Please let me know if any of the problems are unclear or have typos.

Exercise 3.1. Fix a category (Top, PL, or Diff) and write out a definition of orientability, for manifolds, in this category. Show that a surface F is orientable if and only if F does not contain a Mobiüs strip. Likewise, show that a three-manifold M is orientable if and only if M does not contain a solid Klein bottle.

Exercise 3.2. Suppose that F is a surface. Give a classification of I–bundles over F, up to homeomorphism. [Hint: $H_1(F, \mathbb{Z}/2\mathbb{Z})$ and Alexander's trick.]

Exercise 3.3.

- Show that every lens space is obtained by gluing a pair of solid tori.
- Show that every prism manifold is obtained by gluing a solid torus to a copy of the orientation I–bundle over K^2.

Exercise 3.4.

- Equip S^3 with the round metric. Prove that $\text{Isom}(S^3) \cong O(3)$ and $\text{Isom}^+(S^3) \cong \text{SO}(3)$.
- Equip $S^2 \times \mathbb{R}$ with the usual product. Prove that $\text{Isom}(S^2 \times \mathbb{R}) \cong \text{Isom}(S^2) \times \text{Isom}(\mathbb{R})$.

Exercise 3.5. [Hard.] Classify the finite subgroups of $SO(4)$ which act freely on S^3. [See, for example, Wolf's book "Spaces of constant curvature".]

Exercise 3.6. Suppose that M is a three-manifold whose universal cover $\widetilde{M}$ is homeomorphic to S^3. Prove that M is orientable. [There is a proof that avoids geometrisation and thus avoids Exercise 3.4.]

Exercise 3.7. Classify the subgroups of $\text{Isom}(S^2 \times \mathbb{R})$ which act freely and properly on $S^2 \times \mathbb{R}$. Use this to classify the manifolds with $S^2 \times \mathbb{R}$ geometry, both up to isometry and up to homeomorphism.

Exercise 3.8. Show how to obtain the compact manifolds with $S^2 \times \mathbb{R}$ geometry by gluing a pair of disk bundles (over the circle) to each other, or gluing one such to itself.

Exercise 3.9.

- Suppose that M is a compact connected oriented irreducible three-manifold. Prove that M is prime.
- However, prime does not imply irreducible. Find all counter-examples.