2024-11-25 LECTURE 25 SAVL SCHLEIMER MA3F1 D EXAMPLES THE REAL PROJECTIVE SPACE: RECALL S"= { x = R ** | | | | | = 1]. DEFINE RIP = S"/R-- THE N-DIM'L PROJECTIVE STACE. EXERCISE: $\mathbb{RP}^{n} \cong \mathbb{R}^{n+1} - \{0\}$ $2 \sim 7 \times \mathbb{RP}$ $\mathcal{A} \in \mathbb{R} - \{0\}$ THE LATTER IS THE "SPACE of LINES IN R""." STMILARLY, S" IS THE "SPACE of DIRECTIONS IN R""." HERE IS A CELL STRUCTURE FOR S²: n_{z1} n_{z2} N_{z2} NWE TAKE THE QUOTTENT RE-S/XM-X. THIS HAS CELL STRUCTURE DOES NOT EMBED JN R³ THE FAKE ROSE : NOTE THAT R~ s¹. SO T, (K, K) = Z. ALSO SVK IMPLIES THAT $\pi_{1}(R, \lambda_{0}) \equiv \pi_{1}(R_{2})/N \cong \Gamma_{1}(R_{2})/N$

WHERE F, JS GEN BY a, b AND N=<<>>>.
50 $\mathbb{Z}_{\mathfrak{T}}(\mathbb{R}) \cong \langle \mathfrak{q}, \mathfrak{b} \rangle \mathfrak{b} \rangle$
THE SPHERE, AGAJN
$- \frac{1}{2} \left(\frac{1}{2} \right) \left(\frac$
$C_{n} = \frac{1}{2} C_{n} = \frac{1}$
D = 1 = 1, (D, N) = (V, V, V, V, V) = (V, V) =
N= <<4,417 = <<417.
γ
(2) COMPUTING TI, of CW COMPLEXES
PROP (1.26) [FOR CW COMPLEXES]
SUFPORE X TS CWI COMPLEX ETV & AX'
FOR @ 10 SUPPOSE COX IS A SUBCOMPLEX WITH TOt C (0).
FOR @ + () SUPPOSE COX IS A SUBCOMPLEX WITH TOt C(0). LET I: C -> XO DE THE INCLUSION MAP. SUPPOSE
$FOR @ \cdot @$ SUPPOSE COX IS A SUBCOMPLEX WITH TOt C ⁽⁰⁾ . LET I: C \rightarrow X. DE THE INCLUSION MAP. SUPPOSE X-C= e, IS A SINGLE N-CELL.
FOR $@ \cdot @$ SUPPOSE COX IS A SUBCOMPLEX WITH TOt $C^{(o)}$. LET $L:C \longrightarrow X_0$ DE THE INCLUSION MAP. SUPPOSE $X-C=e_X^n$ IS A SINGLE $n-CELL$. SUPPOSE $Q:S_X^{(i)} \longrightarrow C$ IS THE ATTACHING MAP.
FOR $@ \cdot @$ SUPPOSE COX IS A SUBCOMPLEX WITH TOt $C^{(o)}$. LET $L: C \longrightarrow X_0$ DE THE INCLUSION MAP. SUPPOSE $X - C = e_A^n$ IS A SINGLE $n - CELL$. SUPPOSE $Q: S_A^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A + S_A^{n'} \longrightarrow C$ IS THE ATTACHING MAP.
FOR $@ \cdot @$ SUPPOSE COX IS A SUBCOMPLEX WITH $\tau_0 \in C^{(0)}$. LET $:: C \longrightarrow X_0$ DE THE INCLUSION MAP. SUPPOSE $X - C = e_A^n$ IS A SINGLE $n - CELL$. SUPPOSE $Q_i : S_A^{(i)} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{(i)} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{(i)} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{(i)} \longrightarrow C$ IS A PATH FROM τ_0 TO s_A
FOR $@ \cdot @$ SUPPOSE CCX IS A SUBCOMPLEX WITH $\tau_0 \in C^{(0)}$. LET $\iota: c \longrightarrow X_0$ DE THE INCLUSION MAP. SUPPOSE $X - C = e_A^n$ IS A SINGLE $n - CELL$. SUPPOSE $Q: S_A^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n'} \longrightarrow C$ IS A PATH FROM τ_0 TO s_A (Q) SUPPOSE $n=2$ TOTALTEY $S_A^{n'} = [Q_A \cap T]/A$
FOR $@ \cdot @$ SUPPOSE COX IS A SUBCOMPLEX WITH $\tau_0 \in C^{(0)}$. LET $\iota: C \to X_0$ DE THE INCLUSION MAP. SUPPOSE $X - C = e_X^n$ IS A SINGLE $n - CELL$. SUPPOSE $Q_i: S_X^{n-1} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n-1} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n-1} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n-1} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n-1} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n-1} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_A \in S_A^{n-1} \longrightarrow C$ IS A PATH FROM τ_0 TO s_A (a) SUPPOSE $n=2$. IDENTIFY $S_A = [0,1]/0-1$ AND ETRAM $X = q_A + Q_A = q_A$
FOR $@ \cdot @$ SUTPOSE COX IS A SUBCOMPLEX WITH TOFC ⁽⁰⁾ . LET I: C \rightarrow X, DE THE INCLUSION MAP. SUPPOSE X-C= e_{χ}^{n} IS A SUNGLE M-CELL. SUPPOSE $Q_{i}: S_{\chi}^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $Q_{i}: S_{\chi}^{n'} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_{\chi} \in S_{\chi}^{n'}$ AND $g_{\chi}: I \longrightarrow C$ IS A PATH FROM TO S_{χ} @ SUPPOSE $n=2$. IDENTIFY $S_{\chi}^{i} = [0,1]/_{0,n-1}$ AND FORM $S_{\chi} = g_{\chi} * U_{\chi} * g_{\chi}$ & LOOPS (C, To). THEFTIES AND $S_{\chi} = g_{\chi} * U_{\chi} * g_{\chi}$ & LOOPS (C, To).
FOR $@ \cdot @$ SUPPOSE COX IS A SUBCOMPLEX WITH $\tau_0 \in C^{(0)}$. LET $1: C \rightarrow X_0$ DE THE INCLUSION MAP. SUPPOSE $X - C = e_A^n$ IS A SINGLE $n - CELL$. SUPPOSE $Q_a: S_a^{n'} \rightarrow C$ IS THE ATTACHING MAP. SUPPOSE $S_a \in S_a^n$ AND $g_A: I \rightarrow C$ IS A PATH FROM τ_0 TO s_A (a) SUPPOSE $n=2$. IDENTIFY $S_a' = [0,1]/_{0,n-1}$ AND FORM $Y_a = g_a * Q_a * \overline{g}_a \in LOOPS(C, \tau_0)$. DEFINE $N = 44 \times 5477$.
FOR Q (D) SUPPOSE COX IS A SUBCOMPLEX WITH TO C ^(a) . LET I: C >> Xo DE THE INCLUSION MAP. SUPPOSE X-C= e_{χ}^{n} IS A SINGLE M-CELL. SUPPOSE Q: $S_{\chi}^{n,1} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE Q: $S_{\chi}^{n,1} \longrightarrow C$ IS THE ATTACHING MAP. SUPPOSE S_C $S_{\chi}^{n,1} \longrightarrow C$ IS A PATH FROM To SQ Q SUPPOSE N=2. IDENTIFY $S_{\chi}^{n} = [0,1]/0.1$ AND FORM $\chi_{\chi} = g_{\chi} * Q_{\chi} * \overline{g}_{\chi} \in LOOPS(C, To).$ DEFINE N=48 8277.



(b) SUPPOSE 1.73. THEN L_{*} IS AN ISOMORPHISM. (c) THE INCLUSION $\chi^{(2)} \longrightarrow \chi$ INDUCES ISOMORPHISM $T_{*}(\chi^{(2)}, \kappa_{0}) \longrightarrow T_{*}(\chi, \kappa_{0})$

PROOF: @ FOLLOWS FROM () + INDUCTION JF X IS FINITE LATTACH THE CELLS of X-X" ONE BY ONE] IF X IS NOT FINITE WE REDUCE TO THE FINITE CASE USING AD AS FOLLOWS. LET t X(2) -> X. WIS to IS ISOMORPHISM. SURJECTIVE : FIX [FJEJEJ.(X, x.) SO BELOOPS(X, x.) SINCE I IS COMPACT GO IS TOLI). (AT) IMPLIES THERE IS A FINITE SUBCOMPEX COX SO THAT

V(I) CC. NOTE TOE V(I) SO TOE C"

WE HAVE INCLUSIONS:

(b) IMPLIES St IS ISOMORPHISM, LET $[\sigma'] = (s_{\star})^{-}([\sigma])$. NOTE $\sigma' \in LOOPS(C^{k}, \kappa_{\circ})$ AND $\gamma' \in \sigma$. NOTE $(t_{\star} \circ P_{\star})[\sigma'] = [\sigma]$. SINCE σ WAS ARB IN LOOPS(κ, τ_{\circ})

t, IS SURJECTIVE /

INDECTIVE: SUTPOSE $y \in LOOPS(X^{(r)}, n_0)$. SUTPOSE $[t] = [e] \in \pi_1(X, r_0)$. SU THERE IS $F: I \times I \longrightarrow X$ A BASED NULL-HONOTOPY GIVING $f_{a}^{a}e$. SO $F(I \times I)$ IS COMPACT, SO GUNTAINED T F x_0 IN SOME FIN. SUBCOMPLEX. THUS [t] = [e] x_0 IN $\pi_1(C, \pi_0)$. SO [t] = [e] IN $\pi_1(C^{2}, \times_0)$ BT (6). [THAT IS: THE ISOMORPH. s_{a} IMPLIES THERE IS SOME $F': I \times I \longrightarrow C^{(r)}$ SO THAT $t^{a} = e$ IN $c^{(r)}$. SINCE $C^{(r)} c X^{(c)}$ WE ARE TONE WITH (6) (A) (6) NENT JIME!