

$N + 1$ topics in Mechanics

R.S.MacKay

1 A mechanical Anosov system: the triple linkage

We prove that for suitable parameters the free motion of Thurston and Weeks' triple linkage is mixing Anosov on every positive energy level, thus bringing Anosov systems into mechanical relevance.

- T.J.Hunt, R.S.MacKay, Anosov parameter values for the triple linkage and a physical system with a uniformly chaotic attractor, *Nonlinearity* 16 (2003) 1499-1510

2 A mechanism to drive biomolecular conformation changes

We propose that many biomolecular conformation changes, such as that of the power stroke of myosin, are driven by a single molecule (e.g. phosphate ion) expanding a trap.

- R.S.MacKay, D.J.C.MacKay, Ergodic pumping: a mechanism to drive biomolecular conformation changes, submitted to *Physica D* for Aubry's 60th birthday issue

3 Poincaré's second species orbits

For the circular restricted 3-body problem with small secondary, we prove existence of large uniformly hyperbolic subshifts of orbits which repeatedly approach near collision with the secondary.

- SV Bolotin, RS MacKay, Periodic and chaotic trajectories of the second species for the n -centre problem, *Cel Mech Dyn Astron* 77 (2000) 49-75
- SV Bolotin, RS MacKay, Non-planar second species periodic and chaotic orbits for the circular restricted three-body problem, in preparation

4 Some robustly mixing fluid flows

In the class of C^3 divergence-free vector fields on a domain of R^3 with smooth boundary, vanishing on the boundary, I make examples which are mixing and remain so for all small perturbations.

5 Safety limits for a walking robot

M.Ghaffari and I constructed a method to determine amounts of inhomogeneity in surface or external forces for which the stable walking cycle of a bipedal robot model has a locally unique continuation. It is an extension of the method of:

- Z Bishnani, RS MacKay, Safety criteria for aperiodically forced dynamical systems, *Dyn Sys* 18 (2003) 107-129

6 An observation selection bias on redshift

C.P.Rourke and I propose that there is an observation selection bias effect on cosmological redshift: if two world-lines have an average recession velocity of z_0 then the average redshift observed is $z_0 + \langle (z - z_0)^2 \rangle / (1 + z_0)$. Thus the expansion of the universe may be over-estimated.

7 Interaction of two charges in a magnetic field

D.Pinheiro and I prove that the motion of two charges in a plane with uniform magnetic field is bounded unless the charges sum to zero, the case of equal charge:mass ratio is integrable, and that all cases with opposite signs of charge contain suspended chaotic subshifts.

8 Mode conversion in the cochlea

To explain Kemp's echo, a pulse that comes out of the ear several milliseconds after an incident pulse, I propose the cochlear wave must undergo "mode conversion" (a phenomenon in which waves in an inhomogeneous medium reach a place where they turn into another type of wave in the reverse direction, without the wavelength going to 0 or infinity).

9 Unzipping DNA

For a class of models of DNA, we show that there are order L equilibria if the strands at one end are separated by a distance L , and examine their effect on the statistical mechanics.

- N Theodorakopoulos, M Peyrard, RS MacKay, Nonlinear structures and thermodynamic instabilities in a one-dimensional lattice system, Phys Rev Lett 93 (2004) 258101

10 A vortex discrete breather

We prove existence and ℓ_2 linear stability of some vortex breathers in a triangular lattice.

- V Koukouloyannis, RS MacKay, Existence and stability of 3-site breathers in a triangular lattice, J Phys A 38 (2005) 1021–30

11 Instabilities of a strained vortex

I explain the results of Tsai and Widnall (examined partially also by Guckenheimer and Mahalov) on instabilities generated by interactions of Kelvin waves on a vortex in a weak straining field.

12 Arrays of discommensurations

C.Baesens and I prove that if the set of minimum energy advancing discommensurations of mean spacing p/q for a Frenkel-Kontorova chain has phonon gap then all minimum energy states with mean spacing ω just above p/q are approximated exponentially well in $q\omega - p$ by concatenations of advancing p/q discommensuration.

13 Overdamped dynamics of tilted Frenkel-Kontorova chains

For overdamped 2nd order dynamics of a tilted Frenkel-Kontorova chain and each spatial periodicity class with no equilibrium, we prove existence of a globally attracting periodically sliding solution.

- C Baesens, RS MacKay, A novel preserved partial order for cooperative networks of units with overdamped second order dynamics, and application to tilted Frenkel-Kontorova chains, *Nonlinearity* 17 (2004) 567–580

14 Quantum statistical mechanics of Frenkel-Kontorova chains

We extend the fixed point of renormalisation for the depinning transition of incommensurate minimum energy states of Frenkel-Kontorova models to one for the quantum statistical mechanics at temperature $T = 0$, Planck's constant $\hbar = 0$ but $\hbar/T = \infty$. In addition to the previously known unstable directions, there is a new one for non-zero Planck's constant, with eigenvalue $\kappa = J/\varepsilon \approx 2.630$, where J is the scaling of energy and ε is the scaling of the bottom of the phonon spectrum.

- NR Catarino, RS MacKay, Renormalization and quantum scaling of Frenkel-Kontorova models, *J Stat Phys*, to appear in Feigenbaum's 60th birthday issue

15 Hamiltonian slow manifolds

Given a C^∞ Hamiltonian system with an almost invariant symplectic submanifold containing slow motion compared to the linearised normal dynamics, or nearly periodic motion with period avoiding resonance with the linearised normal dynamics, I iteratively improve to reduce the deviation from invariance to arbitrary order in the ratio of timescales. This is relevant to explaining energy transfer between discrete breathers (including travelling discrete breathers) and construction of balanced models in atmospheric science.

- RS MacKay, Slow Manifolds, in “Energy Localisation and Transfer”, eds T Dauxois, A Litvak-Hinenzon, RS MacKay, A Spanoudaki (World Sci, 2004), 149–192

16 Cerbelli and Giona's map

I show that a piecewise affine area-preserving map proposed by Cerbelli and Giona is pseudo-Anosov. This enables me to compute many properties, in particular the dimension scaling function for the measure produced by a leaf of the contracting foliation.

17 Non-existence of an invariant Lagrangian section

For a 2dof Hamiltonian flow ϕ_t with a field of “vertical” Lagrange planes V that is tilted positively by the flow, I show that if there is an invariant Lagrangian submanifold through a given point x , transverse to the vertical planes, then $D\phi_t(V(\phi_{-t}(x))) > D\phi_{-s}(V(\phi_s(x)))$ for all $s, t > 0$ in the partial order on Lagrange planes at x induced by $V(x)$.