The sporadic finite subgroups of SL(3,C) and their McKay quivers

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1. Introduction

The 2D McKay Correspondence states that the finite subgroups G of SL(2,C) are in bijective correspondence with the Dynkin diagrams A_n, D_n, E_6, E_7, E_8 by noting that the resolution diagram of the Du Val singularity C^2/G (each of which is a Dynkin diagram) is the truncated McKay quiver of G.

More generally, the higher dimensional McKay correspondence relates the representation theory of a linear group G with the geometry of the resolution of singularities of the orbit space X = C^n/G.

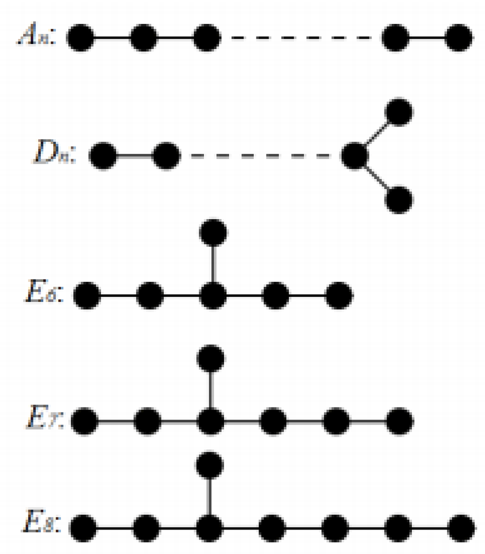


Fig. 1. Dynkin diagrams

The McKay quiver Q of a linear group G is a directed graph which is calculated by computing irreducible decompositions of tensor products of the inclusion representation with the irreducible representations. That is, if G is a linear group with a given representation θ, then the McKay quiver of G has a vertex for each irreducible representation ρ of G and the number of arrows between irreps ρ and ρ′ is dim $\mathrm{Hom}_G(\rho', \rho \otimes \theta)$.

In this project we determine the McKay quiver for each of the sporadic cases of finite subgroups of SL(3,C) as a first step into the further study of the higher dimensional McKay correspondence.

2. Examples

There are many interesting finite subgroups of SL(3,C), including the symmetry groups of the dodecahedron and the Klein quartic.

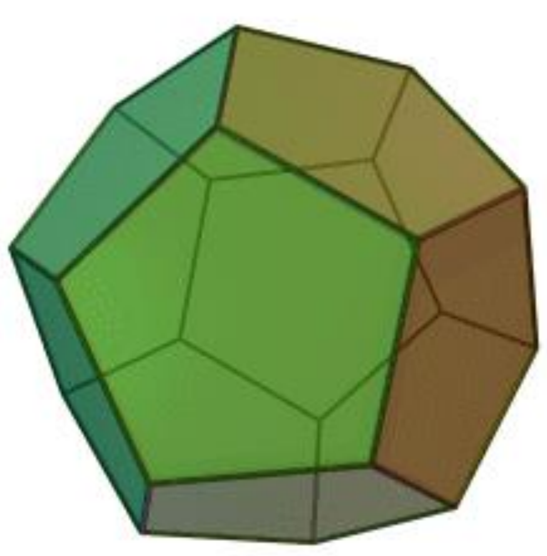


Fig. 2.a. Dodecahedron

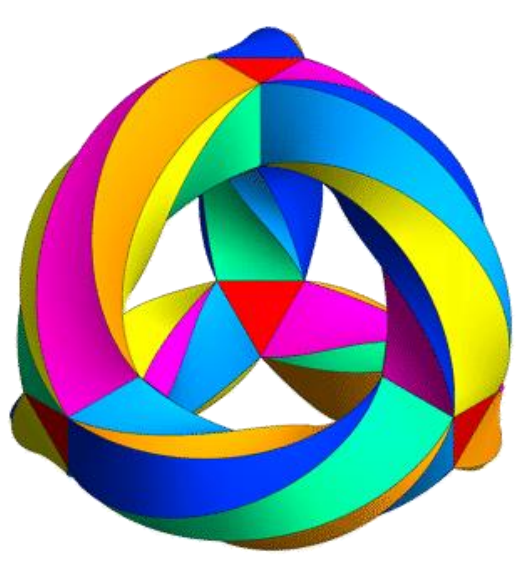


Fig. 2.b. Klein quartic

Images from http://math.ucr.edu/home/baez/klein.html

3. Method

Character theory provides a more manageable framework in which to make computations since the tensor product of representations translates to a more simple pointwise product for characters.

The irreducible characters form an orthonormal basis for the space of class functions CF(G) with the following sesquilinear form on C[G]:

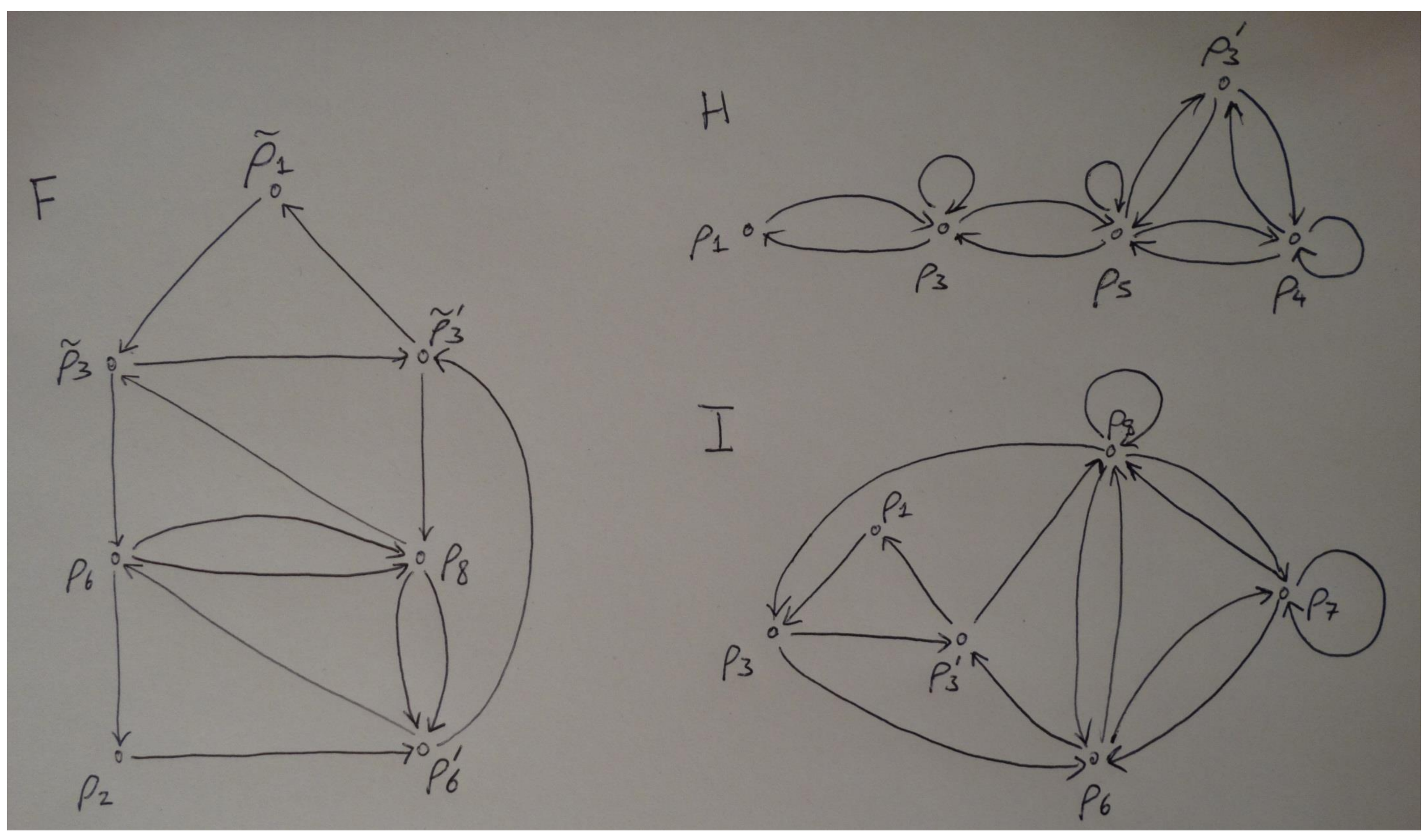
$$(f_1, f_2)_G = \frac{1}{|G|} \sum_{x \in G} f_1(x) \overline{f_2(x)}$$

The above formula allows us to compute the irreducible decomposition of any representation, and hence the McKay quiver, provided that the character table for G is known.

We implemented the groups of interest using the computer algebra system Magma based on the classification of finite subgroups of SL(3,C) given in [YY], and then proceeded to extract their character tables which we used to subsequently determine their McKay quivers.

4. Results and conclusion

The McKay quivers were computed for the eight sporadic cases of finite subgroups of SL(3,C). Invariance under dual operation and action by degree 1 representations was noted.
Below is a sample of the quivers determined.
In particular, F is the sporadic subgroup of SL(3,C) of order 216, and H and I are the simple groups of order 60 (dodecahedron) and 168 (Klein quartic) respectively.



References: [YY] S. Yau and Y. Yu, Gorenstein Quotient Singularities in Dimension Three, 1993

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