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ANISOTROPIC TURBULENCE OF SHEAR-ALFVÉN WAVES

S. GALTIER
Institut d'Astrophysique Spatiale, Université Paris-Sud–CNRS, Bâtiment 121, F-91405 Orsay Cedex, France; sebastien.galtier@ias.fr
S. V. NAZARENKO
Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK; snazar@maths.warwick.ac.uk
A. C. NEWELL
Department of Mathematics, University of Arizona, P.O. Box 210089, 617 North Santa Rita, Tucson, AZ 85721; anewell@math.arizona.edu
AND
A. POUQUET
ASP/NCAR, P.O. Box 3000, Boulder, CO 80307-3000; pouquet@ucar.edu
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ABSTRACT

Weak turbulence of shear-Alfvén waves is considered in the limit of strongly anisotropic pulsations that are elongated along the external magnetic field. The kinetic equation thus derived agrees with the Galtier et al. formulation of the full three-dimensional helical case when taking the proper limit. This new approach allows for significant simplification, and, as a result, the applicability conditions for the weak turbulence theory are now more transparent. It thus provides an attractive theoretical framework for describing anisotropic MHD turbulence in astrophysical contexts where a strong magnetic field is present and for which shear-Alfvén waves are important.

Subject headings: ISM: general — MHD — turbulence — waves

1. INTRODUCTION

Iroshnikov (1963) and Kraichnan (1965) (hereafter IK) proposed independently a phenomenology for incompressible, homogeneous, isotropic MHD turbulence based on three-wave processes of interacting and counterpropagating Alfvén waves embedded in a local mean magnetic field. The main differences with hydrodynamic turbulence were a slowing down of the energy transfer to small scales and a $k^{-3/2}$ scaling for the energy spectrum. While in hydrodynamic turbulence, the Kolmogorov $k^{-5/3}$ energy spectrum prediction is well supported by many experimental and numerical observations (Frisch 1995), in magnetized flows there is still a debate about the predicted scaling either in the absence of a uniform magnetic field $\boldsymbol{b}_0$ (Politano, Pouquet, & Sulem 1995; Biskamp & Müller 2000) or in the anisotropic case (Cho & Vishniac 2000; Maron & Goldreich 2001). Indeed, the effect of Alfvén waves on the dynamics is still a subject of discussion (Biskamp 2000), e.g., for the solar wind or the interstellar medium (ISM). Interstellar scintillation is interpreted as the scattering of radio waves by electron density fluctuations in the ionized ISM whose turbulent power-law spectrum extends over many decades (Amstrong, Rickett, & Spangler 1995). The anisotropic scattering of radio waves (Frail et al. 1994) suggests that such fluctuations are preferentially perpendicular to $\boldsymbol{b}_0$, which is often much greater than the fluctuating part (Spangler 1999), and that therefore a theory of interstellar turbulence should be built on anisotropic MHD. Moreover, observational data analyses of the Jovian magnetosphere (Saur et al. 2001) show that the Alfvén time is often (much) smaller than the nonlinear transfer time, which strongly suggests that weak Alfvénic turbulence is a plausible theory for astrophysical flows in an appropriate range of scales.

As is well known, the presence of a large-scale magnetic field leads to bidimensionalization of the turbulence, leading to slower transfer along $\boldsymbol{b}_0$. This can be seen from numerical simulations of incompressible (Montgomery & Turner 1981; Shebalin, Matthaeus, & Montgomery 1983; Oughton, Priest, & Matthaeus 1994; Ng & Bhattacharjee 1996; Milano et al. 2001), reduced (Kinney & McWilliams 1998), and compressible MHD (Oughton, Matthaeus, & Ghosh 1998; Mac Low 1999). If this anisotropy is taken into account in the IK dimensional analysis, a $k_\perp^{-2}$ energy spectrum is obtained, where "⊥" denotes the direction perpendicular to $\boldsymbol{b}_0$ (Ng & Bhattacharjee 1997). Goldreich & Sridhar (1997) predicted this result as well, but they call this regime intermediate turbulence. Recently, it was shown (Galtier et al. 2000) that the $k_\perp^{-2}$ energy spectrum is actually an exact finite flux Kolmogorov solution of the weak wave turbulence kinetic equations at the level of three-wave interactions. When the four-wave interactions are dominant, the predicted energy spectrum is $k_\perp^{-7/3}$ (Goldreich & Sridhar 1995). For strong anisotropic MHD turbulence, there is still a debate in the absence of a rigorous theory; either $k_\perp^{-5/3}$ (Sridhar & Goldreich 1994) or $k_\perp^{-3/2}$ (Nakayama 2001) is predicted, using the ad hoc EDQNM closure or the Lagrangian DIA approximation.

In this Letter, we address a different but related question: we wish to consider *first* the physical limit corresponding to having only shear-Alfvén waves in the fluid, and *then* take the limiting case of weak turbulence of that ensemble of interacting waves. Our motivation stems from the fact that the complexity in the algebra of deriving the kinetic equations for the eight correlators involved in the full case of weak MHD turbulence may hinder one's understanding of the underlying physics. Such a complexity makes the general case harder to use and analyze, and to establish conditions for its applicability. On the other hand, the kinetic equations are easier to analyze in the limit of nearly bidimensional turbulence, $k_\parallel \ll k_\perp$. We are able to show here that making an assumption about strong anisotropy ($k_\parallel \ll k_\perp$) in the *original* MHD equations allows for a reliable control of the assumptions made during the derivation and results in transparency of the applicability conditions. The latter point is especially important because of the active debate around the role of three-wave versus four-wave processes (Goldreich & Sridhar 1995; Ng & Bhattacharjee

1996; Galtier et al. 2000; Nazarenko, Newell, & Galtier 2001). The resulting kinetic equation is simple, and it provides an attractive theoretical framework for applying anisotropic MHD turbulence to astrophysics.

2. DERIVATION OF THE KINETIC EQUATION

To describe Alfvén waves, let us introduce "perturbed" Elsässer variables $\epsilon \boldsymbol{z}^s = \boldsymbol{v} + s(\boldsymbol{b} - \boldsymbol{b}_0)$, where $\boldsymbol{b}_0 = b_0\hat{\boldsymbol{e}}_\parallel$ is a strong external magnetic field (with $|\hat{\boldsymbol{e}}_\parallel| = 1$), $s = \pm 1$ is a polarization factor indicating the wave propagation direction, and ϵ is a small parameter measuring the intensity of the wave turbulence. Units are such that the $\boldsymbol{v}$ and $\boldsymbol{b}$ are the fluid and Alfvén velocities. The inviscid three-dimensional incompressible MHD equations written in terms of the "perturbed" Elsässer variables are

$$(\partial_t - sb_0\partial_\parallel)z_j^s = -\epsilon\partial_{x_m} z_m^{-s} z_j^s - \partial_{x_j} P_*, \tag{1}$$

where P_* is the total pressure and $\partial_\parallel$ is the derivative along $\hat{\boldsymbol{e}}_\parallel$. Fourier transforming equation (1) and separating the fast and slow time dependencies, we have (Galtier et al. 2000)

$$\partial_t a_j^s(\boldsymbol{k}) = -i\epsilon k_m P_{jn} \int a_m^{-s}(\kappa) a_n^s(\boldsymbol{L}) e^{i(-s\omega_k - s\omega_\kappa + s\omega_L)t} \delta_{\boldsymbol{k},\kappa\boldsymbol{L}} d_{\kappa\boldsymbol{L}}, \tag{2}$$

where $d_{\kappa \boldsymbol{L}} = d\kappa d\boldsymbol{L}$, $\delta_{\boldsymbol{k},\kappa\boldsymbol{L}} = \delta(\boldsymbol{k} - \kappa - \boldsymbol{L})$, $z_j^s(\boldsymbol{x}, t) = \int a_j^s(\boldsymbol{k}, t) e^{i(\boldsymbol{k}\cdot\boldsymbol{x} + s\omega_k t)} d\boldsymbol{k}$, and $P_{jn}(k) = \delta_{jn} - k_j k_n/k^2$ is the projection operator which ensures incompressibility; the Alfvén waves frequency is $\omega(\boldsymbol{k}) = \boldsymbol{b}_0 \cdot \boldsymbol{k} = b_0 k_\parallel$. We now simplify equation (2) assuming the turbulence to be strongly anisotropic, i.e., with $k_\parallel \ll k_\perp$, as expected in the presence of a strong $\boldsymbol{b}_0$. In this Letter, we will only be concerned about the shear-Alfvén waves that, in this case, are described by the transverse fields $\boldsymbol{a}_\perp^s = (a_1^s, a_2^s)$. The divergence-free condition allows one to express a_2^s in terms of a_1^s as $a_2^s = -(k_1/k_2)a_1^s - (k_\parallel/k_2)a_\parallel^s \approx -(k_1/k_2)a_1^s$. Let $a^s = a_1^s$, and neglect terms with factors $k_\parallel$ on the right-hand side of equation (2) and obtain

$$\partial_t a^s(\boldsymbol{k}) = -i\epsilon \int \frac{k_2}{\kappa_2 L_2 k_\perp^2} (\boldsymbol{k}_\perp \cdot \boldsymbol{L}_\perp)(\boldsymbol{k} \times \kappa)_\parallel a^{-s}(\kappa) a^s(\boldsymbol{L}) e^{-2isb_0\kappa_\parallel t} \delta_{\boldsymbol{k},\kappa\boldsymbol{L}} d_{\kappa\boldsymbol{L}}, \tag{3}$$

using $k_\parallel = \kappa_\parallel + L_\parallel$ in the exponential term. Note that the integration in this equation is still over the three-dimensional vectors κ and $\boldsymbol{L}$ and that a^{-s} depends on all three of the wavenumber components. Let us introduce the wave turbulence spectra $q^s(\boldsymbol{k})$ as $\langle a^s(\boldsymbol{k}) a^{s'}(\boldsymbol{k}') \rangle = q^s(\boldsymbol{k})\delta(\boldsymbol{k} + \boldsymbol{k}')\delta(s - s')$, where the averaging is taken over an initial ensemble. Spatial homogeneity means space averaging is equivalent. Further, because of the linear dynamics, the initial ensemble evolves to a state for which the random phase approximation holds. Here, as usual, $\delta(\boldsymbol{k} + \boldsymbol{k}')$ appears due to the turbulence homogeneity and $\delta(s - s')$ is due to the fast decorrelation of the oppositely propagating waves. Following standard weak turbulence approach (Benney & Newell 1969), we write successively equations for the second- and third-order moments:

$$\begin{aligned}\partial_t \langle a^s(\boldsymbol{k}) a^s(\boldsymbol{k}') \rangle = \partial_t q^s(\boldsymbol{k})\delta(\boldsymbol{k} + \boldsymbol{k}') = -i\epsilon \int \frac{k_2}{\kappa_2 L_2 k_\perp^2} (\boldsymbol{k}_\perp \cdot \boldsymbol{L}_\perp)(\boldsymbol{k} \times \kappa)_\parallel \langle a^{-s}(\kappa) a^s(\boldsymbol{L}) a^s(\boldsymbol{k}') \rangle e^{-2isb_0\kappa_\parallel t} \delta_{\boldsymbol{k},\kappa\boldsymbol{L}} d_{\kappa\boldsymbol{L}} \\ -i\epsilon \int \frac{k_2'}{\kappa_2 L_2 k_\perp'^2} (\boldsymbol{k}_\perp' \cdot \boldsymbol{L}_\perp)(\boldsymbol{k}' \times \kappa)_\parallel \langle a^{-s}(\kappa) a^s(\boldsymbol{L}) a^s(\boldsymbol{k}) \rangle e^{-2isb_0\kappa_\parallel t} \delta_{\boldsymbol{k}',\kappa\boldsymbol{L}} d_{\kappa\boldsymbol{L}},\end{aligned} \tag{4}$$

$$\begin{aligned}\partial_t \langle a^{-s}(\boldsymbol{k}) a^s(\boldsymbol{k}') a^s(\boldsymbol{k}'') \rangle = -i\epsilon \int \frac{k_2}{\kappa_2 L_2 k_\perp^2} (\boldsymbol{k}_\perp \cdot \boldsymbol{L}_\perp)(\boldsymbol{k} \times \kappa)_\parallel \langle a^s(\kappa) a^{-s}(\boldsymbol{L}) a^s(\boldsymbol{k}') a^s(\boldsymbol{k}'') \rangle e^{2isb_0\kappa_\parallel t} \delta_{\boldsymbol{k},\kappa\boldsymbol{L}} d_{\kappa\boldsymbol{L}} \\ -i\epsilon \int \frac{k_2'}{\kappa_2 L_2 k_\perp'^2} (\boldsymbol{k}_\perp' \cdot \boldsymbol{L}_\perp)(\boldsymbol{k}' \times \kappa)_\parallel \langle a^{-s}(\kappa) a^s(\boldsymbol{L}) a^{-s}(\boldsymbol{k}) a^s(\boldsymbol{k}'') \rangle e^{-2isb_0\kappa_\parallel t} \delta_{\boldsymbol{k}',\kappa\boldsymbol{L}} d_{\kappa\boldsymbol{L}} \\ -i\epsilon \int \frac{k_2''}{\kappa_2 L_2 k_\perp''^2} (\boldsymbol{k}_\perp'' \cdot \boldsymbol{L}_\perp)(\boldsymbol{k}'' \times \kappa)_\parallel \langle a^{-s}(\kappa) a^s(\boldsymbol{L}) a^{-s}(\boldsymbol{k}) a^s(\boldsymbol{k}') \rangle e^{-2isb_0\kappa_\parallel t} \delta_{\boldsymbol{k}'',\kappa\boldsymbol{L}} d_{\kappa\boldsymbol{L}}.\end{aligned} \tag{5}$$

The fourth-order moment above decomposes into the sum of a triple product of second-order moments and a fourth-order cumulant. The latter does not contribute to long time behavior. Second-order moments that involve amplitudes with different values of s are zero because of the independence of phases of such amplitudes. Therefore, the first term in the above equation is zero, and both the second and the third terms consist of only one combination of the second-order moments each. Let us integrate the equation for the third-order moments over a time period T, which is much greater than the linear period $1/\omega = 1/(b_0 k_\parallel)$ but considerably smaller than $1/(\epsilon^2 b_0 k_\perp)$ (characteristic time of the three-wave processes), i.e., $1/(b_0 k_\parallel) \ll T \ll 1/(\epsilon^2 b_0 k_\perp)$. Then, one can neglect the slow time dependence of the amplitudes a^s during the integration, which gives

$$\langle a^{-s}(\boldsymbol{k})a^s(\boldsymbol{k}')a^s(\boldsymbol{k}'')\rangle = -i\epsilon\delta_{kk'k''}\left\{\frac{k'_2}{k_2k''_2k'^2_\perp}(\boldsymbol{k}'_\perp\cdot\boldsymbol{k}''_\perp)(\boldsymbol{k}'\times\boldsymbol{k})_\parallel q^{-s}(\boldsymbol{k})q^s(\boldsymbol{k}'') + \frac{k''_2}{k_2k'_2k''^2_\perp}(\boldsymbol{k}''_\perp\cdot\boldsymbol{k}'_\perp)(\boldsymbol{k}''\times\boldsymbol{k})_\parallel q^{-s}(\boldsymbol{k})q^s(\boldsymbol{k}')\right\}\Delta(2b_0sk_\parallel), \quad (6)$$

where $\Delta(x) = \int_t^{t+T}\exp(ix\tau)d\tau = \exp(ixt)[\exp(ixT)-1]/(ix)$. Substitution of equation (6) into equation (4) gives

$$\partial_t q^s(\boldsymbol{k}) = -\epsilon^2\int\frac{k_2}{k_\perp^2\kappa_2^2L_2}(\boldsymbol{k}_\perp\cdot\boldsymbol{L}_\perp)^2(\boldsymbol{k}_\perp\times\kappa_\perp)^2_\parallel\left[\frac{L_2}{k_2L_\perp^2}q^s(\boldsymbol{k}) - \frac{k_2}{L_2k_\perp^2}q^s(\boldsymbol{L})\right] \times q^{-s}(\kappa)\{\Delta(2b_0s\kappa_\parallel)\exp(-2ib_0s\kappa_\parallel t) + \Delta(-2b_0s\kappa_\parallel)\exp(2ib_0s\kappa_\parallel t)\}\delta_{\boldsymbol{k},\kappa\boldsymbol{L}}d_{\kappa\boldsymbol{L}}. \quad (7)$$

Here the first and second terms in the braces correspond to the first and second terms on the right-hand side of equation (4), respectively. The large time behavior of the kinetic equation is given by the Riemann-Lebesgue lemma: for $T\to+\infty$, we have $\exp(-ixt)\Delta(x)\to\pi\delta(x)+i\mathcal{P}(1/x)$, where $\mathcal{P}$ is the principal value of the integral. Because of the condition on T, one can only take the limit $T\to\infty$ if

$$k_\parallel/k_\perp \gg \epsilon^2. \quad (8)$$

This is the first condition of applicability of the weak turbulence description, a condition that was already discussed in Galtier et al. (2000). This condition can be satisfied at any finite wavenumber for sufficiently weak turbulence. On the other hand, for any turbulence intensity ϵ, there always exists a region of small $k_\parallel$ in which the condition is violated; this corresponds to the *nonuniform* validity of the kinetic equation. Such behavior for general three- and four-wave interactions is discussed in Newell, Nazarenko, & Biven (2001). Another applicability condition is that the spectrum must change slowly when crossing the wavenumber cone (8): the spectrum $q^s(\boldsymbol{k})$ must stay approximately constant as a function of $k_\parallel$ in the range $-\epsilon^2k_\perp \ll k_\parallel \ll \epsilon^2k_\perp$ at fixed $k_\perp$. Indeed, the function $\Delta(x)$ can only be treated as a delta function if the rest of the integrand changes more slowly than $\Delta(x)$, the latter having a characteristic width $w\sim 1/T > \epsilon^2b_0k_\perp$. As was discussed in Galtier et al. (2000), such an absence of "spikes" in the spectrum corresponds to an absence of long spatial correlations along the external magnetic field. Assuming that both applicability conditions are satisfied, using the Riemann-Lebesgue lemma and the symmetry of the equation for the spectrum, we obtain

$$\partial_t q^s(\boldsymbol{k}) = \frac{\pi\epsilon^2}{b_0}\int\frac{k_2(\boldsymbol{k}_\perp\cdot\boldsymbol{L}_\perp)^2(\boldsymbol{k}\times\kappa)^2_\parallel}{k_\perp^2L_2\kappa_2^2}q^{-s}(\kappa)\left[\frac{k_2}{L_2k_\perp^2}q^s(\boldsymbol{L}) - \frac{L_2}{k_2L_\perp^2}q^s(\boldsymbol{k})\right]\delta(\kappa_\parallel)\delta_{\boldsymbol{k},\kappa\boldsymbol{L}}d_{\kappa\boldsymbol{L}}. \quad (9)$$

The energy spectrum $e^s(\boldsymbol{k})$ of the shear-Alfvén waves is the sum of the perpendicular components of the energy tensor or, using the divergence-free condition, $e^s(\boldsymbol{k}) = (k_\perp^2/k_2^2)q^s(\boldsymbol{k})$; it obeys the following kinetic equation:

$$\partial_t e^s(\boldsymbol{k}) = \frac{\pi\epsilon^2}{b_0}\int\frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{L}_\perp)^2(\boldsymbol{k}\times\kappa)^2_\parallel}{k_\perp^2L_\perp^2\kappa_\perp^2}e^{-s}(\kappa)[e^s(\boldsymbol{L}) - e^s(\boldsymbol{k})]\delta(\kappa_\parallel)\delta_{\boldsymbol{k},\kappa\boldsymbol{L}}d_{\kappa\boldsymbol{L}}. \quad (10)$$

Equation (10), our main result, coincides with equation (46) of Galtier et al. (2000) obtained from the general kinetic equations of weak Alfvénic turbulence in the limit $k_\parallel \ll k_\perp$. This shows that statistical averaging and the $k_\parallel/k_\perp\to 0$ limit commute. The presence of $\delta(\kappa_\parallel)$ in the kinetic equation (10) is the consequence of the three-wave frequency resonance condition. It means that $k_\parallel$ is only an external parameter, and thus there is no energy exchange between modes with different $k_\parallel$-values, which leads to even stronger anisotropy $k_\parallel \ll k_\perp$. Physically, one can view such a process as a Bragg-like scattering of finite $k_\parallel$ waves off a two-dimensional turbulent "grating" (i.e., $k_\parallel = 0$ modes). One of the well-known consequences of the three-wave process for Alfvén wave turbulence in the case $k_\parallel \ll k_\perp$ is the $k_\perp^{-2}$ energy spectrum (see also Bhattacharjee & Ng 2001 for a model of weak turbulence); it is an exact constant-flux solution of equation (10); moreover, such stationary constant-flux solutions may have their exponents anywhere in the range from -1 to -3 depending on the degree of asymmetry of forcing of the $s = \pm1$ waves (Galtier et al. 2000). Such asymmetric wave pumping is very common in astrophysics (e.g., there are more Alfvén waves traveling away from the Sun than toward it in the solar wind; Matthaeus, Goldstein, & Roberts 1990).

3. DISCUSSION

This simple derivation of the weak turbulence kinetic equation allows us to appreciate the conditions ($k_\parallel/k_\perp \gg \epsilon^2$ and the absence of long-range correlations in the $\boldsymbol{b}_0$ direction) under which the theory is valid. The first one can always be checked on the basis of the solution of the kinetic equation. The second condition cannot be checked on the basis of the weak turbulence theory itself because this theory is invalid near $k_\parallel = 0$ and, therefore, cannot be used to see if any spikes are present in this region or not. However, at present it seems unlikely that any strong turbulence mechanism could lead to long parallel correlations and to a condensation of turbulence near $k_{\parallel} = 0$. Future numerical simulations and observations might shed more light on this issue. Note also that an analysis of data from the Jovian magnetosphere (Kivelson et al. 1997) may provide observational support for the present theory (Saur et al. 2001).

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