martin boundary ∂G,

$$\mathbb{E}(\sharp \text{ edges } xy \text{ in } R_n \text{ with } x \text{ 'close to' } X \text{ and } y \text{ 'close to' } Y)$$

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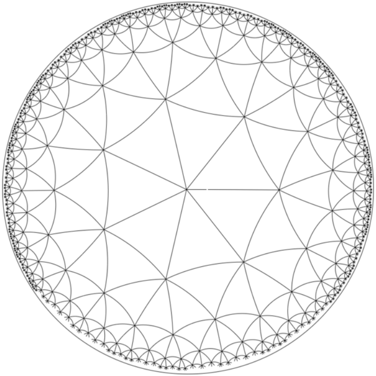

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We use the limit to define a measure on $\partial G \times \partial G$ via

$$C(X, Y) := lim \mathbb{E}(\sharp \text{ edges ...})$$

Effective conductance

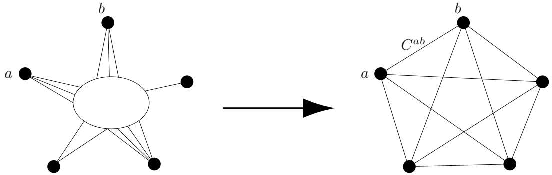
We call C the *effective conductance measure*, because

Theorem (G & V. Kaimanovich '12-'16+)

For every locally finite network G, and every harmonic function h, we have

$$E(h) = \int_{\partial G \times \partial G} \left(\widehat{h}(\eta) - \widehat{h}(\zeta)\right)^2 dC(\eta, \zeta).$$

Finite version: $E(h) = \sum_{a,b \in B} (h(a) - h(b))^2 \, C_{ab}$



Long range percolation

(Joint work in progress with O. Angel, G. Ray,
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How large is $R_\infty^\lambda(T)$?

The expected size of the TWRG

Let C_o^λ denote the component of a uniformly random vertex of $R_n^\lambda(T)$ (or $R_\infty^\lambda(T)$).

Theorem (G & Haslegrave, state of the art 14/1/16)

$$Ae^{a\lambda} \leq \mathbb{E}(|C_o^\lambda|) \leq Be^{e^{b\lambda}}.$$

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Conjecture:

$$\mathbb{E}(|C_o^\lambda|) \sim \lambda^\lambda$$

(backed by simulations)

Not covered today

Connectivity phase transition

Relation to Sznitmann's Random Interlacements

Relation to the Green function via the Naim kernel

Outlook

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Thank you!

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