# Some Nonlinear Research Problems

R. S. MacKay

October 1, 2000

**Abstract**

Some of these are easy, some are hard longstanding challenges. Suitability is indicated by following codes:

**U** Undergraduate final year project

**M** MSc dissertation

**I** Initial project for a PhD student (I usually recommend tackling a relatively simple problem first before trying a main PhD project).

**T** PhD thesis

**F** Research Fellowship project (postdoctoral)

**X** Fields medal or Nobel prize.

There is also a status code (which may change at any time!): * means "in progress", though this may or may not exclude further people being involved.

# 1 Classical Discrete Breathers

## 1.1 Localised modes on discrete breathers [TF*]

Try to complete work of Baesens&Kim&MacKay (incomplete manuscript), to explain the observed numbers of localised modes on discrete breathers, as functions of $\epsilon$ and $\omega_b$, possibly using Cretegny et al extension of Levinsons's theorem (Phys D 119 (1998) 73). Connect with Kim&Kim "perfect transmission" mode (preprint)?

## 1.2 Breathers in systems with acoustic phonons

This can go in a variety of directions, as follows:

### 1.2.1 Breathers in a DNA model [IT]

Forinash et al Phys Rev E 55 (1997) 4740. Flesh out a viewgraph I wrote on this model for Crete 96 conference.

### 1.2.2 Prove existence of Page and Sievers&Takeno breathers in FPU chains [TF*]

Link with impurity mode of linearised defective system, cf. Kivshar (maybe find a reference in the Flach&Willis review, Phys Rpts 295 (1998) 181), or continue from the purely quartic case of Flach (Phys Rev E 51 (1995) 1503), or do a variational proof, cf. Friesecke&Wattis (CMP 161 (1994) 391), using approaches of A.Weinstein or others.

### 1.2.3 Redo breathers for diatomic chains at constant action [MI]

Livi&Spicci&MacKay (Nonlin 10 (1997) 1421) did continuation at constant period; constant action would be better. Understand which breathers have frequencies in the gap.

### 1.2.4 Linear stability of diatomic chain breathers [MIT*]

Apply linear stability analysis of MacKay&Sepulchre (Phys D 119 (1998) 148) to the Livi&Spicci&MacKay breathers (Nonlin 10 (1997) 1421).

### 1.2.5 Extend Diatomic chain breathers to Euclidean invariant networks [T*]

Use continuation of equilibria of Euclidean invariant systems subsection 3.6, to make concrete applications of my 'Optic discrete breathers', I J Nonlin Sci Num Sim 1 (2000).

## 1.3 Connect Anderson localisation to breathers [TF]

Build on Archilla et al (Phys D 134 (1999) 406-418) and Kopidakis&Aubry papers (Phys D (1999) and in press), e.g. 2D cases. Alan Bishop also claims to have some connections (see reference list he wrote for me).

## 1.4 Nekhoroshev estimates for breathers [TF]

Redo (?) Bambusi's proof (Nonlin 9 (1996) 433), to see to what extent it follows the plan sketched in MacKay&Aubry (Nonlin 7 (1994) 1623). Compare comments in de Jong's thesis.

## 1.5 Explicit estimates for breathers [TF*]

Extend Kadelburg's explicit $\epsilon_0(\omega_b)$ for continuation of breathers to other examples, and to $H^{1/2}$ (cf. MacKay in XI ICMP Paris'94, ed Iagolnitzer D). Correct the problem Kadelburg found with norms giving quadratic estimates near resonance.

## 1.6 Free boundary conditions in FPU chains implies softening? [MI]

Look into claim of Page and Sievers (private communication, Lyon) and see what hypotheses are required on asymmetry of the potential.

## 1.7 Compute the distribution of breather sizes in thermal equilibrium [TF*]

Declare any site with energy larger than $K\epsilon$ to be a breather. Compute density of breathers from $e^{-\beta H}$ and the distribution of amplitudes, cf. theses of Aubry and Dauxois, and section in my Hong Kong conf proc.

## 1.8 Travelling kinks and breathers (with tails?) [TF*]

Find an integrable system (Ablowitz-Ladik or Toda lattice) with travelling breathers or kinks, with or without tails. Choose speed/frequency so that they are skew-periodic orbits. Linearise the one-period map. If only double eigenvalue $+1$ then continue to non-integrable cases, cf. Carr and McLeod withdrawn preprint, Iooss&Kirchgassner and Iooss preprints.

## 1.9 One-phase averaging for breathers [TF*]

Reduce the error estimates in Ahn et al (Nonlin Dyn, in press), e.g. using subsection 6.33. Show this can lead to approximate travelling breathers. cf. Houle, Phys Rev E 56 (1997) 3657

## 1.10 Instability of breathers under two-phonon resonance? [TF]

MacKay&Sepulchre (Phys D 119 (1998) 148) prove $\ell_2$-linear stability of a class of breathers provided $n\omega_b \neq \omega_{ph1} + \omega_{ph2}$ for any two phonon frequencies. Cretegny et al (Phys D 119 (1998) 73) find inelastic scattering if this condition fails. Does $\ell_2$-linear stability also fail?

## 1.11 Quasiperiodic breathers [TF]

Prove generic non-existence? but see de Jong's thesis.

## 1.12 Fragmentation of proteins [TF*]

Derrick and Giannacopoulos (Chem dept) observe preferential fracture of proteins in certain places under impact by water molecules. Seek to understand this in terms of regions in which energy spreads and regions in which it remains localised, on the basis of FPU models with pieces of third nearest neighbour interaction ($\alpha$-helices). Might DB also be relevant to hydrocarbon cracking?

# 2 Quantum network problems

## 2.1 Persistence of spectral projections [TF*]

Extend results of my Hong Kong proceedings to get spatial decorrelation estimates, and apply to correlated electron models, e.g. attractive Hubbard model, and maybe to Anderson impurity model.

## 2.2 Holstein model

### 2.2.1 Excited states [UMITF]

Compute Betti numbers of the general invariant varieties of critical points at the uncoupled limit, to establish existence of lots of excited states for weak coupling. Then extend to Holstein-Hubbard model, i.e. complete Baesens&MacKay, J Phys A 31 (1998) 10065, using my persistence of spectral projections result.

### 2.2.2 Holstein-Hubbard bipolarons [TF]

Use my persistence of spectral projections results to prove existence, following Baesens&MacKay, Nonlin 7 (1994) 59

### 2.2.3 Quadrisinglet bipolarons [MI]

Understand region where Proville's quadrisinglet is minimum energy in terms of singularity theory and Maxwell sets.

### 2.2.4 Minimum energy states for 1D adiabatic Holstein [TF]

Try to prove they are rotationally ordered.

## 2.3 Bipolarons in SSH model [F]

Su-Schrieffer-Heeger (PRL 42 (1979) 1698), and then in models of PPV (Greenham&Friend, Solid State Phys 49 (1995) and Friend, Nature 397 (1999) 121), cf. Shimoi&Abe, PRB 50 (1994) 14781 Maybe extend to cuprate superconductors, cf. Phys World Feb 2000 and Proville & Aubry.

## 2.4 Semiclassical mechanics of breathers [TF]

Extend Aubry&Proville draft paper. Connect with my rigorous approach (Hong Kong).

## 2.5 Quantum statistical mechanics of F-K chains [TF*]

Extend my classical stat mech paper, J Stat Phys 80 (1995) 45, to quantum case by converting to classical statistical mechanics of a chain of elastic loops of length $\beta\hbar$ at temperature $\hbar$, cf. Tognetti, Feynman (Lectures on Stat Mech).

## 2.6 Energy conversion in cells

e.g. Rees&Howard, J Mol Bio 293 (1999) 343. In particular,

### 2.6.1 Understand how photosynthesis works [TF]

cf. Lord Porter's talk at JJ Thompson centenary, and Xu et al, PNAS 95 (1998) 5935, and EC TMR net www.biophys.leidenuniv.nl

### 2.6.2 Molecular motors [UMI]

Understand how muscle and other motors work, e.g. Kitamura et al, Nature 397 (1999) 129; Howard, Nature 365 (1993) c.710; Hua et al, Nature 388 (1997) 390; Schnitzer&Block, Nature 388 (1997) 386; Yasuda et al, Cell 93 (1998) 1117; Astumian, Science 276 (1997) 917; Phil Trans Roy Soc B 355, 29 April 2000; Rayment et al, Science 261 (1993) 58; Spudich, Nature 372 (1994) 515. Also EC RTN 2000-00091.

## 2.7 Carbon nanotubes [MT]

Work on some aspect (mechanical or electronic) of carbon nantubes, e.g. Jan 1998 and July 2000 issues of Phys World, and Yao et al, Nature 402 (1999) 273

## 2.8 4-methyl pyridine breathers [TF*]

Understand Fillaux's experiments, and his interpretation in terms of quantum sine-Gordon equation (cf. Bullough), and try out an explanation in terms of quantum version of rotobreathers (cf. Aubry).

# 3 Other problems in networks

## 3.1 Equilibria of Mallet-Paret's image processing network [MI]

in 'Stochastic and spatial aspects of dynamical systems', eds van Strien and Verduyn Lunel. All those of simplest type were found by MacKay and Sparrow. Find the rest. Compare Abell&Humphries&vanVleck preprint. Define $W^s_{\hat{\theta}}$ for equilibria in 2D lattice for Mallet-Paret.

### 3.2 Gibbs potential for coupled map lattices [MI*]

For a coupled map lattice $f$ on a network S, $tr(\log Df^u)_{ss}$ provides a potential whose Gibbs states are natural invariant measures. Compute it for some examples, and make examples with interesting Gibbs potentials. cf. project with M.Jiang.

### 3.3 CMLs with phase transition [UMI]

Simulate Gielis&MacKay CMLs. Try to make examples where space-time corresponds to just space for some stat mech problem. Understand in what sense Kuramoto's globally coupled rotors (in: Araki H (ed), SLNP 39 (1975) 420, also see D.Watts, 'Small worlds', Ch 9 (1999)) shows phase transition

### 3.4 CMLs equivalent to 2D Ising [IT]

Jiang&Pesin (ref) claim that they can construct a chain of horseshoe maps for which the potential $(\log Df)_{ss}$ looks like the 2D Ising potential. Fill in the details and simulate it numerically to check. cf W.Just

### 3.5 Measures of space-time chaos [T]

Reconstruct Gibbs potentials from observations.

### 3.6 Continuation of equilibria of Euclidean invariant systems with respect to defects [UMIT]

The problem here is that the spectrum for frequencies of small vibrations accumulates at zero. But one can choose a different norm on the space of configurations modulo isometries, with respect to which the potential energy for realistic systems is $C^2$ and has non-degenerate minima. Check this out for some examples, e.g. 2D triangular lattice. Requires solving a minimax optimisation problem. Study which distributions of defects permit uniform continuability. cf. critical thickness for epitaxy. Find pruning rules for distributions of defects, application to Si-Ge.

### 3.7 Screening for algebraically decaying coupling [TF]

Understand how screening effects modify the Baesens&MacKay decay results (Helv Phys Acta 72 (1999) 23).

## 3.8 Persistence of normally hyperbolic invariant bundles for networks [TF]

Prove persistence of normally hyperbolic invariant submanifolds uniformly in system size, and the same for more general bundles, e.g. Cantor over a circle. Check Hale (old books), Kevin Campbell (paper) and Rick Moeckel (Equadiff talk). Derive Kuramoto phase equations.

## 3.9 Formulation of Hamiltonian dynamics in $\ell_\infty$ [TF]

Problems of conservation of energy when infinite! cf. Slijepcevic, Mielke, and Eyink (in Fourier space)

## 3.10 Tilted Frenkel-Kontorova chains

Several directions:

### 3.10.1 Hull function [TF]

Prove existence of hull function for sliding incommensurate states of overdamped inertial F-K models, study its limit to rationals.

### 3.10.2 Continuity [MI]

Prove continuity of sliding speed with respect to rotation symbol.

### 3.10.3 AC forcing [TF]

Investigate effects of AC forcing and transition from over- to under- damped.

### 3.10.4 Saddle-node on cycle [MI]

Prove the saddle-node on cycle picture for the depinning transition.

### 3.10.5 Sliding discommensurations [TF*]

Floria&Mazo (Adv Phys 45 (1996) 505) reckon that if you tilt a Frenkel-Kontorova chain with a discommensuration in then once it starts sliding there is a unique time-periodic sliding discommensuration solution. Prove this using Mallet-Paret's theory (two preprints).

#### 3.10.6 Nucleation of defects [TF]

In the regime where periodic state is pinned but discommensurations are not, compute the rate of nucleation of kink-antikink pairs by noise, using Wentzell-Freidlin theory, and hence the conductivity in this regime. Try to extend it either side of the regime.

## 3.11 Infinite Newton's cradle [MI]

cf. Hinch numerics and heuristics and my paper Phys Lett A 251 (1999) 191.

## 3.12 Multiprocessor networks [TF]

Predict functionality of multiprocessor networks, e.g. British Telecom, internet (Phil Trans RS, Aug 2000), Alzheimer's disease, Parkinson's disease (e.g. Roffler&Tarlov, Nature 307 (1984) 62). Try some combination of my 'Dynamics of networks' paper and Kelly's stochastic papers. Extend my paper to dynamical coupling, cf. Stark on membrane dynamics. Think about the role of small worlds, cf. D.Watts book

## 3.13 Network of saddle-node on cycle neurons [MI]

Simulate and see whether it exhibits "intelligence"! (Dynamics of networks, in Stochastic and spatial structures in dynamics, eds van Strien and Verduyn Lunel). See Izhikevich (I J Bif Chaos 10 (2000)), or Keener & Sneyd (Math Physiology book), for bursting neurons.

## 3.14 Josephson junction arrays [MI]

Review literature on Josephson junction supercomputing technology, e.g. Nature 398 (29 April 1999), and assess the role of discrete breathers.

## 3.15 Relaxation rate for coupled Markov processes [TF*]

Prove persistence uniformly in network size.

## 3.16 Multi-dimensional F-K models [MITF]

Understand minimum energy states (cf. Frank&van der Merwe, PRSA 198 (1949) 216; de la Llave et al, DCDS 3 (1997) 135; Blank, Nonlinearity 2 (1989) 1; Vallet's thesis) and/or pinning of dislocations (cf. Jon Chapman talk, and Sinclair et al, Nature 298 (1982) 127). Investigate applications to heteroepitaxy (e.g. Heteroepitaxy on silicon, eds HK Choi et al, Materials Res Soc symp 116 (1988) [esp papers by Showalter and Bean]; Heterostructures on silicon, eds

Nissin YI&Rosencher E, Kluwer (1989); SiGe, D Paul, Phys World 13 no 2 (Feb 2000) p.27).

### 3.17 Conformational disease [MI]

e.g. Davis et al, Nature 401 (1999) 376 and refs; Cohen, J Mol Bio 293 (1999) 313; Carrell&Lomas, Lancet 350 (1997) 134. Also Schutte, Berlin? Roy Soc meeting Mar 00 (proceedings should appear in Phil Trans B)

### 3.18 Conductivity of CDW materials [T]

CDW = Charge density wave. Assess standard theory, e.g. Gruner, RMP, and Phelps thesis. Consider role of Holstein models, cf. Aubry. Nice 2D CDW experiment in Han et al, PRB 50 (1994) 14746.

### 3.19 Ferroelectric memory [MI]

e.g. Park et al, Nature 401 (1999) 682. Apply Sepulchre&MacKay networks of bistable units result?

### 3.20 Self-organised criticality [UM]

Understand Bak's book (1997) and application to magnetosphere (Chapman).

## 4 Aperiodically forced systems

### 4.1 Safety criteria [TF*]

Extend Bishnani-MacKay to effects of forcing on non-equilibrium attractors of autonomous systems, cf. snapshot attractors e.g. Romeiras et al Phys Rev A 41 (1990) 784, and Schmalfuss, L Arnold, Colonius...

### 4.2 Flux and barriers [MI]

cf. my two lectures at the Newton Institute in Autumn 96, but Wiggins already wrote something about this. Application to magnetosphere, Chapman, J Geophys Res 99 (1994) 5977.

### 4.3 Approximate invariant circles [TF]

Understand effect of aperiodic forcing on invariant circles of Hamiltonian systems, cf. Shuckburgh.

# 5 Continuous-time anti-integrable problems

## 5.1 Periodically forced light systems [T*]

Extend Bolotin&MacKay, Nonlin 10 (1997) 1015 by using implicit function theorem instead of minimisation.

## 5.2 Poincaré's "orbites à chocs" [TF*]

H Poincare, Les methodes nouvelles de la mecanique celeste, vol III, ch XXXII. I don't think he has a proof. We made one in the planar circular restricted case (submitted to Cel Mech). Extend to the 3D case, elliptic case and unrestricted problem, maybe using Alexeyev.

## 5.3 $\delta$-billiards [MI]

Continue orbits of delta-billiards to hard convex scatterers.

## 5.4 Continuation of hard collisions to smooth potentials [IT]

cf. Tel, Ott scattering from potential hills (use Silnikov&Turaev), and Rom-Kedar&Turaev, and pairs of conjugate points as criterion for a bounce? cf. Wintgen

# 6 Low-dimensional Hamiltonian dynamics

## 6.1 Breakup of tori in 3D volume-preserving maps and 4D symplectic maps [MITR]

cf. MacKay,Meiss&Stark idea (Nonlin 2 (1989) 555) and Chandre&Jauslin, PRL 81 (1998) 5125 and Chandre&MacKay (2000).

## 6.2 Visualisation of objects in 4D symplectic space [UMI]

Code the mouse to perform an SU(2) action.

## 6.3 Positive metric entropy for typical smooth area-preserving maps [X]

cf. Kosygin&Sinai, my renormalisation idea and Knill's failed attempt.

### 6.4 Renormalisation of vector fields [TF*]

Make my procedure (in 'Three topics in Hamiltonian dynamics') into a proof of existence of critical fixed point, cf. Koch (ETDS 1999), Govin,Chandre&Jauslin

### 6.5 Numerical test for an orbit closure to have positive measure [MI]

Formulate and test a method, cf. Rua Murray? and apply to area-preserving maps.

### 6.6 Bifurcations of cantori (including bimodal circle maps) [F*]

Develop a theory of annihilation of a cantorus by a homoclinic.

### 6.7 Energy-period diagrams [TF*]

Flesh out my Tokyo proceedings on this.

### 6.8 Relate Maslov index to band diagram [MI]

Follow the theme of Aubry, Phys D 103 (1997) 201 and Salamon&Robbin.

### 6.9 Adiabatically pulled Frenkel-Kontorova chains [TF]

Boundary circles and accelerating solutions (cf. Doeff&Misiurewicz Nonlin 10 (1997) 1755 for latter). Build on Slijepcevic (Nonlin) to derive scaling laws for avalanches.

### 6.10 Saddle-centre periodic orbits on approach to a saddle [MI]

Understand the sequences of saddle-centre bifurations observed in many classical models of vibration of molecules cf. Farantos (Int Rev Phys Chem 15 (1996) 345, J Chem Phys 104 (1996) 2921), Child (private comm) and my explanation to Child

### 6.11 Rapid symmetry breaking branches [MI]

Explain rapidly moving branches of asymmetric solutions observed for periodic orbits of multiharmonic FK models (Ketoja&MacKay) and breathers (Cretegny)

## 6.12 Numerical method to compute spectrum of Hamiltonian and symplectic matrices [IF]

The spectrum is symmetric (under $\lambda \mapsto -\lambda$ or $1/\lambda$, respectively). Devise numerical methods to exploit this. One starting point is FoCM99 abstract of Heike Fassbender. Another is Leimkuhler and van Vleck, Num Math (1997).

## 6.13 Toroidal chaos for potentials on a torus [TF]

Find a condition which ensures toroidal chaos for energy above the second saddle, cf. Geisel et al, Z Phys B 71 (1988) 117, and methods of Bolotin.

## 6.14 Sierpinski gaskets for symplectic maps [TF]

Try to embed Norton's example?

## 6.15 Geodesics in Minkowski metrics [TF]

Start from ideas in Denvir&MacKay (Trans AMS 350 (1998) 4553) and by some other researchers (Itturiaga, Contreras?)

## 6.16 Anti-integrable fixed points of renormalisation [TF]

There appear to be anti-integrable fixed points of renormalisation for area-preserving maps, but make a proper mathematical formulation.

## 6.17 Compare diffusion rates in Nekhoroshev's models [TF]

Look at Nekhoroshev's paper

## 6.18 Prove Riemann's hypothesis [X]

Relate to some Hamiltonian system, cf. Berry, Keating

## 6.19 Saddle-centre bifurcation [MI]

Calculate a concrete example of a 2 degree of freedom system with a saddle-centre periodic orbit

## 6.20 Understand confinement time in plasmas [TF]

Huge number of factors, but assess their relative importance. Start from Hazeltine&Meiss book or Sheffield, RMP 66 (1994) 1015, and Richard Dendy. Develop diffusion model from fluxes (could be a Gibbsian one rather than Markov); could try to solve the steady state problem with sources and sinks to avoid problem of side-branching into islands.

## 6.21 Survey Hamilton's papers [M]

Collected papers, vol 2 is on mechanics

## 6.22 Prove Melnikov asymptotics for rapidly forced pendulum [MI]

using Angenent method (in Symplectic Geometry, D Salamon ed), as opposed to Delshams etc (Nonlin).

## 6.23 Prove Mather's conditions for double pendulum [MI]

in some field with some time-periodic forcing

## 6.24 No-slip billiards [UM]

Read Broomhead and Gutkin, Phys D 67 (1993) 188, and study some other examples.

## 6.25 Magnetic geodesics [T]

Play around with them

## 6.26 Jacobian conjecture [X]

Every area-preserving polynomial map of $R^2$ has polynomial inverse. See survey in Dullin&Meiss preprint

## 6.27 Spherical magnetic pendulum [I]

Find 3 periodic orbits to apply Denvir-MacKay result (TAMS 350 (1998) 4553), or choose some other systems

## 6.28 Find a universal Turing machine in the 3-body problem [X]

C.Moore, Nonlinearity 4 (1991) 199, constructs a 2D diffeomorphism with a universal computer. Try to embed it in a typical nonintegrable dynamical systems like the circular restricted 3-body problem.

### 6.29 Converse KAM for non-standard foliations [T*]

Extend idea of MacKay-Percival (CMP 98 (1985) 469) and MacKay (Phys D 36 (1989) 64) to other foliations than vertical, but without changing coordinates. Apply to 3-body problem.

### 6.30 Physical Anosov systems [T]

Find physical Anosov systems, like Hunt and Mackay's proof for Thurston 3-cam link (Sci Am, 1984?).

### 6.31 Non-twist maps [T]

Prove a weak twist KAM theorem to explain why invariant circles seem more robust near zero shear. Look for non-trivial examples of Le Calvez boundary continua.

### 6.32 Gromov's camel [MI]

Look for applications of Gromov's camel (e.g. McDuff&Salamon book, or Hofer&Zehnder book), e.g. space debris, tokamaks.

### 6.33 Slow manifolds [TF*]

Develop general theory of slow manifolds in Hamiltonian systems.

### 6.34 Mather's $\Delta W$ [TF]

Redo Mather's $\Delta W$ paper (Publ Math IHES (1986?)) taking the sum over orbits of gaps rather than the maximum, to link to flux, e.g. prove continuity with respect to rotation symbol. Prove that our constructions have locally minimal flux (absolute minimum subject to including the cantorus).

### 6.35 Nonlinear normal modes of $C_{60}$[UMI]

Compute the symmetry types of linear normal modes of a classical model of $C_{60}$ by representation theory and then apply Montaldi, Roberts & Stewart ideas to deduce how many of them have nonlinear continuations, and of which stabilities. Think about the possibility of localised vibrations.

### 6.36 Guiding centre motion [UM]

Complete MacKay, On the motion of guiding centres, in: S Benkadda et al (eds), Transport, chaos and plasma physics (World Sci, 1994). Compare with Misguich et al, EUR-CEA-FC report 1438 (1992).

### 6.37 Temperature [M]

Understand temperature for non-high dimensional Hamiltonian systems. cf. Rugh, J Phys A 31 (1998) 7761 and Berdichevsky, Thermodynamics of chaos and order.

## 7 Low-dimensional non-Hamiltonian dynamics

### 7.1 Boundary of Siegel balls [TF*]

see Fornaess book? Use Bochner tube theorem. Visualise by SU(2) rotations in $C^2$.

### 7.2 Loss of smooth conjugacy for families of torus diffeomorphisms [TF]

Try to locate where it happens and find scaling laws.

### 7.3 Realise a Plykin attractor in electronics [M]

Tim Hunt worked out some ODEs for it. Make a real physical demo.

### 7.4 Unfolding of time-1 map of saddle-node of periodic orbits [TF*]

Chenciner's bubbles in a non-local context. Develop theory and numerics started by Baesens-Kim-MacKay a long time ago.

### 7.5 Compute Ruelle's derivative of SRB measure [TF]

Ruelle preprint. e.g. for dissipative perturbation of toral automorphisms, heat conductivity of a 1D chain

### 7.6 Falkner-Skan, bifurcations from infinity and third order systems generally [TF*]

Complete work of Llibre&MacKay on Poincare compactification of Falkner-Skan and hence maybe cleaner version of Sparrow&Swinnerton-Dyer (JDE 119 (1995) 336).

### 7.7 Bifurcations of flows on a 2-torus [MI]

cf. BGKM (Phys D 49 (1991) 387) and Glendinning paper

## 7.8 Make horseshoes visible [UMI]

Use OGY method (Ott et al, Phys Rev Lett 64 (1990) 1196) to stabilise any orbit on a horseshoe, cf. their paper on stabilising random orbits in an attractor

## 7.9 Stabilise or optimise an invariant measure [IT*]

Extend work of Bonanno, Yuan&B.Hunt (Nonlin 12 (1999) 1207).

## 7.10 Understand dynamics of $\ddot{y} = y^2 - a^2t^2$ [M]

This is equivalent to Airy equation and differential Galois theory says there is no formula for the solutions in standard terms (Morales Ruiz book)

## 7.11 Investigate bicritical circle map renormalisation [MI]

Follow up on my paper Phys Lett A 187 (1994) 391

## 7.12 Think about saddle-node on cycle in vibration problems [MI]

Often a vibrating motor, e.g. ship or aeroplane has a vibration whose amplitude and frequency modulate periodically. Why? cf avergaed equations for van der Pol?

## 7.13 Multi-dimensional Zeeman map [MI]

cf. his 2D map and analysis by Cushman

## 7.14 Relative depth of attractors [MT]

Develop idea of relative depth of attractors in a weak noise field, coming from Wentzell-Freidlin quasipotential.

## 7.15 Design pricing system to stabilise supply chains [MI]

Sainsbury's Innovation centre says their store managers all order lots of the same commodity at once or none at all for a long period. Correct this situation by an internal pricing system depending on supply.

## 7.16 Analyse Ellis's 5D cosmological model [MI]

See Wainwright-Ellis book 'Dynamical systems in cosmology'

### 7.17 Giudici's bifurcation [MI]

Giudici et al, Phys Rev E 55 (1997) 6414. I don't think he makes the right interpretation of the experimental results. Work out a more appropriate bifurcation scenario.

### 7.18 Hilbert's 16th problem [X]

Bound the number of limit cycles for a polynomial vector field in the plane. cf Ilyashenko & Yakovenko (eds), AMS Transl (2) 165 (1995)

### 7.19 Suspension of noninvertible maps [TF]

Try to use Perez-Marco&Yoccoz, Asterisque 222 (1994) idea with non time-like foliations

### 7.20 Generalised LaSalle [MI]

Deduce the consequences of a distance function which is non-increasing and decreases off some submanifold. Apply to Fokker-Planck and chemical reactions.

### 7.21 Millenium Bridge [UM]

Interpret the problems of the Millenium Bridge in terms of an instability involving synchronisation of walkers.

## 8 PDEs

### 8.1 Derive incompressible Euler equations from compressible Euler [TF]

Read P-L Lions vol. 2 first. Use section 6.33.

### 8.2 Hamiltonian variational PDEs [TF]

fit Bridges multisymplectic ideas into Cartan framework and deduce some general properties, cf. Marsden-Shkoller

### 8.3 Slow manifolds for PDEs [TF*]

Apply to atmospheric dynamics to deduce equations eliminating inertia-gravity waves

## 8.4 Instability of a vortex core [UMI]

Apply theory of MacKay, Dyn Stab Sys 2 (1987) 55 on interaction between positive and negative energy waves to Tsai&Widnall problem, J Fluid Mech 73 (1976) 721 But first read Guckenheimer&Mahalov, PRL 68 (1992) 2257 and Mahalov, Phys Fluids A 5 (1993) 891

## 8.5 Amplitude equations [F]

Show invalidity of Newell's amplitude equations for roll patterns because of existence of preferred phase at defects

## 8.6 Q balls [TF*]

Understand interaction of Q balls. Use section 6.33. Extend to localised objects in other field theories.

## 8.7 Nonlinear optics [MI]

Understand the problems of Turitsyn and Littlewood.

## 8.8 Cochlear mechanics [T*]

Investigate explanation in terms of mode conversion, cf. A Huxley, Lloyd Watts. Look at recent refs in Duke preprint.

## 8.9 Nodal lines [MT]

Investigate the generic shapes and movement of nodal lines in microwave ovens and devise a strategy to heat food more uniformly by modulating the frequency. Start from J Nye.

## 8.10 Coherent structures [MI]

e.g. hurricanes, Willoughby, Nature 401 (1999) 649; Emanuel

## 8.11 Pipe flow [X]

Derive relation between flux and pressure gradient for pipes, cf. Kempe's year book, log law of the wall, Nazarenko, Barenblatt, Srinivassan

## 8.12 Giraffe pattern [TF]

Model as cracking of mud? Contrast Koch&Meinhardt, RMP 66 (1994) 1481

### 8.13 Honeycomb weathering [MI]

Smith, Nature 298 (1982) 121, cf water over ice?

### 8.14 Finite-time blowup for Navier-Stokes [X]

Prove existence of a smooth initial condition for a bounded domain which develops non-differentiability in finite time. For some starting points, see Kitchenassamy, JDE 125 (1996) 299.

### 8.15 Reconnection [T]

Understand reconnection of magnetic field/vortex lines in weakly resistive/viscous plasmas/fluids. One starting point: JG Lyon, Science 288 (16 June 2000) p.1987

## 9 Quantum mechanics

In addition to problems under section 2:

### 9.1 Nonlinearity in quantum mechanics [TF]

e.g. tunnelling as analogue of separatrix splitting (e.g. Creagh), non-quadratic terms in creation-annihilation operators, Birkhoff normal forms?

### 9.2 Derive the fine structure constant [X]

$\alpha \approx 1/137.0360$

### 9.3 Suppression of transport [MF]

Study Koch's critique of MacKay-Meiss and try to justify it by comparison with opacity of a grid to EM radiation of wavelength less than spacing. cf Geisel

### 9.4 Resonances [MI]

Compute statistics of resonances between two quantum systems $E_n + E'_m \sim E_{n'} + E'_{m'}$. cf. Krikorian, Marklof

## 10 Statistics

### 10.1 Bayesian time series analysis [MT]

modelling, forecasting, e.g. number of frequencies

## 10.2 Function fitting

e.g. with asymmetric cost for errors: Pesaran

## 10.3 Neutron scattering [TF]

Infer spatial structure of correlations in vibrations from neutron scattering dynamic structure function. Start from Lovesey's book and then Wang et al (PRL 76 (1996) 3598) and Taraskin&Elliott (PR B 55 (1997) 117). cf X-ray diffraction (eg Bricogne, in Buck &Macaulay, Maximum Entropy in action)

# 11 Chemical Reactions

## 11.1 Prediction of rates [TF]

e.g. from classical mechanics of molecules, stochastic ratchet ideas etc., cf. Hanggi et al, RMP 62 (1990) 251

## 11.2 Prove closed reactions go to unique equilibrium [MI]

cf. Gatermann&Huber and earlier work of Feinberg. Use subsection 7.20. Extend to non-mass action and temperature dependence.

## 11.3 Gene expression [MTF]

Investigate the dynamics of gene expression (e.g. refs in Gatermann&Huber preprint) and possible relation to cancer. See JD Watson et al, Molecular biology of the gene (4th ed, Benjamin-Cummings, 1987). Baverstock.

## 11.4 Enzyme kinetics [MI]

e.g. Segel IH (Wiley, 1975). Also, MIMI day 1 Aug 2000

# 12 Other

## 12.1 Analytic prediction [UM]

Simplify the work of J Schmidt along lines I suggested to him. cf. Fritz John, CPAM 13 (1960) 551

## 12.2 Optimal scheduling [MIT]

Apply our paper Nonlin 13 (2000) 257 to some economics examples and compare with data.