Lecture XVI:

Suppose $v, v' \in h \in H$ are adjacent, $v < v'$

Say σ, σ' are the transition slices for $v, v' \in h$ if

$\begin{Bmatrix} (h,v) \\ (h,v') \end{Bmatrix}$ is the bottom pair of $\begin{Bmatrix} \sigma \\ \sigma' \end{Bmatrix}$, $\begin{Bmatrix} \sigma \text{ is terminal} \\ \sigma' \text{ is initial} \end{Bmatrix}$

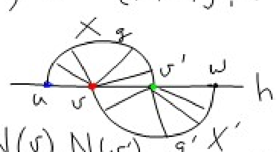
and for all $(k,\omega) \in \begin{Bmatrix} \sigma \\ \sigma' \end{Bmatrix}$ if $g \in H$, $D(g)$ is a component domain of $(D(k), \omega)$ then g appears in $\begin{Bmatrix} \sigma \\ \sigma' \end{Bmatrix}$

iff $\begin{Bmatrix} D(g)|_{v'} \\ D(g)|_{v} \end{Bmatrix} \neq \emptyset$.

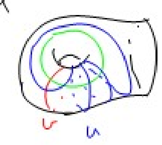
Easy examples: If $Y = D(h)$

(1) $Y \cong A^2$, $\sigma = \{(h,v)\}$, $\sigma' = \{(h,v')\}$

(2) $Y \cong S_{1,1}$.



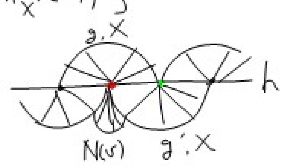
Let $X, X' = N(v), N(v')$

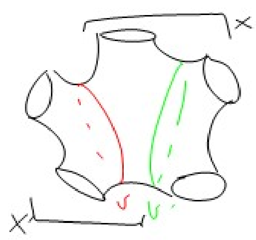


$\sigma = \{(h,v), (g, \pi_X(v'))\}$

$\sigma' = \{(h,v'), (g', \pi_{X'}(v))\}$

(3) $Y \cong S_{0,5}$





X, X' is the compt domain of $Y \setminus v$, $Y \setminus v'$ that is $\cong S_{0,4}$

$\sigma = \{(h,v), (g, v')\}$

$\sigma' = \{(h,v'), (g', v)\}$

Lemma [Transition] Suppose $v, v' \in h$, σ, σ' are the transition slices, $\xi(D(h)) \geq 2$.

Then $\text{trans}(\mu_\sigma) = \text{trans}(\mu_{\sigma'}) = \emptyset$ and $\text{base}(\mu_\sigma) = \text{base}(\mu_{\sigma'})$, the unused domains of σ, σ' are exactly the compts of $D(h) \setminus (v \cup v')$.

Pf omitted // Point: σ' is a rearrangement of σ.

Elementary moves:

$\tau, \tau' \in V(H) = \{\text{complete slices}\}$

$v, v' \in h \in H$ adjacent.

Write $\tau \to \tau'$ [Element. move]

if σ, σ' are transition slices for v, v'

$\tau \setminus \sigma = \tau' \setminus \sigma'$, $\sigma \le \tau$, $\sigma' \le \tau'$.

Lemma: If $\tau \to \tau'$ then $\tau <_s \tau'$

Pf: $\sigma \neq \sigma'$ so $\tau \neq \tau'$ $((h,v) <_p (h,v'))$

Fix any $(k,w) \in \tau \setminus \tau'$ [Want to find $(k',w') \in \tau'$ s.t. $(k,w) <_p (k',w')$]

So: $(k,w) \in \sigma \Rightarrow D(k) \subseteq D(h)$

$\Rightarrow D(k)|_{v'} \neq \emptyset$

if $k = h$ then $w = v'$ and we are done.

If not: $\max \phi_h(k) = v$ so

by def of $<_p$ $(k,w) <_p (h, v')$ //

Def: Let $|H| = \sum'_{h \in H} |h|$

Prop: Any complete hier. H admits a resolution $\{\tau_i\}_{i=0}^{N}$

⊛ $\mu_{\tau_0} = I(H)$ and τ_0 is initial

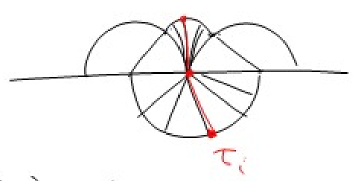
⊛ $\mu_{\tau_N} = T(H)$ and τ_N is terminal.

⊛ $\tau_i \to \tau_{i+1}$

⊛ $N \le |H|$.

Pf: Induct on slice order. //

Remark: $V(H)$ may be much larger than $|H|$.



Thm: $\exists B = B(S)$ s.t.

$$d_M(I(H), T(H)) \le B|H|$$

Pf: Build H, choose a resolution $\{\tau_i\}$

Let $\mu_i := \mu(\tau_i)$. Exercise:

There is a uniform $B = B(S)$ s.t.

$\forall i \quad d_M(\mu_i, \mu_{i+1}) \le B$. //