Please let me know if any of the problems are unclear or have typos.

Exercise 5.1. Revise the definitions and then prove the following. Suppose that K is a field and $V = \{V_i, \partial_i\}$ is a finite, exact sequence of vector spaces over K.

- Prove that V splits as a direct sum of exact sequences of length two.
- Deduce that the alternating sum $\sum(-1)^i \dim V_i$ vanishes.
- Show by means of an example that the splitting need not be canonical.

Exercise 5.2. Find an example for, or disprove, the following statement.

- There is a connected, compact three-manifold M so that $\partial M \cong P^2$.

Exercise 5.3. Suppose that $K \subset S^3$ is a knot and let $X_K = S^3 - n(K)$ be its exterior. Show that X_K is irreducible.

Exercise 5.4. Suppose that $K \subset S^3$ is a knot. Let $N_K = N(K)$ be a closed tubular neighborhood of K; take $n(K)$ to be its interior. Set $X_K = S^3 - n(K)$ and take $T = \partial X_K = \partial N_K$. We define the *meridian* and *longitude* to be oriented simple closed curves μ and λ in T which die in $H_1(N_K)$ and $H_1(X_K)$ respectively.

- Prove that μ and λ are well-defined (up to orientation and isotopy).
- Prove that the algebraic intersection number $\mu \cdot \lambda$ is plus or minus one.

Exercise 5.5. For each knot K with at most six crossings compute $g(K)$, its *minimal Seifert genus*. Show that $g(K) = 0$ if and only if K is the unknot.

Exercise 5.6. Compute the mapping class group (homeomorphisms up to isotopy) of the solid torus $D^2 \times S^1$.

Exercise 5.7. Suppose that K and K' are non-trivial oriented knots. Define $M = X_K \cup_A X_{K'}$ by gluing a meridional annulus of ∂X_K to a meridional annulus of $\partial X_{K'}$. Show that M is again a knot complement; the resulting knot L is called the *connect sum* $K \# K'$. Draw L and show it is a satellite knot. (Some care with orientations is needed to make the connect sum well-defined.)