Please let me know if any of the problems are unclear or have typos.

Exercise 10.1. [Medium.] Suppose that A is a set. The *free group generated by* A, denoted $\mathbb{F}_A$, is the free product of copies of $\mathbb{Z}$, one for each element of A. The *rank* of $\mathbb{F}_A$ is defined to be $|A|$, the cardinality of A. Show that $\mathbb{F}_A \cong \mathbb{F}_B$ if and only if $|A| = |B|$. (You may assume that A is finite. When it is, we may use the notation $\mathbb{F}_n$ for $\mathbb{F}_A$, where $n = |A|$.)

Exercise 10.2. [Page 85, of Hatcher.]

- Suppose that G is a graph and $p\colon G' \to G$ is a covering map. Show that G' is homeomorphic to a graph.
- [Nielsen-Schreier.] Suppose that $H < \mathbb{F}_A$ is a subgroup. Show that H is isomorphic to a free group.

Exercise 10.3. [Easy.] Suppose that $H < \mathbb{F}_n$ is a subgroup of index $k < \infty$. Compute the rank of H. Give a concrete example of an index three subgroup of $\mathbb{F}_2$.

Exercise 10.4. For any non-zero integer p we define the topological space L_p as follows.

$$L_p \ = \ D^2 \sqcup S^1 \big/ {z \sim z^p}$$

Check that $L_1 \cong D^2$. In general, find the fundamental group $\pi_1(L_p)$ and a universal cover $\widetilde{L_p}$. Provide illustrative figures. (For the completist: Do the same for L_0.)

Exercise 10.5. Set $\zeta = \exp(\pi i/n)$. Let D_n be the regular $2n$–gon in the complex plane $\mathbb{C}$, with vertices at the points $\{\zeta^k\}_{k=0}^{2n-1}$. Thus D_n is a closed, two-dimensional disk with $2n$ vertices and $2n$ edges. Let e_k be the edge with vertices ζ^k and ζ^{k+1}. Let $d_n = |1 + \zeta|$. We now form a quotient space $Q_n = D_n/\sim$. Identify two points $x, y \in D_n$ if

(∗) for some k, we have $x \in e_k$, $y \in e_{n+k}$, and $|x - y| = d_n$.

Show that Q_n is a surface. Using the induced CW structure, find a presentation of $\pi_1(Q_n)$. Give careful illustrations of the cases $n = 2$ and $n = 3$. (Challenge: Prove $Q_{2m} \cong Q_{2m+1}$.)

Exercise 10.6. [Exercise 14, page 80, of Hatcher.] List all connected covers of $P^2 \vee P^2$. Prove your list is complete, up to isomorphism of covers.

Exercise 10.7. [Picture-hanger's problem.] We identify our living-room wall with $\mathbb{C}$ and hammer in a pair of nails at 0 and 1. It is straight-forward to hang a picture P from these nails so that, after removing just one of them, P does *not* fall to the ground. Find a way to hang the picture so that, after removing either nail, P *does* fall. (Challenge: Suppose that we hammer in n nails at $0, 1, \ldots, n-1$. Find a way to hang P so that removing any one nail causes P to fall.)

Exercise 10.8. Explain the game of skill *fast-and-loose*, also called the *endless chain*, shown here: `http://youtu.be/pwO_u9E3ihU?t=1m27s`