Please let me know if any of the problems are unclear or have typos. Please let me know if you have suggestions for exercises. For some of the problems I have given a (very vague) level of difficulty. Finally, if you want to do just part of a problem, let me know.

Exercise 5.1. Suppose that $K \subset S^3$ is a knot. Suppose that $n(K)$ is a small open regular neighbourhood of K. Define the *knot exterior* to be $X_K = S^3 - n(K)$. Show that X_K is irreducible.

Exercise 5.2. Suppose that Δ is a model tetrahedron with vertices labelled by 0, 1, 2, and 3. Let ϕ be the pairing taking the face 012 to the face 123 (in that order). Let $T = \{\Delta, \phi\}$ be the resulting triangulation. Let $M = |T|$.

- Prove that M is a three-manifold with boundary.
- Prove that $M \cong S^1 \times D^2$.
- Draw the resulting triangulation of $\widetilde{M}$, the universal cover of M.
- Realise a meridian disk of M as a normal surface.

Exercise 5.3. Suppose that Δ is a model tetrahedron with vertices labelled by 0, 1, 2, and 3. Let ϕ_0 be the pairing taking the face 012 to the face 123 (in that order). Let ϕ_1 be the pairing taking the face 013 to the face 023 (in that order). Let $T = \{\Delta, \phi_j\}$ be the resulting triangulation. Let $M = |T|$.

- Prove that M is a three-manifold.
- Prove that $M \cong S^3$.
- [Medium.] Note that the two edges of the one-skeleton $T^{(1)}$ each give a loop (and thus a knot) in S^3. Identify these knots.
- [Hard.] By removing a point from the interior of Δ we may realise $T^{(1)}$ as a graph in $\mathbb{R}^3$. Draw this graph.

Exercise 5.4. Suppose that F is a surface. Show that an I–bundle T over F is determined (up to bundle isomorphism) by the associated *monodromy* homomorphism $\rho_T \colon \pi_1(F) \to \mathbb{Z}_2$. [This was only claimed in lecture; give the details.]

Exercise 5.5. Let M be the result of removing a small open ball from the real projective three-space. Prove that M is homeomorphic to the orientation I-bundle over P^2, the real projective plane.

Exercise 5.6. [Medium.] Suppose that U and V are copies of the orientation I–bundle over K^2, the Klein bottle. Suppose that $\phi \colon \partial U \to \partial V$ is a homeomorphism. Set $M_\phi = U \cup_\phi V$. Determine how the Thurston geometry of M_ϕ depends on ϕ. In particular, find all $\mathbb{E}^3$ manifolds that have this form.

Exercise 5.7. [Medium.] Enumerate all triangulations consisting of one tetrahedron. Decide which of these are three-manifolds and determine their homeomorphism type.

Exercise 5.8. [Open.] Determine the *Matveev complexity* (minimal triangulation number) of the lens spaces $L(p, q)$.