Commutative Algebra II Assignment 2

Due: midday Friday 5th November

Question 1

Show that a p-adic integer is rational if and only if its expansion (as a power series in p) is eventually recurrent.

Sidenote: for lead-up questions on p-adic integers, see the full example sheet.

Question 2

Let A be a Noetherian ring. Prove that $A[\![x]\!]$ is Noetherian.

Question 3

Let A be a Noetherian ring. Prove that Krull dim $A = 0$ implies A is Artinian, using the following steps:

1. Show that Krull dim $A = 0$ implies the intersection J of all maximal ideals equals the nilradical.
2. Use the Noetherian property of A to prove $J^N = 0$ for some N.
3. Construct a filtration of A by ideals so that each I_n/I_{n-1} is isomorphic to the residue class field $k(m_i) = A/m_i$, where the m_i are the maximal ideals of A.