Please let me know if any of the problems are unclear or have typos.

Exercise 2.1. [Medium] Give an example of a metric space (X, d) which does not isometrically embed into the euclidean plane $\mathbb{E}^2$: that is, into $\mathbb{R}^2$ with the usual metric.

Exercise 2.2. State the definition of arccos. Now suppose $u, v \in \mathbb{R}^n$ are vectors. We define the angle between u and v to be $\theta_{u,v} = \arccos(u \cdot v/|u||v|)$. Show that $\theta_{u,v}$ is well-defined. Now compute the angles between the following pairs of vectors in $\mathbb{R}^2$.

- $(1, 0)$ and $(0, 1)$.
- $(0, 1)$ and $(-1/2, \sqrt{3}/2)$.
- $(-1/2, \sqrt{3}/2)$ and $(-1, 1)$.
- $(-1, 1)$ and $(-1, 0)$.

Exercise 2.3. For $a, b \in \mathbb{R}^n$ we define

$$[a, b] = \left\{ c \in \mathbb{R}^n \,\middle|\, \exists t \in [0, 1] \text{ so that } c = ta + (1 - t)b \right\}.$$

Suppose $x, y, z \in \mathbb{R}^n$ have $x \in [y, z]$, $y \in [z, x]$, and $z \in [x, y]$. What can you deduce? Justify your answer.

Exercise 2.4. Set $X = [-1, 1]$ and, for $x, y \in X$ define $d(x, y) = |x - y|$. Verify this is a metric. Find the group $\mathrm{Isom}(X)$ and justify your answer.

Exercise 2.5. [Exploration] Fix $n \geq 2$. For $p \geq 1$ and for $x, y \in \mathbb{R}^n$ define

$$d^p(x, y) = \left(\sum_i |x_i - y_i|^p \right)^{1/p}.$$

Verify that $(\mathbb{R}^n, d^p)$ is a metric space. Show that $(\mathbb{R}^n, d^p)$ is isometric to $(\mathbb{R}^n, d^q)$ if and only if $p = q$.