Magnetic field generation by coherent turbulence structures

Demosthenes Kivotides

Mathematics Department

UNIVERSITY OF NEWCASTLE

Magnetic field generation...

- The talk is based on the paper:

Magnetic field generation...

- The talk is based on the paper:
- *D. Kivotides*, A. J. Mee, C. F. Barenghi, Magnetic field generation by coherent turbulence structures, *submitted*

Magnetic field generation...

- Nonlinear systems of physical mathematics: what is their function within physics?

Magnetic field generation...

- Nonlinear systems of physical mathematics: what is their function within physics?
  - Analytical treatment quite difficult.

Magnetic field generation...

- Nonlinear systems of physical mathematics: what is their function within physics?
  - Analytical treatment quite difficult.
  - Numerics & informatics effective but seek a conceptual understanding beyond formal manipulations.

Magnetic field generation...

- Nonlinear systems of physical mathematics: what is their function within physics?
  - Analytical treatment quite difficult.
  - Numerics & informatics effective but seek a conceptual understanding beyond formal manipulations.
  - Decompose difficult nonlinear problems into a network of simpler problems with known interconnections.

Magnetic field generation...

- Nonlinear systems of physical mathematics: what is their function within physics?
  - Analytical treatment quite difficult.
  - Numerics & informatics effective but seek a conceptual understanding beyond formal manipulations.
  - Decompose difficult nonlinear problems into a network of simpler problems with known interconnections.
  - Take advantage of singular/near singular structures that dominate the system’s energetics:

Magnetic field generation...

- Nonlinear systems of physical mathematics: what is their function within physics?
  - Analytical treatment quite difficult.
  - Numerics & informatics effective but seek a conceptual understanding beyond formal manipulations.
  - Decompose difficult nonlinear problems into a network of simpler problems with known interconnections.
  - Take advantage of singular/near singular structures that dominate the system's energetics:
    - Shocks, detonations, vortex tubes/sheets,

Magnetic field generation...

- Nonlinear systems of physical mathematics: what is their function within physics?
  - Analytical treatment quite difficult.
  - Numerics & informatics effective but seek a conceptual understanding beyond formal manipulations.
  - Decompose difficult nonlinear problems into a network of simpler problems with known interconnections.
  - Take advantage of singular/near singular structures that dominate the system’s energetics:
    - Shocks, detonations, vortex tubes/sheets,
    - particle-segregation patterns,

Magnetic field generation...

- Nonlinear systems of physical mathematics: what is their function within physics?
    - Analytical treatment quite difficult.
    - Numerics & informatics effective but seek a conceptual understanding beyond formal manipulations.
    - Decompose difficult nonlinear problems into a network of simpler problems with known interconnections.
    - Take advantage of singular/near singular structures that dominate the system's energetics:
        - Shocks, detonations, vortex tubes/sheets,
        - particle-segregation patterns,
        - gravitational collapse structures.

Magnetic field generation...

- Turbulent magnetohydrodynamic dynamo $\equiv$ magnetic field amplification by a turbulent flow field.

Magnetic field generation...

- Turbulent magnetohydrodynamic dynamo $\equiv$ magnetic field amplification by a turbulent flow field.
- Two kinds: homogeneous-isotropic and shear.

Magnetic field generation...

- Turbulent magnetohydrodynamic dynamo $\equiv$ magnetic field amplification by a turbulent flow field.
- Two kinds: homogeneous-isotropic and shear.
- Kinematic dynamo

Magnetic field generation...

- Turbulent magnetohydrodynamic dynamo $\equiv$ magnetic field amplification by a turbulent flow field.
- Two kinds: homogeneous-isotropic and shear.
- Kinematic dynamo
- $Re = 10^4$, $Pr_m = \nu/\lambda = 10^{-8} - 0.4$; use an LES method in vorticity space.

Magnetic field generation...

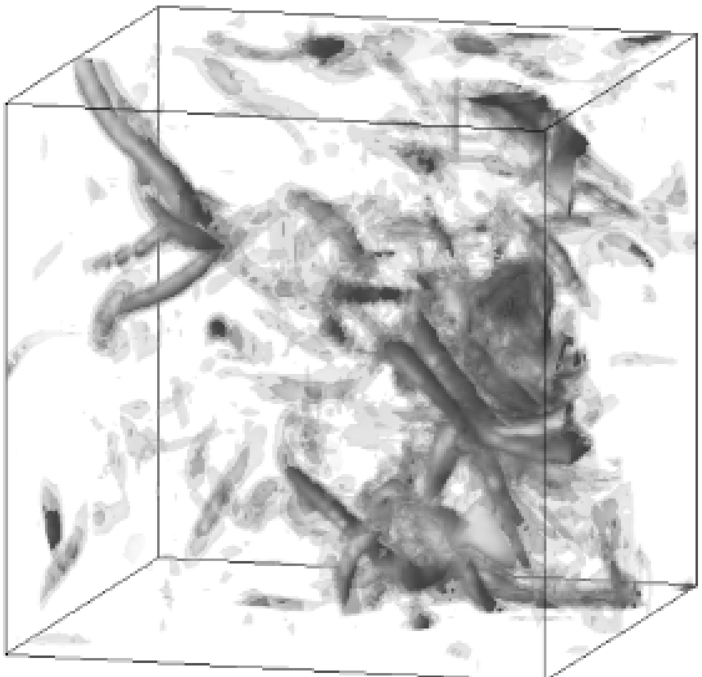
- Turbulent magnetohydrodynamic dynamo $\equiv$ magnetic field amplification by a turbulent flow field.
- Two kinds: homogeneous-isotropic and shear.
- Kinematic dynamo
- $Re = 10^4$, $Pr_m = \nu/\lambda = 10^{-8} - 0.4$; use an LES method in vorticity space.
- Is there a turbulent vortex structures dynamo?

Magnetic field generation...

- Turbulent magnetohydrodynamic dynamo $\equiv$ magnetic field amplification by a turbulent flow field.
- Two kinds: homogeneous-isotropic and shear.
- Kinematic dynamo
- $Re = 10^4$, $Pr_m = \nu/\lambda = 10^{-8} - 0.4$; use an LES method in vorticity space.
- Is there a turbulent vortex structures dynamo?
- What are the structure and statistics of the amplified magnetic field?

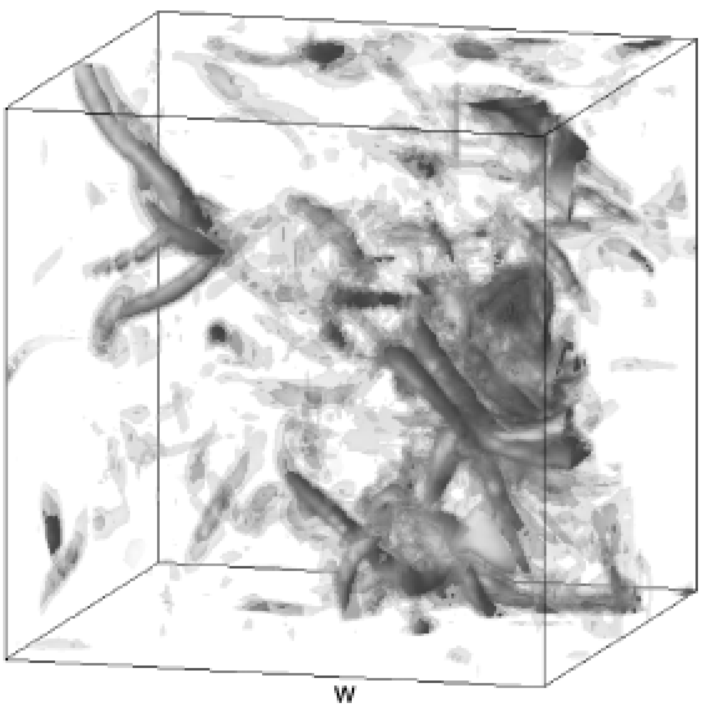
Magnetic field generation...

- Vortex structures in turbulence (Farge et al, PRL 2001).

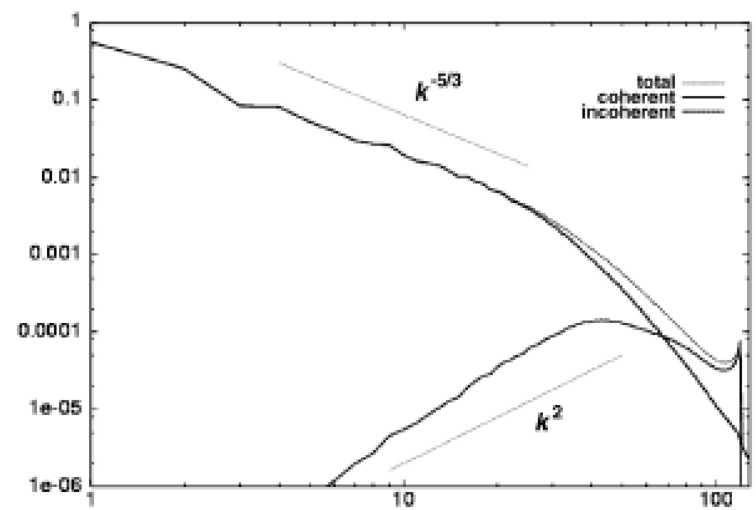


Magnetic field generation...

- Vortex structures in turbulence (Farge et al, PRL 2001).



- Vortex structures spectra (Farge et al, PRL 2001).



Magnetic field generation...

- Vortex tube turbulence model (Kivotides & Leonard, PRL 2003).

$$\frac{\partial \boldsymbol{X}}{\partial t} = \boldsymbol{V},$$

$$\boldsymbol{V}(\boldsymbol{x}) = -\frac{1}{4\pi}\int_{\mathcal{L}} \frac{(\boldsymbol{x}-\boldsymbol{X})\times\boldsymbol{\omega}(\boldsymbol{X})d\boldsymbol{X}}{|\boldsymbol{x}-\boldsymbol{X}|^3},$$

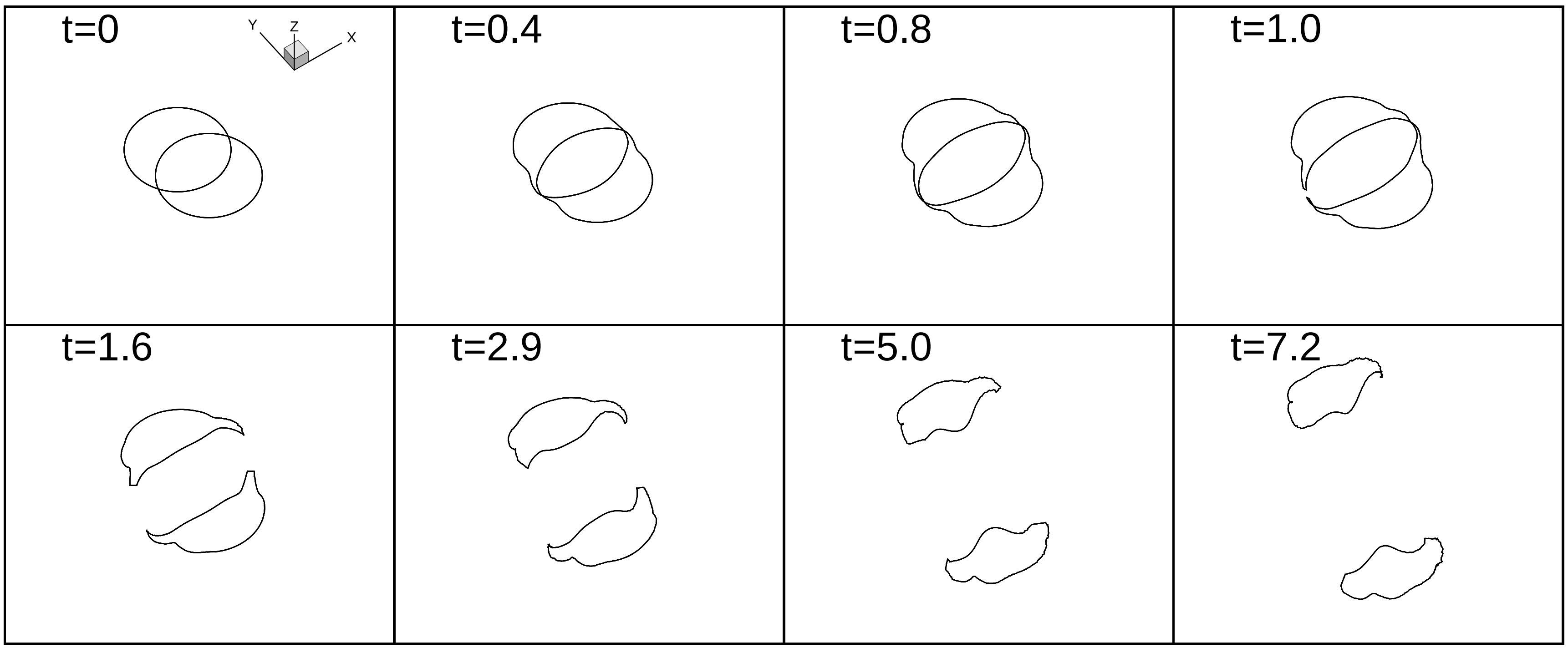
$$\boldsymbol{\omega}(\boldsymbol{x},t) = \gamma\int_{\mathcal{L}} \frac{1}{\sigma(s,t)^3}\zeta\left(\frac{|\boldsymbol{x}-\boldsymbol{X}(s,t)|}{\sigma(s,t)}\right)\left(\frac{\partial \boldsymbol{X}}{\partial s} + \frac{\boldsymbol{x}-\boldsymbol{X}(s,t)}{\sigma(s,t)}\frac{\partial \sigma}{\partial s}\right)ds,$$

$$\frac{d\sigma_i^2}{dt} = 2\gamma\nu,$$

$$\mathcal{T}_{\mathcal{L}}^{t} \longmapsto \mathcal{T}_{\mathcal{L}}^{t+\Delta t}.$$

Magnetic field generation...

- Modeling changes in vortex tangle topology (Kivotides & Leonard, EPL 2003).



Magnetic field generation...

Magnetic field generation...

- Physics of the vortex tube turbulence model:

Magnetic field generation...

- Physics of the vortex tube turbulence model:
  - Incorporates reconnections.

Magnetic field generation...

- Physics of the vortex tube turbulence model:
  - Incorporates reconnections.
  - LES formulation in vorticity space.

Magnetic field generation...

- Physics of the vortex tube turbulence model:
  - Incorporates reconnections.
  - LES formulation in vorticity space.
  - Kolmogorov scalings for second and third order structure functions.

Magnetic field generation...

- Physics of the vortex tube turbulence model:
  - Incorporates reconnections.
  - LES formulation in vorticity space.
  - Kolmogorov scalings for second and third order structure functions.
  - Qualitatively correct alignments between material or vorticity vectors and strain rate eigenvectors.

Magnetic field generation...

- Physics of the vortex tube turbulence model:
  - Incorporates reconnections.
  - LES formulation in vorticity space.
  - Kolmogorov scalings for second and third order structure functions.
  - Qualitatively correct alignments between material or vorticity vectors and strain rate eigenvectors.
  - Two positive, on average, eigenvalues of the strain rate tensor.

Magnetic field generation...

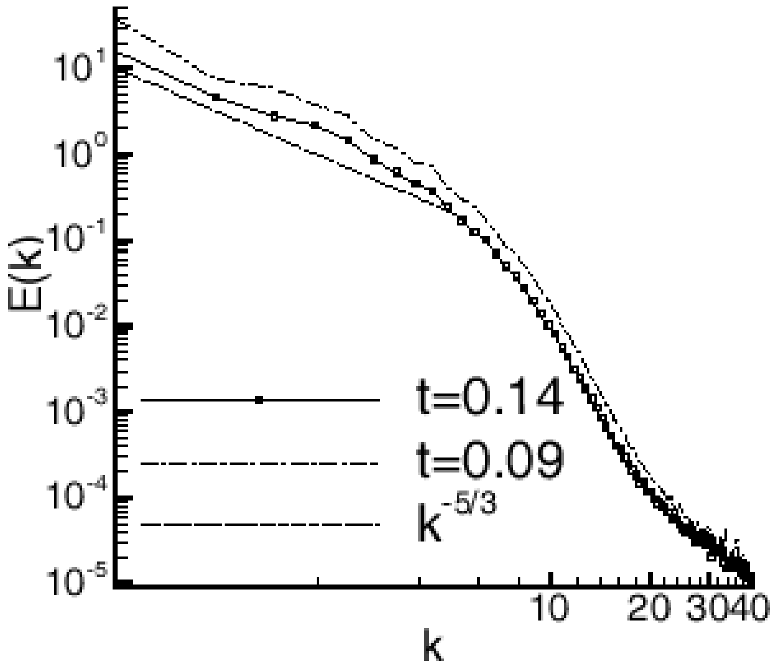
- Physics of the vortex tube turbulence model:
  - Incorporates reconnections.
  - LES formulation in vorticity space.
  - Kolmogorov scalings for second and third order structure functions.
  - Qualitatively correct alignments between material or vorticity vectors and strain rate eigenvectors.
  - Two positive, on average, eigenvalues of the strain rate tensor.
  - Qualitatively correct kinematics.

Magnetic field generation...

- Physics of the vortex tube turbulence model:
  - Incorporates reconnections.
  - LES formulation in vorticity space.
  - Kolmogorov scalings for second and third order structure functions.
  - Qualitatively correct alignments between material or vorticity vectors and strain rate eigenvectors.
  - Two positive, on average, eigenvalues of the strain rate tensor.
  - Qualitatively correct kinematics.
  - Predicted fractal dimension of concentrated vorticity, pdf's of stretched filament radii, spectra of filament curvature and torsion...

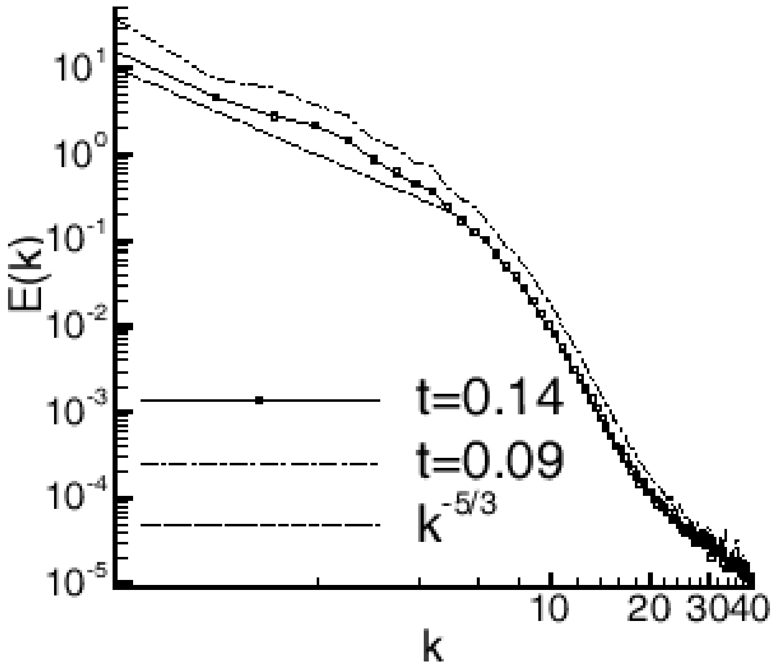
Magnetic field generation...

- Schoinoidal model's spectrum (Kivotides & Leonard, PRL 2003).

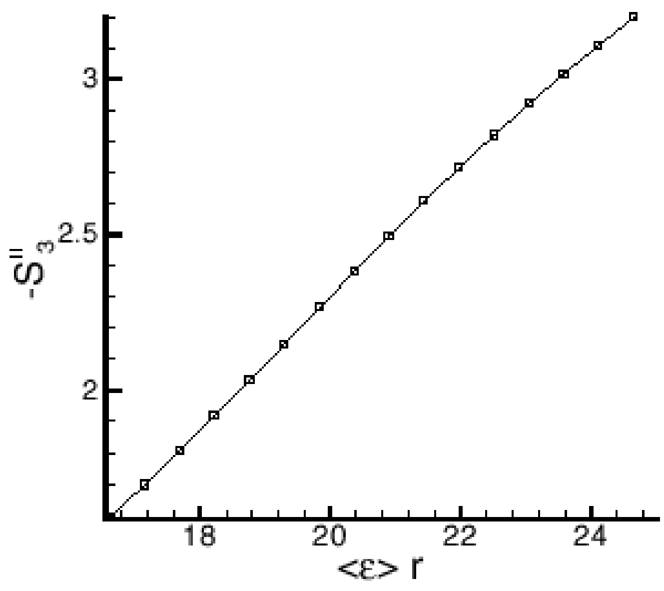


Magnetic field generation...

- Schoinoidal model's spectrum (Kivotides & Leonard, PRL 2003).



- Schoinoidal model's third order structure function (Kivotides & Leonard, PRL 2003).



Magnetic field generation...

- Magnetic field component, linear magnetic induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \lambda \nabla^2 \mathbf{B} + \nabla \times (\boldsymbol{V} \times \mathbf{B}).$$

Magnetic field generation...

- Magnetic field component, linear magnetic induction equation:

$$\frac{\partial \mathrm{B}}{\partial t} = \lambda \nabla^2 \mathrm{B} + \nabla \times (\boldsymbol{V} \times \mathbf{B}).$$

- In a more familiar fluid dynamic form:

$$\frac{\partial \mathrm{B}}{\partial t} + \boldsymbol{V} \cdot \boldsymbol{\nabla} \mathrm{B} = \mathrm{B} \cdot \boldsymbol{\nabla} \boldsymbol{V} + \lambda \nabla^2 \mathrm{B}.$$

Magnetic field generation...

- Magnetic field component, linear magnetic induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \lambda \nabla^2 \mathbf{B} + \nabla \times (\boldsymbol{V} \times \mathbf{B}).$$

- In a more familiar fluid dynamic form:

$$\frac{\partial \mathbf{B}}{\partial t} + \boldsymbol{V} \cdot \boldsymbol{\nabla} \mathbf{B} = \mathbf{B} \cdot \boldsymbol{\nabla} \boldsymbol{V} + \lambda \nabla^2 \mathbf{B}.$$

- The following constraint applies:

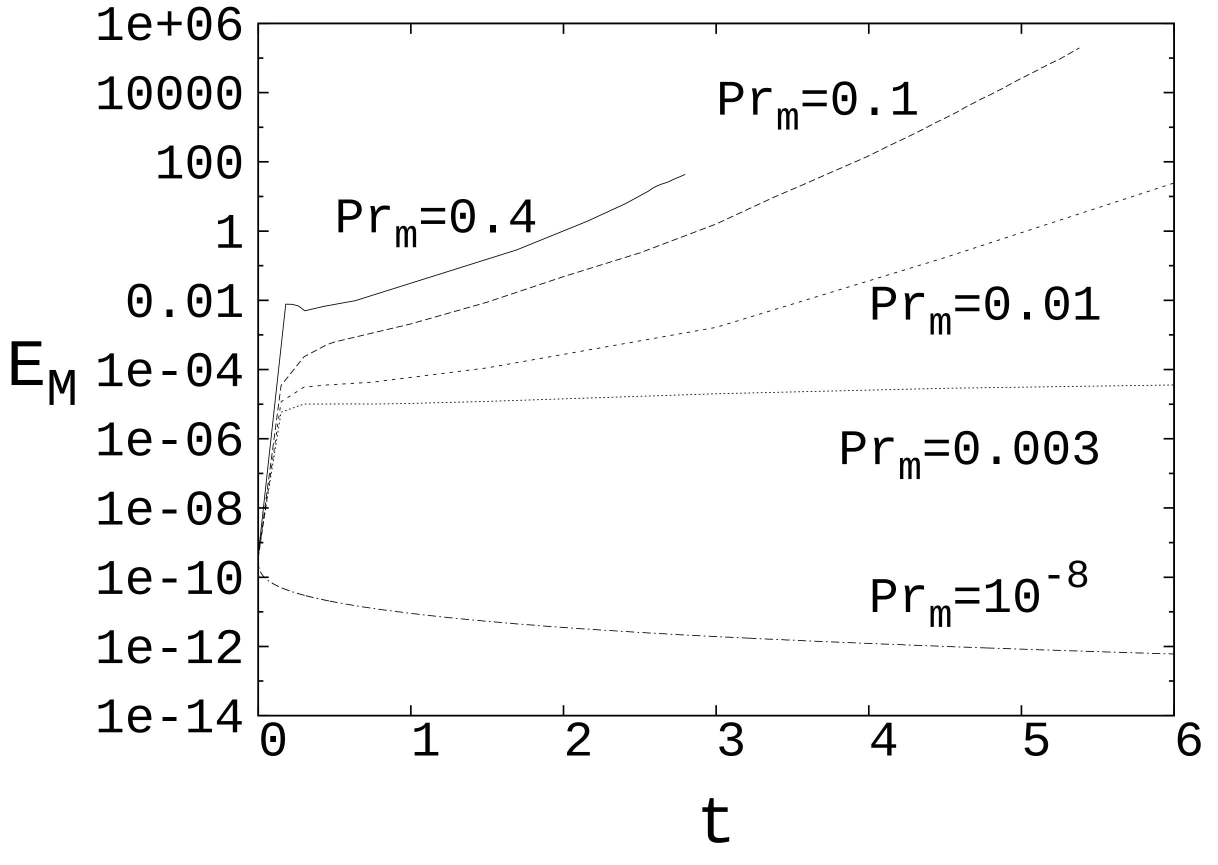
$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0.$$

Magnetic field generation...

- Snapshot of vortex tube turbulence model flow as initial condition.

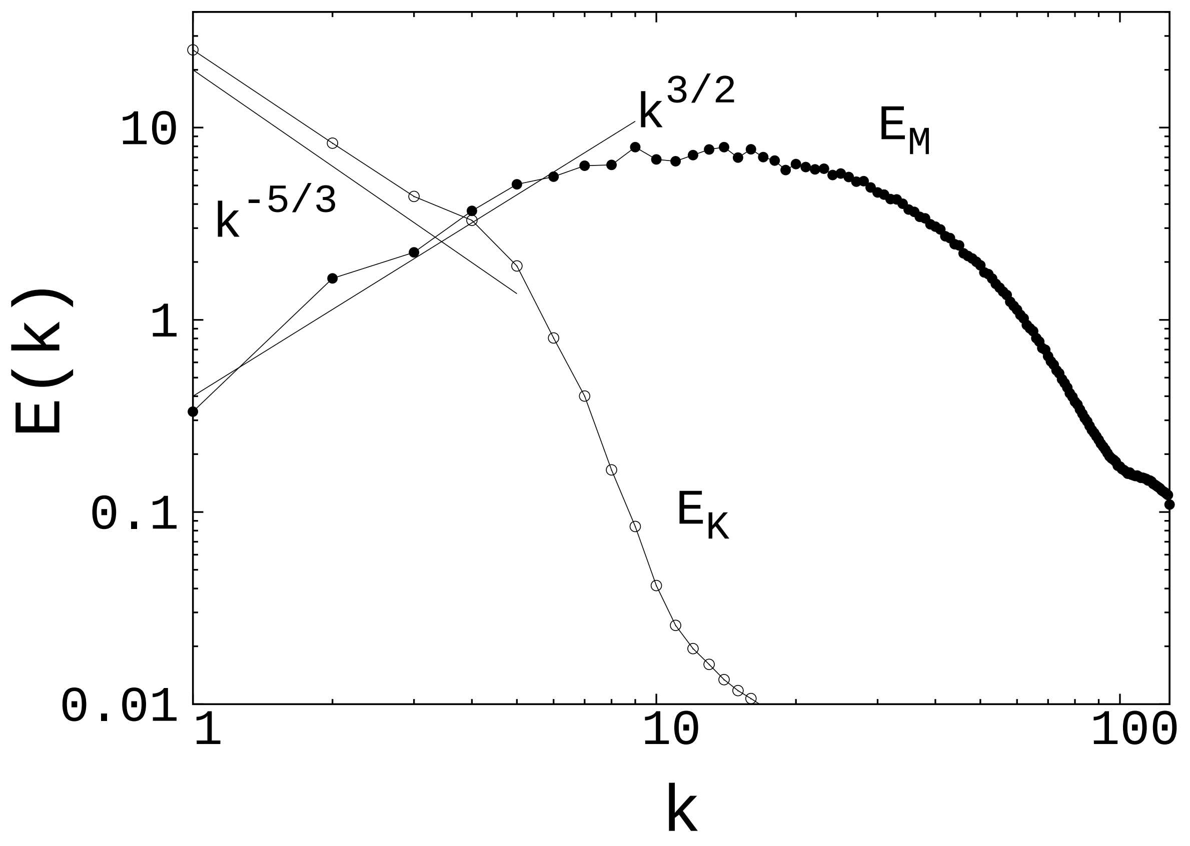
Magnetic field generation...

- Magnetic field generation by turbulent vortex structures; $Re = 10^4$.



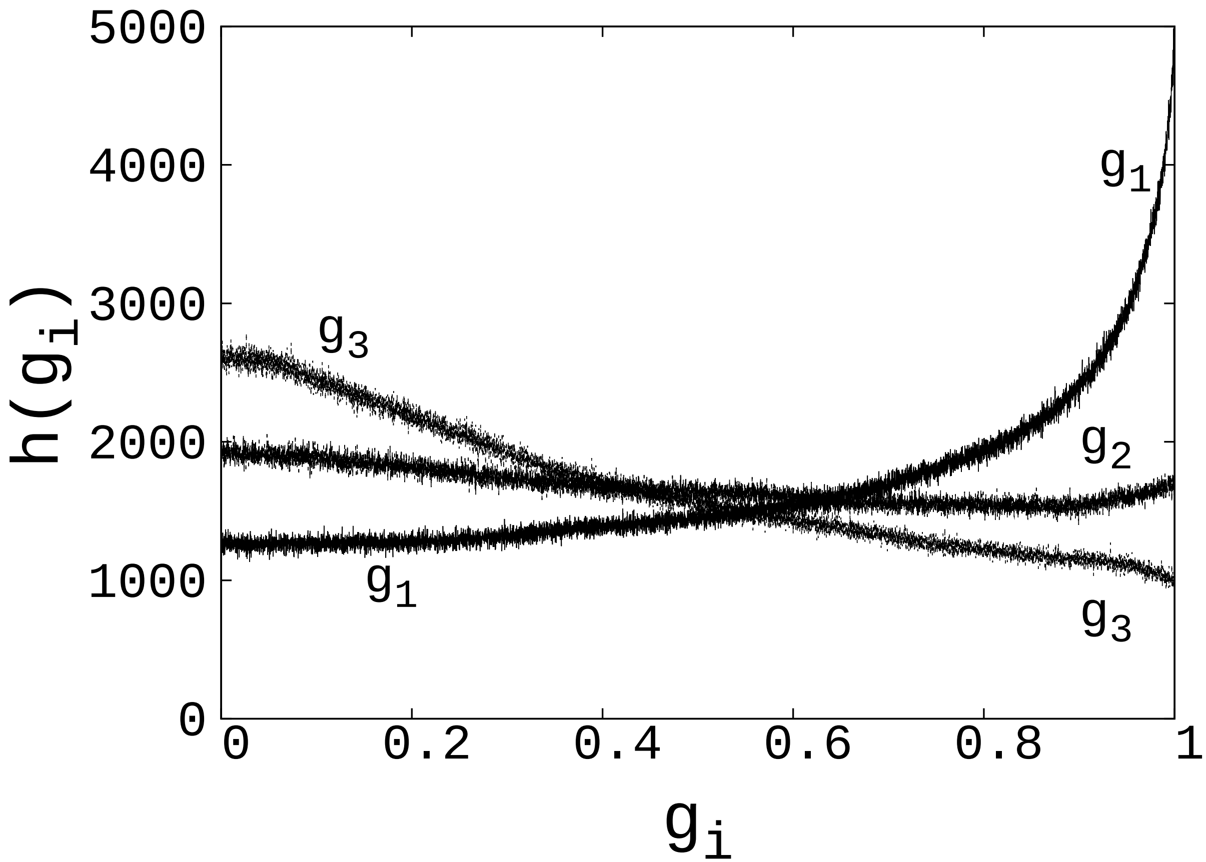
Magnetic field generation...

- Magnetic and fluid spectra; $Re = 10^4, Pr_m = 0.4$.



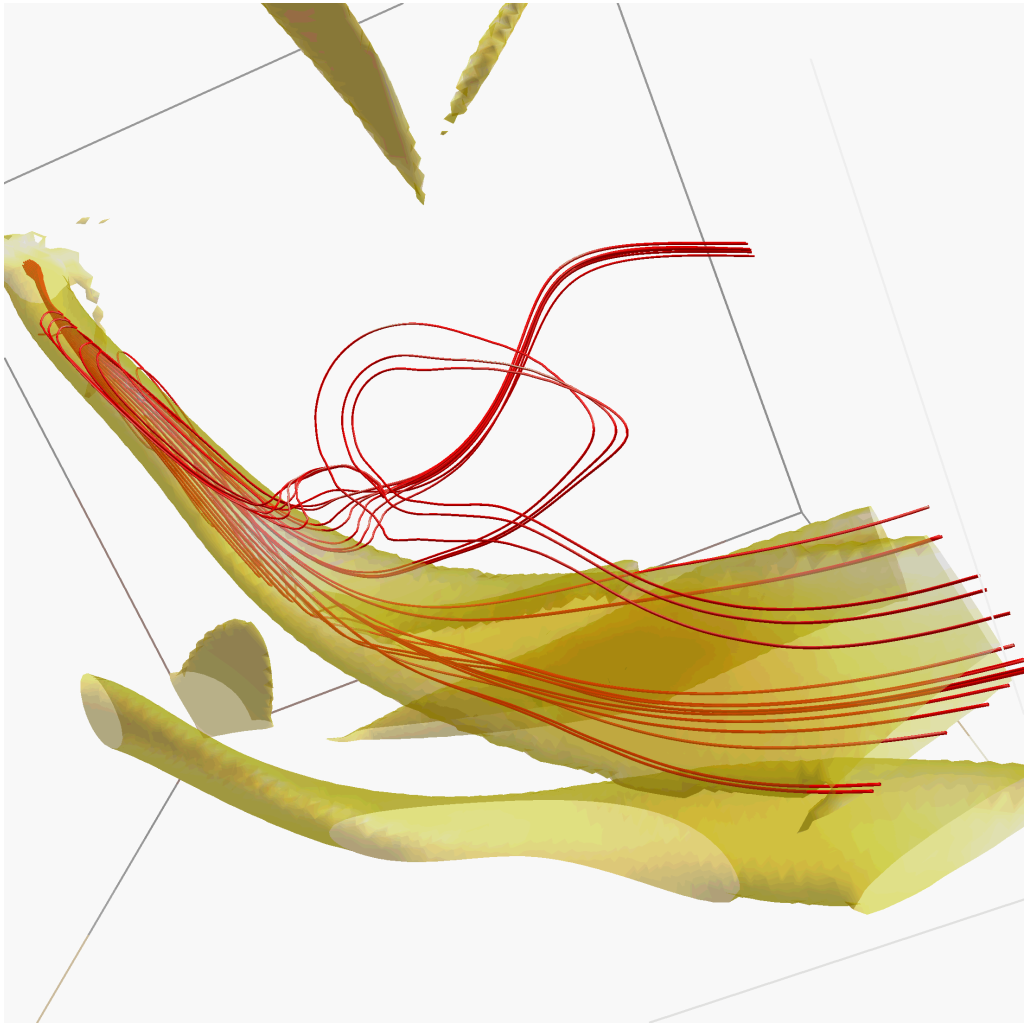
Magnetic field generation...

- Magnetic field - strain rate alignment cosines
  $g_i = |(\mathbf{B} \cdot \boldsymbol{\Lambda}_i)/|\mathbf{B}||\boldsymbol{\Lambda}_i||$ ($i = 1, 2, 3$); $Re = 10^4$, $Pr_m = 0.4$.
  $\langle \Lambda_1 \rangle = 23.284$, $\langle \Lambda_2 \rangle = 0.482$, whereas $\langle \Lambda_3 \rangle = -23.766$.



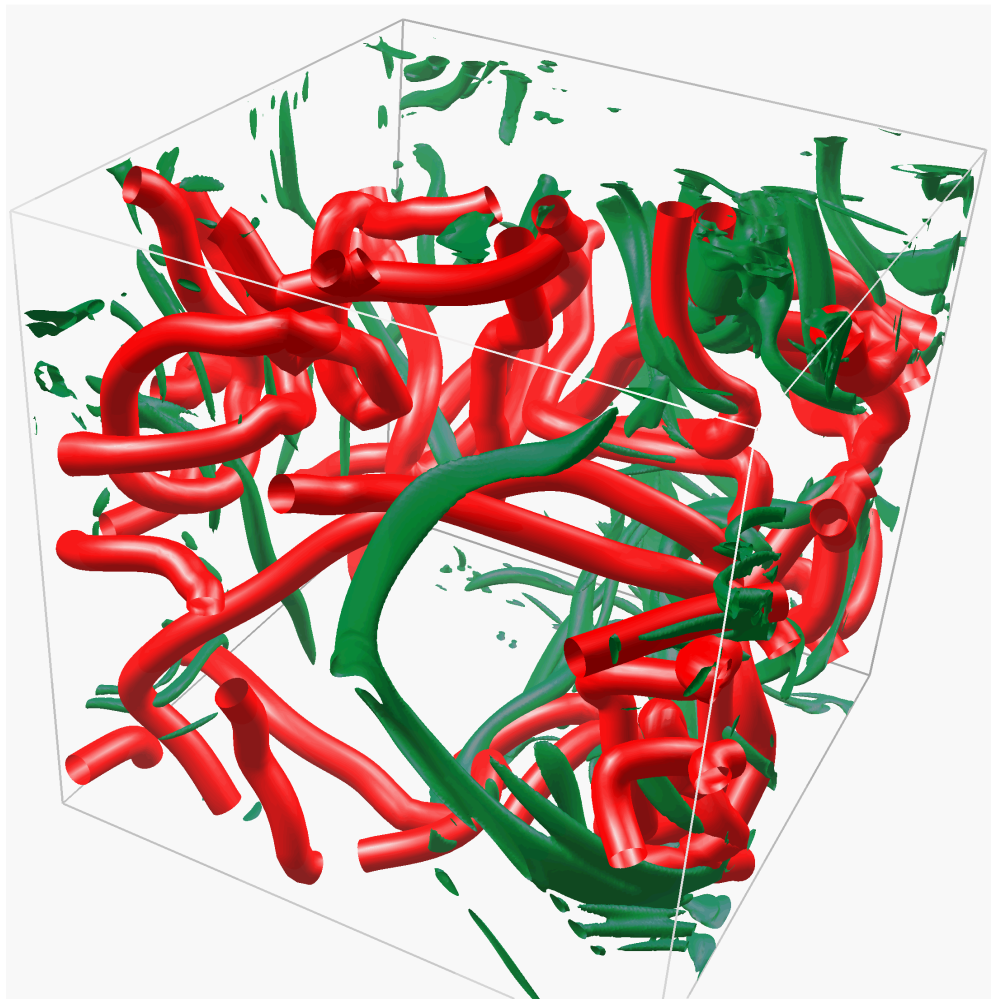
Magnetic field generation...

- Tube to ribbon transition; $Re = 10^4, Pr_m = 0.4$.



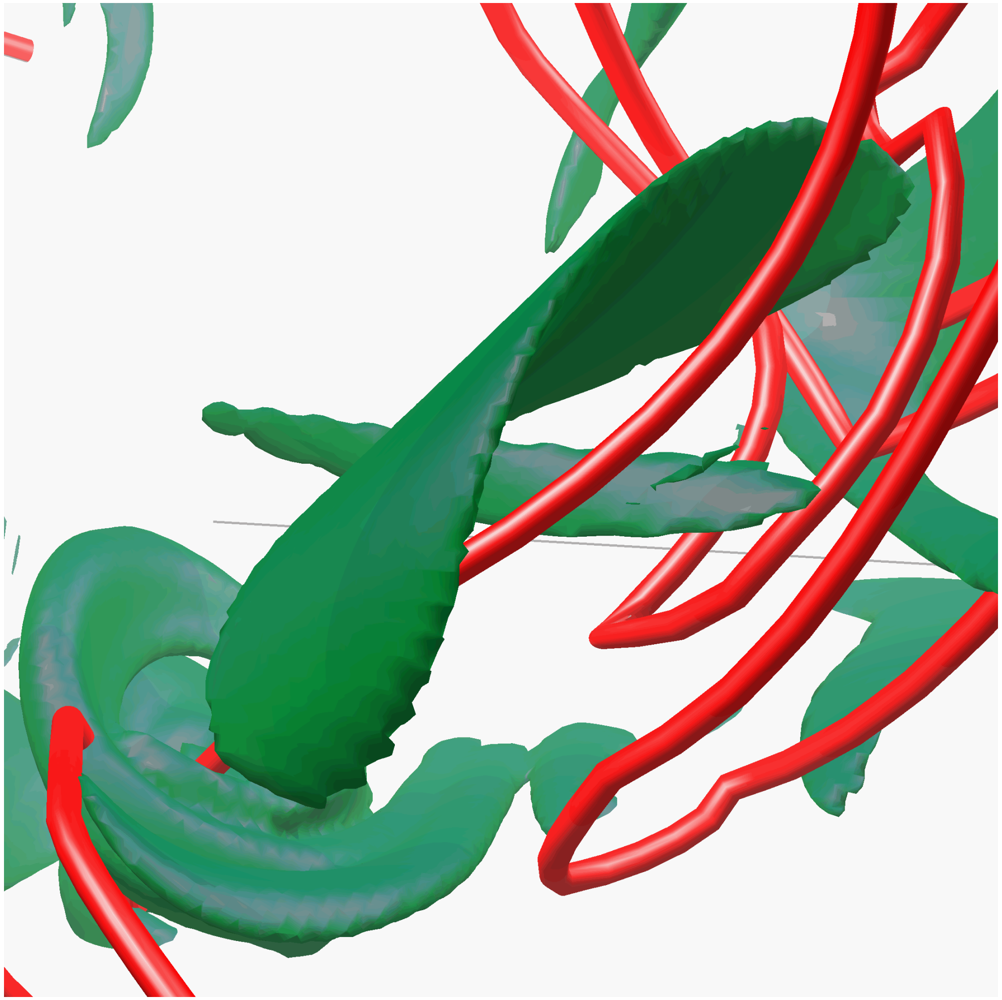
Magnetic field generation...

- Vortex and magnetic structures; $Re = 10^4, Pr_m = 0.4$.



Magnetic field generation...

- Magnetic field spiraling around a vortex; $Re = 10^4, Pr_m = 0.4$.



Magnetic field generation...

- Thermal superfluid $\equiv$ inviscid superfluid $\oplus$ viscous, Navier-Stokes normal-fluid $\oplus$ mutual friction force.

Magnetic field generation...

- Thermal superfluid $\equiv$ inviscid superfluid $\oplus$ viscous, Navier-Stokes normal-fluid $\oplus$ mutual friction force.
- The thermal superfluid dynamo:

Magnetic field generation...

- Thermal superfluid $\equiv$ inviscid superfluid $\oplus$ viscous, Navier-Stokes normal-fluid $\oplus$ mutual friction force.
- The thermal superfluid dynamo:
  - Thermal superfluid $\longleftrightarrow$ Electrically conducting fluid.

Magnetic field generation...

- Thermal superfluid $\equiv$ inviscid superfluid $\oplus$ viscous, Navier-Stokes normal-fluid $\oplus$ mutual friction force.
- The thermal superfluid dynamo:
  - Thermal superfluid $\longleftrightarrow$ Electrically conducting fluid.
  - Navier-Stokes normal-fluid $\longleftrightarrow$ Navier-Stokes fluid.

Magnetic field generation...

- Thermal superfluid $\equiv$ inviscid superfluid $\oplus$ viscous, Navier-Stokes normal-fluid $\oplus$ mutual friction force.
- The thermal superfluid dynamo:
    - Thermal superfluid $\longleftrightarrow$ Electrically conducting fluid.
    - Navier-Stokes normal-fluid $\longleftrightarrow$ Navier-Stokes fluid.
    - Superfluid field $\longleftrightarrow$ Magnetic field.

Magnetic field generation...

- Thermal superfluid $\equiv$ inviscid superfluid $\oplus$ viscous, Navier-Stokes normal-fluid $\oplus$ mutual friction force.
- The thermal superfluid dynamo:
  - Thermal superfluid $\longleftrightarrow$ Electrically conducting fluid.
  - Navier-Stokes normal-fluid $\longleftrightarrow$ Navier-Stokes fluid.
  - Superfluid field $\longleftrightarrow$ Magnetic field.
  - Mutual friction $\longleftrightarrow$ Lorentz force.

Magnetic field generation...

- Superfluid vortex dynamics (Idowu, Kivotides, Barenghi & Samuels, JLTP 2000):

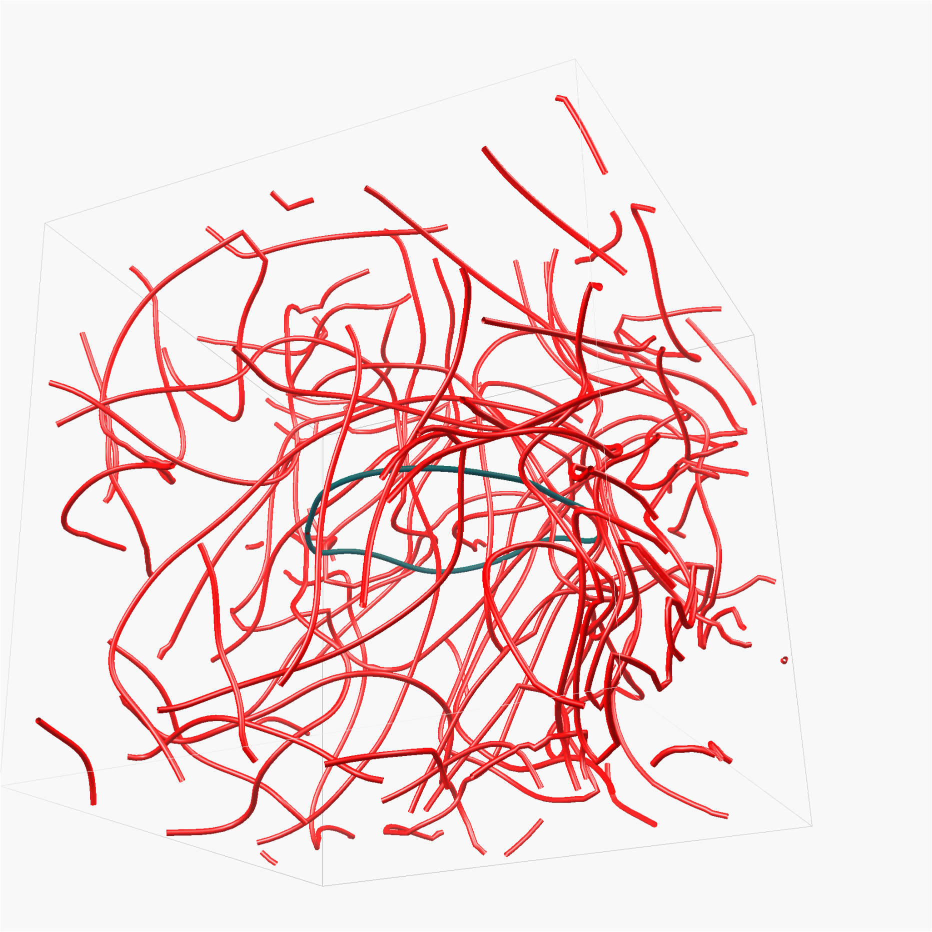
$$\frac{\partial \boldsymbol{X_s}}{\partial t} = \boldsymbol{V_s} + h_0 \boldsymbol{V_s} + h_\times \boldsymbol{X_s'} \times (\boldsymbol{V_n} - \boldsymbol{V_s}) - h_{\times\times} \boldsymbol{X_s'} \times (\boldsymbol{X_s'} \times \boldsymbol{V_n}),$$

$$\boldsymbol{V_s}(\boldsymbol{x}) = -\frac{\kappa}{4\pi} \int_{\mathcal{L}_s} d\xi_s \frac{\boldsymbol{X_s'} \times (\boldsymbol{X_s} - \boldsymbol{x})}{|\boldsymbol{X_s} - \boldsymbol{x}|^3},$$

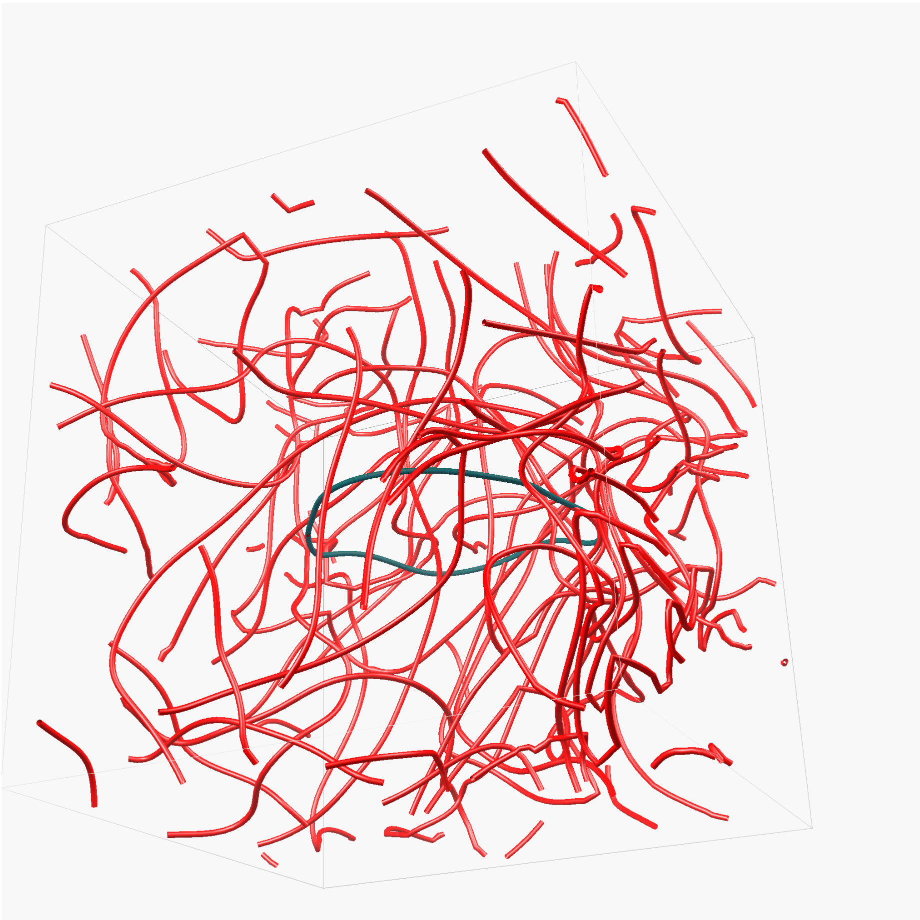
$$\boldsymbol{X_s'} \equiv \partial \boldsymbol{X_s} / \partial \xi_s,$$

$$\mathcal{T}^t_{\mathcal{L}_s} \longmapsto \mathcal{T}^{t+\Delta t}_{\mathcal{L}_s}.$$

Magnetic field generation...

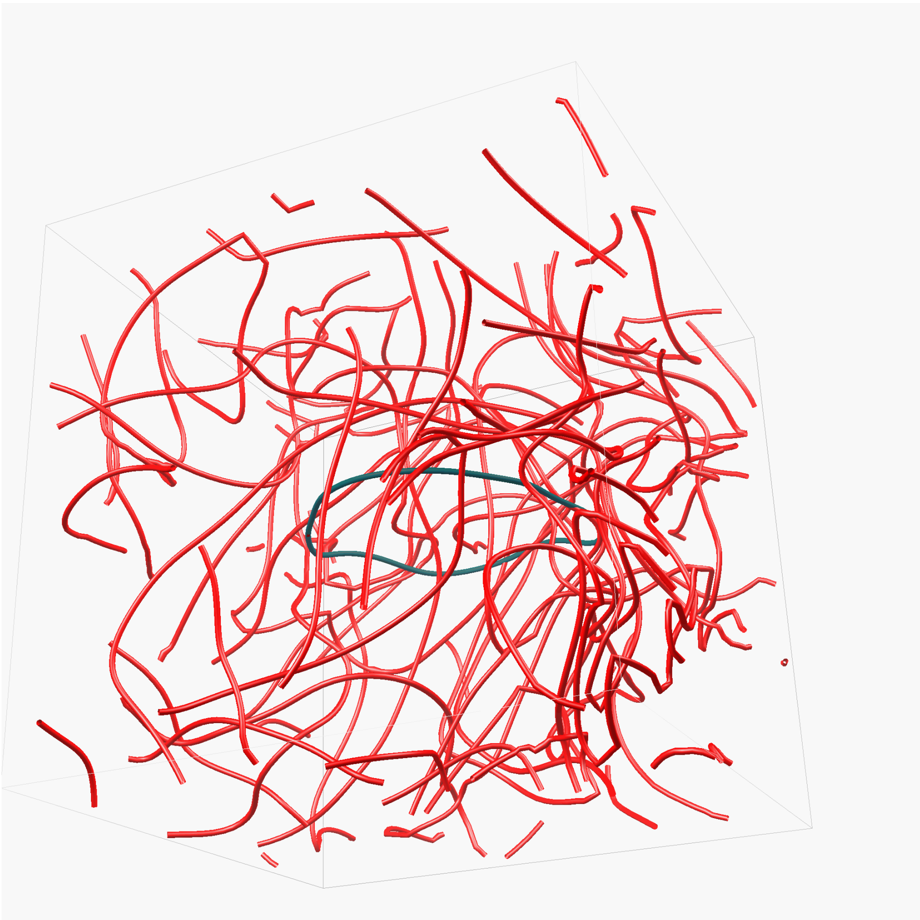


Magnetic field generation...



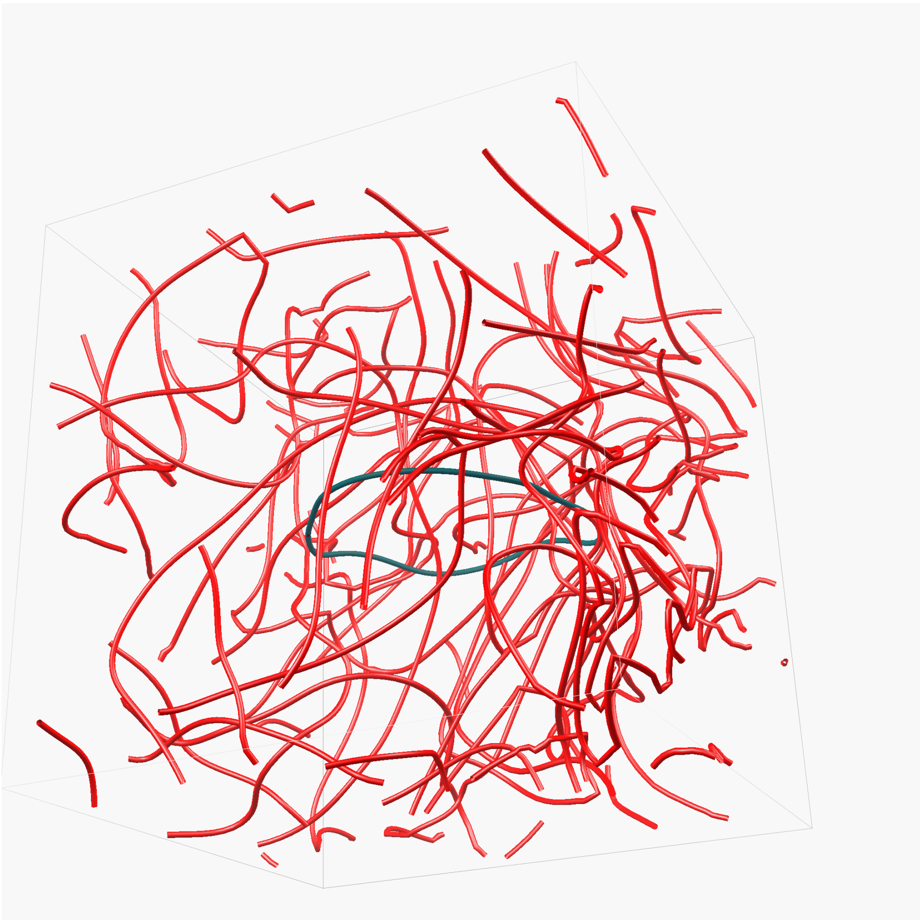
- A snapshot of the vortex tube model normal turbulent flow.

Magnetic field generation...



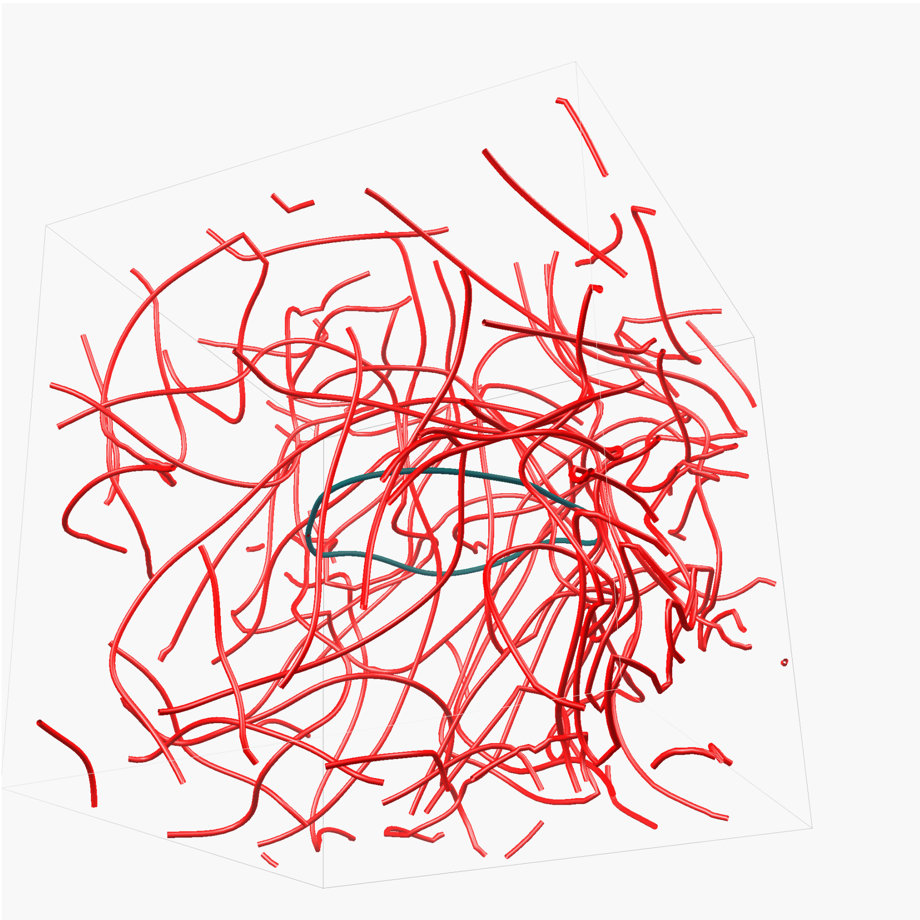
- A snapshot of the vortex tube model normal turbulent flow.
- A seed superfluid vorticity.

Magnetic field generation...



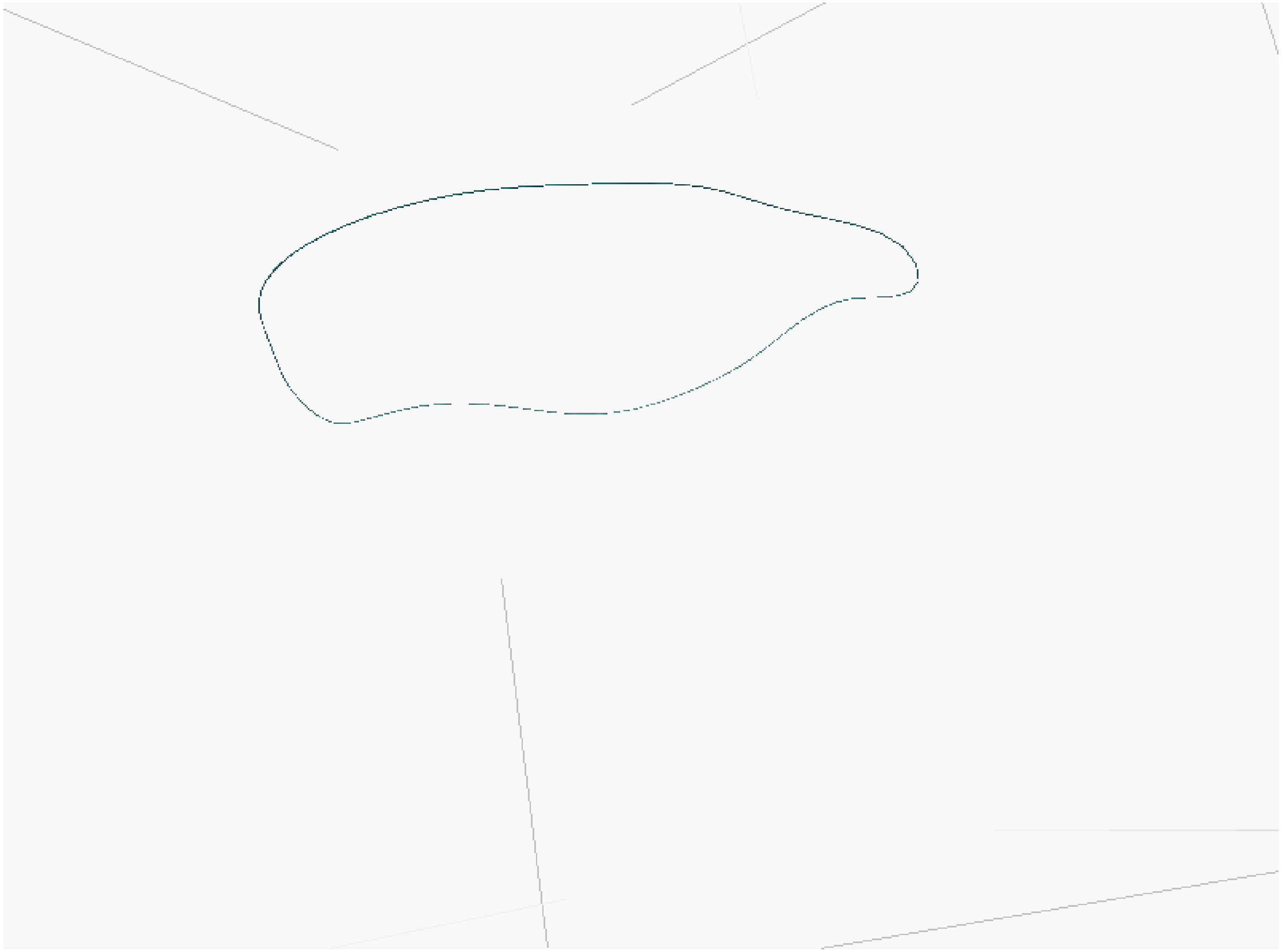
- A snapshot of the vortex tube model normal turbulent flow.
- A seed superfluid vorticity.
- Is there a dynamo? (Kivotides, PRL 2006).

Magnetic field generation...



- A snapshot of the vortex tube model normal turbulent flow.
- A seed superfluid vorticity.
- Is there a dynamo? (Kivotides, PRL 2006).
- $Re = 40$, $\gamma \approx 100\kappa$.

Magnetic field generation...



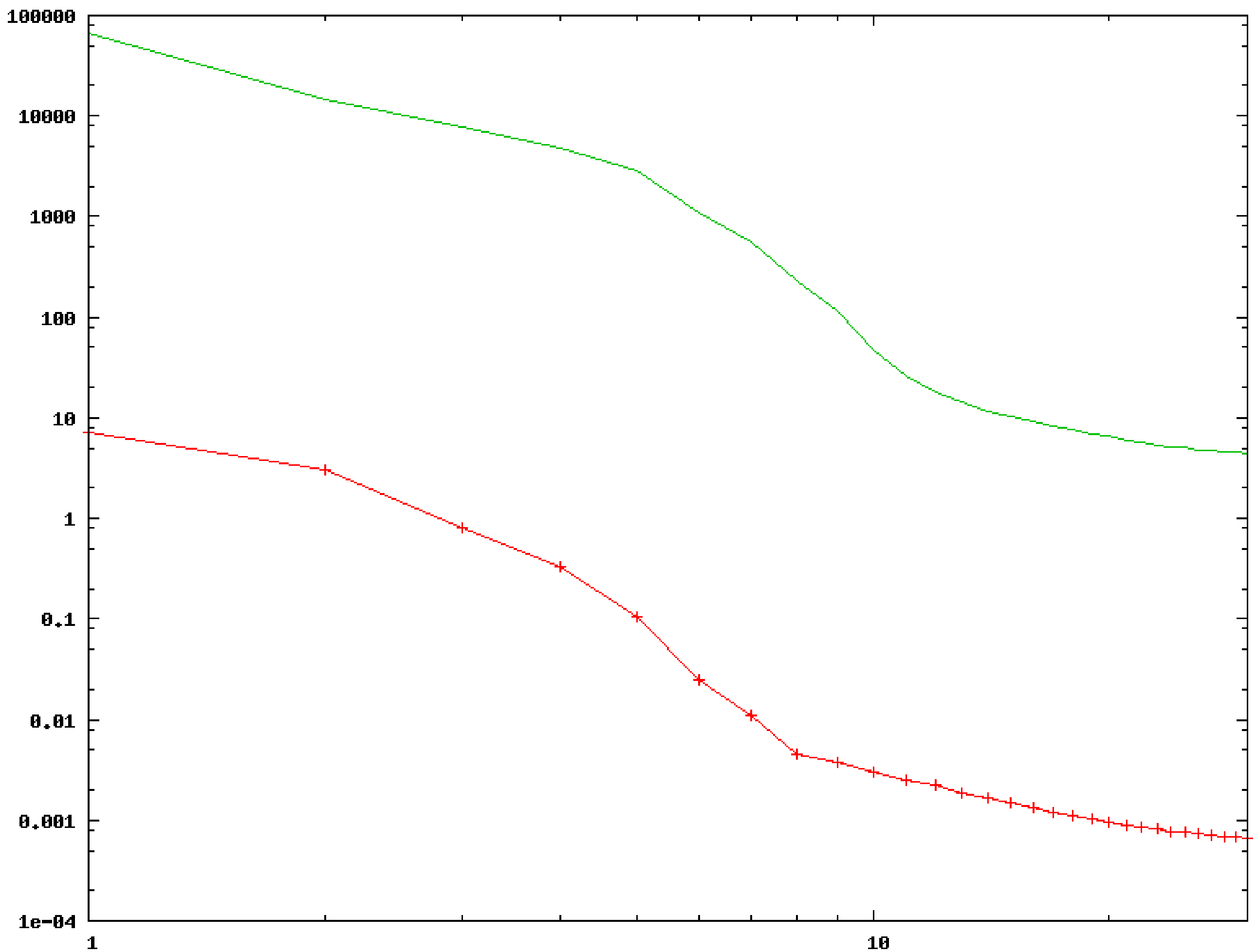
Magnetic field generation...

Magnetic field generation...

- Normal-fluid (green line) and superfluid (red lines) energy spectra.

Magnetic field generation...

- Normal-fluid (green line) and superfluid (red lines) energy spectra.



Thank you for your attention!