Please let me know if any of the problems are unclear or have typos, if you have suggestions for exercises, or if you would like to do just part of a problem. For some of the problems I have given a (very vague) level of difficulty.

Exercise 8.1. Suppose that M is a closed, connected, oriented three-manifold. Suppose further that $\pi_2(M)$ is non-trivial. Prove that M is either a non-trivial connect sum or is homeomorphic to one of P^3 or $S^2 \times S^1$.

Exercise 8.2. Take

$$S = \{z \in \mathbb{C} \mid 0 < \mathrm{Imag}(z), 1 \leq z \leq 2\}$$

to be a vertical strip in the upper half plane model of $\mathbb{H}^2$. Let ϕ be the map (from the vertical edge of S above 1 to the vertical edge above 2) defined by $\phi(z) = 2z$. Let A be the metric completion of S/ϕ.

- Give a sketch of S and of A.
- Show that A has a boundary component γ which is a geodesic loop of length $\ln(2)$.
- Show that any compact arc in A, connecting a point of A to a point of γ, must cross the edges of S infinitely many times.

Exercise 8.3. [Hard?] Give a more formal proof of Lemma 5.4.

Exercise 8.4. Let (M, K) be a compact, connected, oriented and triangulated three-manifold. Suppose that S and T are transverse, oriented, embedded normal surfaces in (M, K). Let $\gamma \subset S \cap T$ be a curve of intersection. Let R be the result of doing an *exchange and round-off* along γ. (That is, let $N = N(\gamma)$ be a small regular neighbourhood of γ. Let A be two non-adjacent annuli components of $\partial N - (S \cup T)$. We form $R = ((S \cup T) - N) \cup A$.) Finally suppose that Δ is a face of $K^{(2)}$.

Prove that if some component of $R \cap \Delta$ is a closed loop then some component of $R \cap \Delta$ is a non-normal arc.