Influence of slow scales of turbulent flows on the dynamo action

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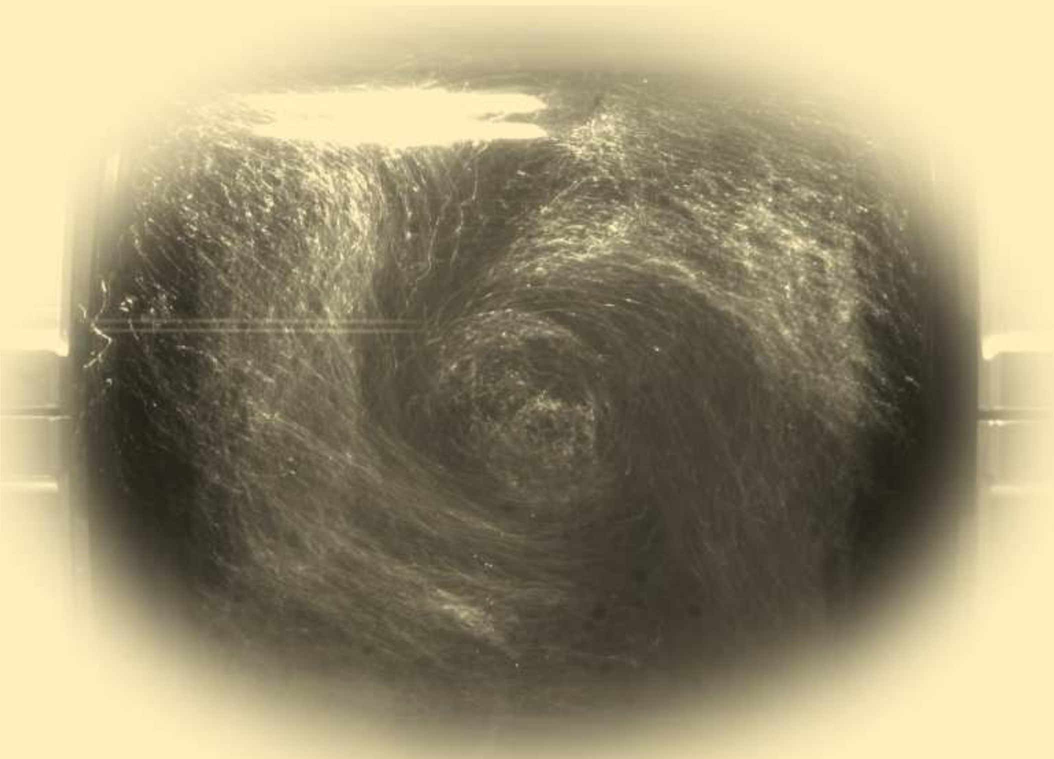

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Outline

- Introduction:
  - $\Rightarrow$ Why slow scales?
- Hydrodynamics:
  - $\Rightarrow$ Water experiment
- MHD:
  - $\Rightarrow$ Kinematic dynamo simulations
- Conclusions

Problem formulation

Governing Equations:

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{u} \times \vec{B} \right) + \eta \nabla^2 \vec{B}$$

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \vec{\nabla} \right) \vec{u} = -\frac{1}{\rho} \vec{\nabla} p + \nu \nabla^2 \vec{u} + \frac{1}{\rho \mu_0} \left(\vec{\nabla} \times \vec{B} \right) \times \vec{B} + \vec{F}_{ext}$$

$$\vec{\nabla} \cdot \vec{u} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

Adimensional numbers:

$$Rm = \frac{UL}{\eta} = UL\mu_0\sigma \qquad Re = \frac{UL}{\nu} \qquad Pm = \frac{Rm}{Re}$$

Problem formulation

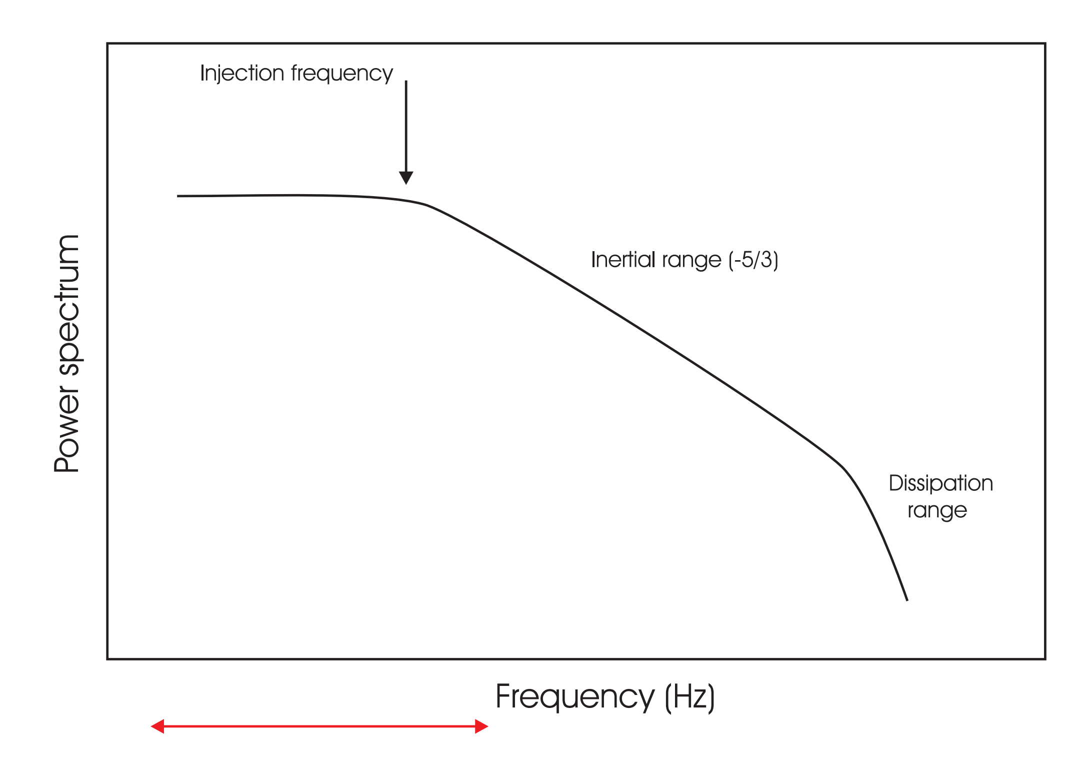
But, $\nu << \eta$ for most neutral conducting fluids...

$$Re >> Rm$$

tipically, $Re \sim 10^5 Rm$

Fully developped turbulence!

Problem formulation



Our study will focus on the slow scales...

Problem formulation

Our approach:

Kinematic dynamo + Hydro Experiments

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{u} \times \vec{B} \right) + \eta \nabla^2 \vec{B}$$

where $\vec{u}$ is measured in water experiments

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \vec{\nabla} \right) \vec{u} = -\frac{1}{\rho} \vec{\nabla} p + \nu \nabla^2 \vec{u} + \frac{1}{\rho \mu_0} \left(\vec{\nabla} \times \vec{B} \right) \times \vec{B}$$

Below the threshold, $\vec{B} = 0 \Rightarrow \vec{u}$ is controlled by hydrodynamics

Previous studies

Analytical / Numerical:

- "Ponomarenko dynamo with time-periodic flow''
  C. Normand,
  Phys. Fluids, **15** (2003) pp. 1606-1611.

- ''The dynamo effect'' S. Fauve, F. Pétrélis,
  COST-P6 meeting, Paris, January 2004 and
  Peyresq lectures on non-linear phenomena,
  ed. J.A. Sepulchre
  World Scientific, Singapore (2003).

$\Rightarrow$ both of them predict threshold variation due
to slow evolution.

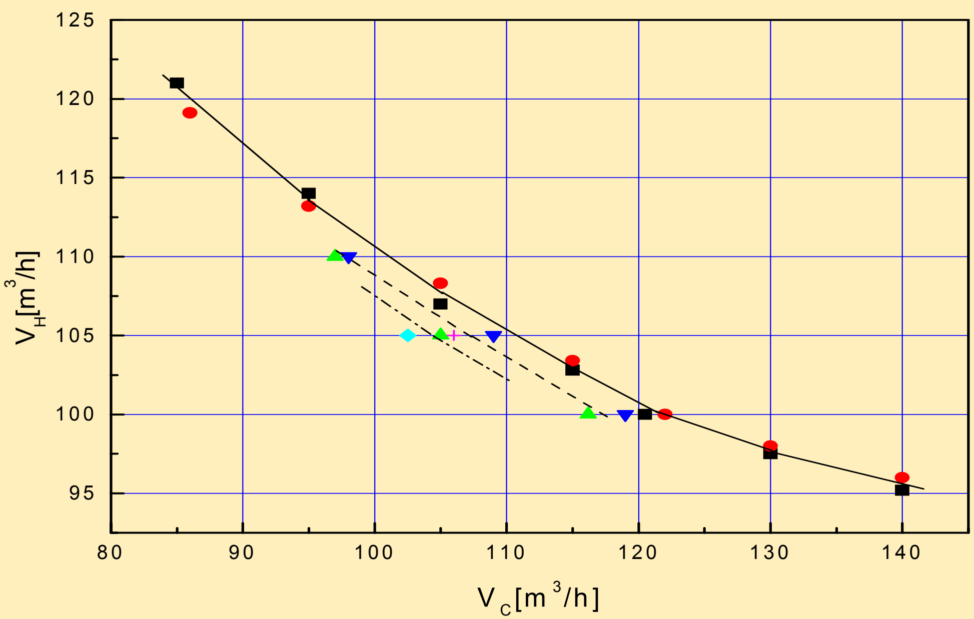
Previous studies

Experimental:

- “Complementary Experiments at the
  Karlsruhe Dynamo Test facility”
  U. Mueller, R. Stieglitz, S. Horanyi, F.Busse
  XXI ICTAM (CD-ROM Proceedings),
  ISBN: 83-89687-01-1
  published by IPPT-PAN Warsaw (2004)

Previous studies

Experimental:



Periodic axial flows with period and amplitudes

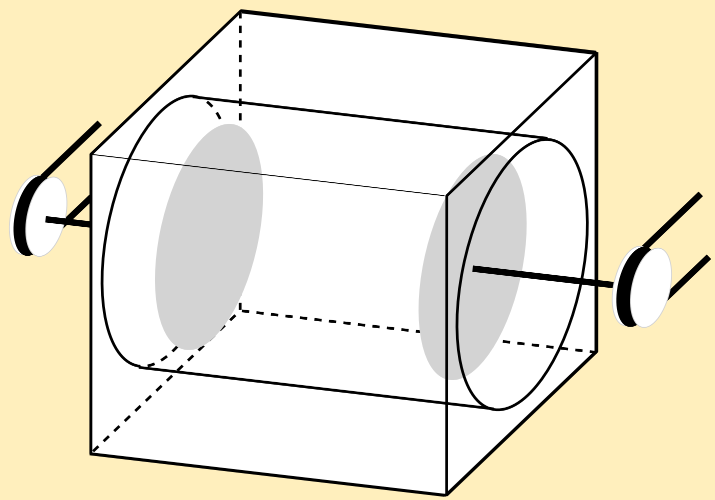
$\tau = [0, 7.5, 20]\mathrm{s}$ $\qquad A = 0, 5, 20\ \%$

They show a **decrease** of the threshold.

Water experiment

Experimental setup:

- Cylindrical volume
  $D = 0.1 - 0.4$m, $H = 0.1 - 0.5$m
- Two counter rotating propellers
- Frequency: $f = 1 - 20$Hz
- Re: propeller frequency and spatial dimensions

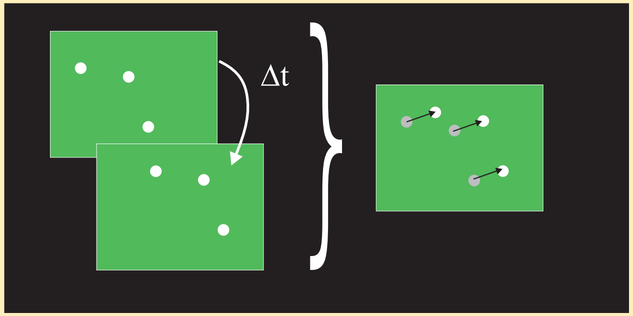


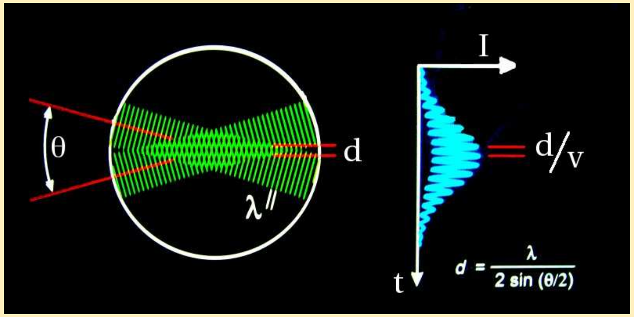
Water experiment

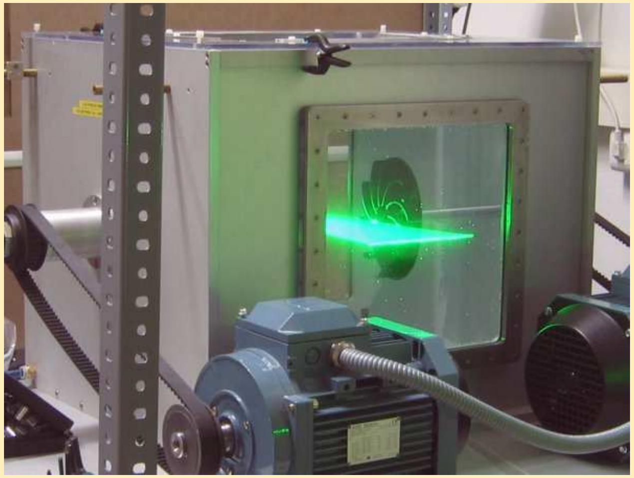
Typical propeller:

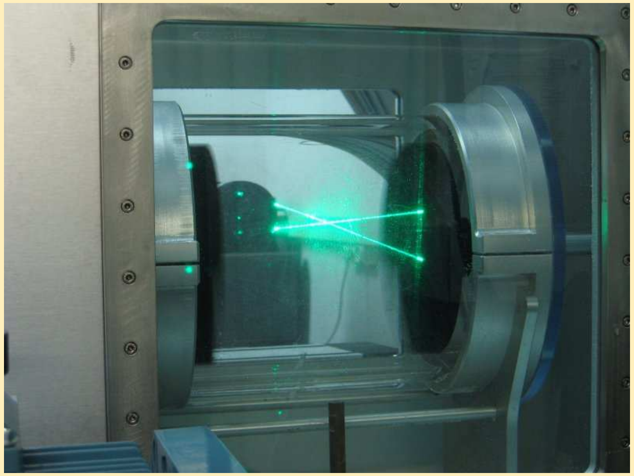
Velocity measurements

PIV (spatial evolution) $\Leftrightarrow$ LDA (temporal evolution)







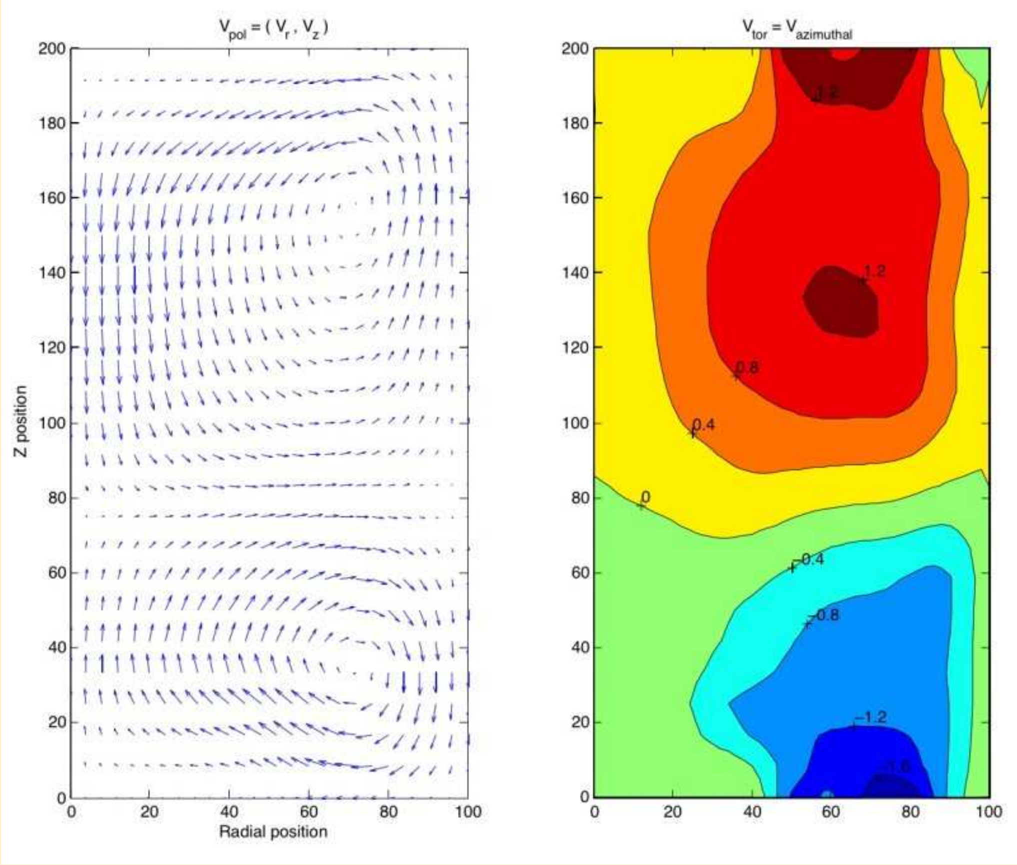


spatial resolution ↑
temporal resolution ↓

temporal resolution ↑
spatial resolution ↓

Measured velocity flow

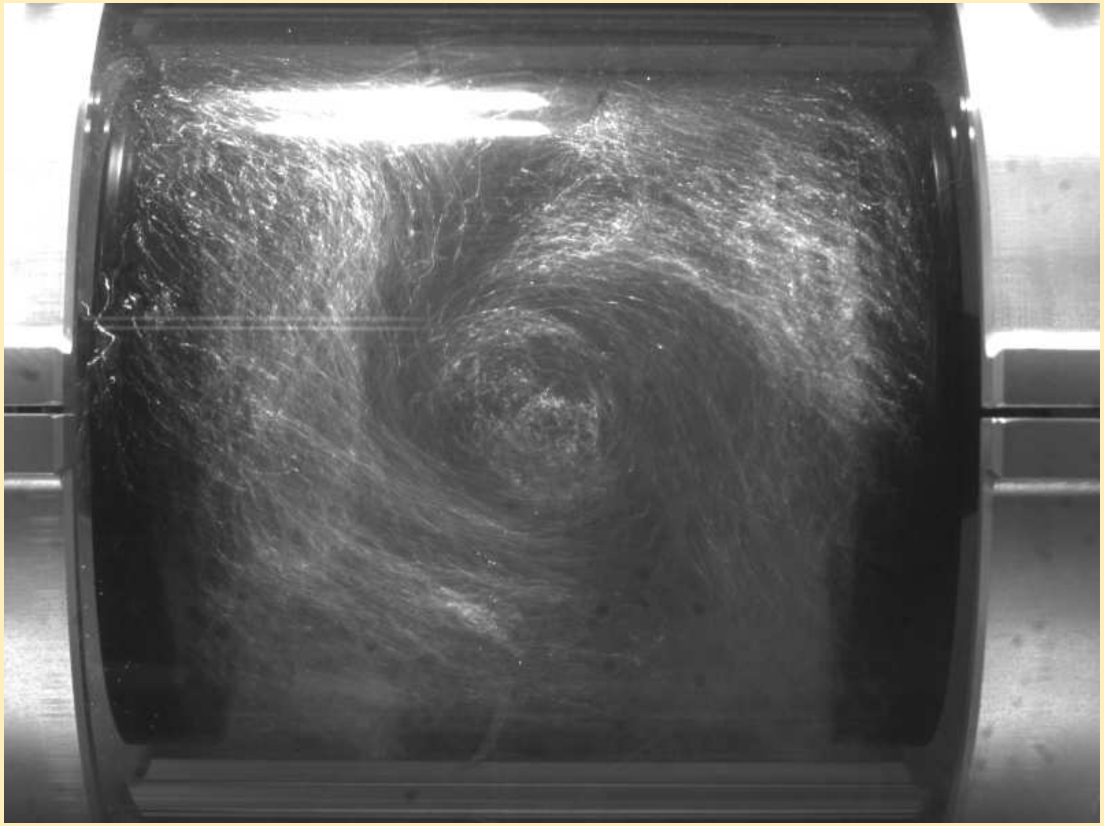
Time averaged (LDA):



Not symmetric around $z = 100$

Vortices

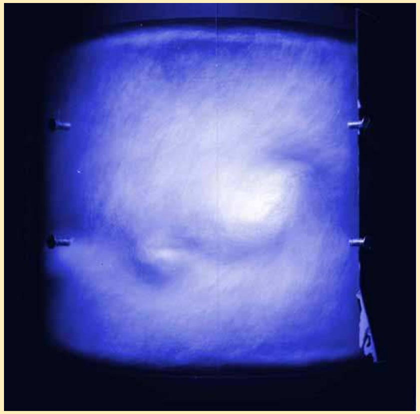
$f_{prop} = 4.75$Hz



Vortices

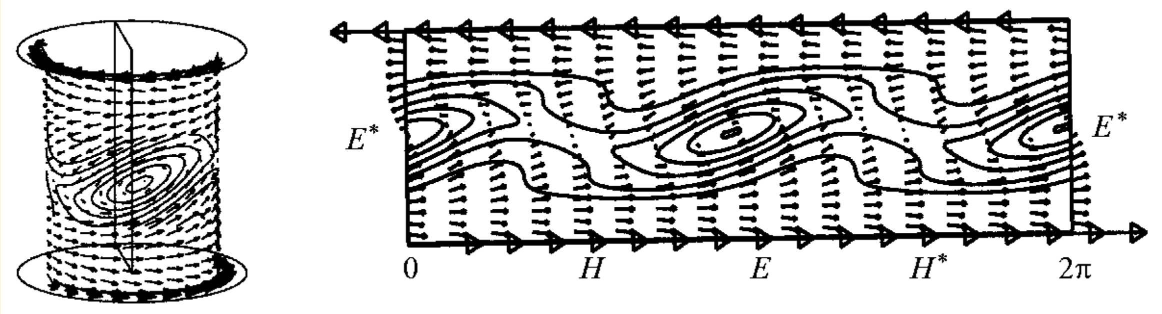
Vortices in similar configurations:

- Experiments:



Louis Marié, Ph.D thesis, CEA / Université Paris 7, France.

- Numerics:



C.Nore, L.S.Tuckerman, O.Daube, S.Xin. J. Fluid Mech **977** (2003) p.51

Dissymmetry

Why is there a dissymmetry?

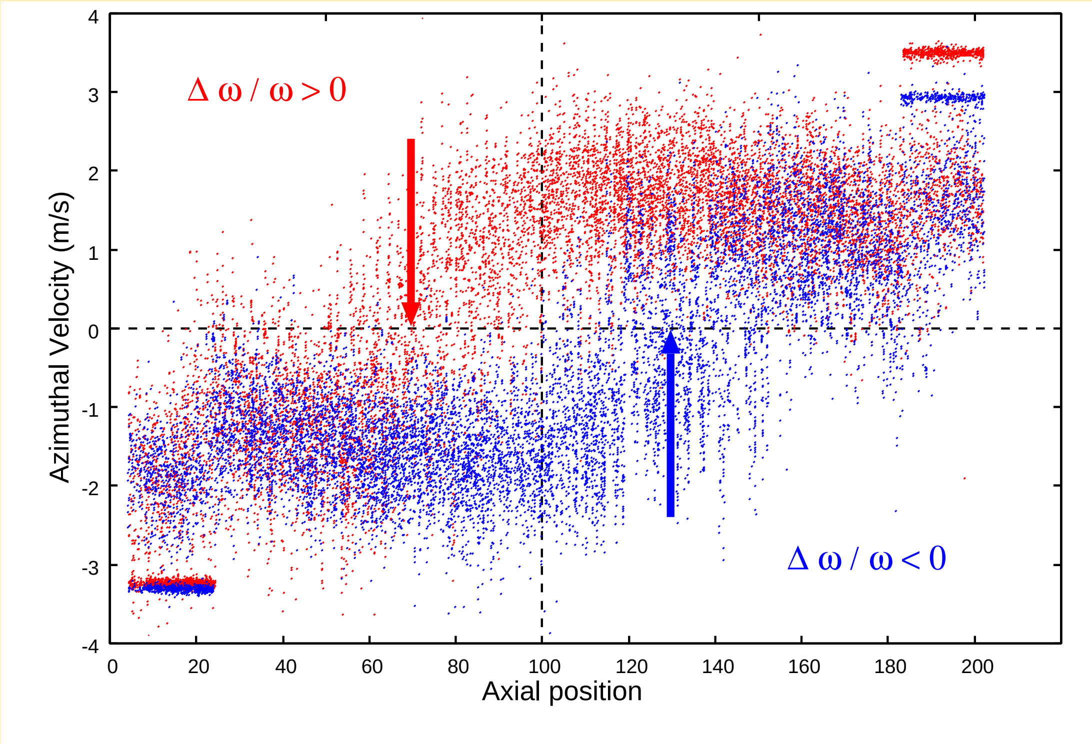
Experimental inhomogeneities?

- Propellers are identical to 10μm
- Propeller velocities are constant in time (fluctuations are less than 0.5%)
- Deviations from real to expected values in spatial dimensions are less than one mm.
- Difference between frequencies of both propellers is below 0.5%

Dissymmetry

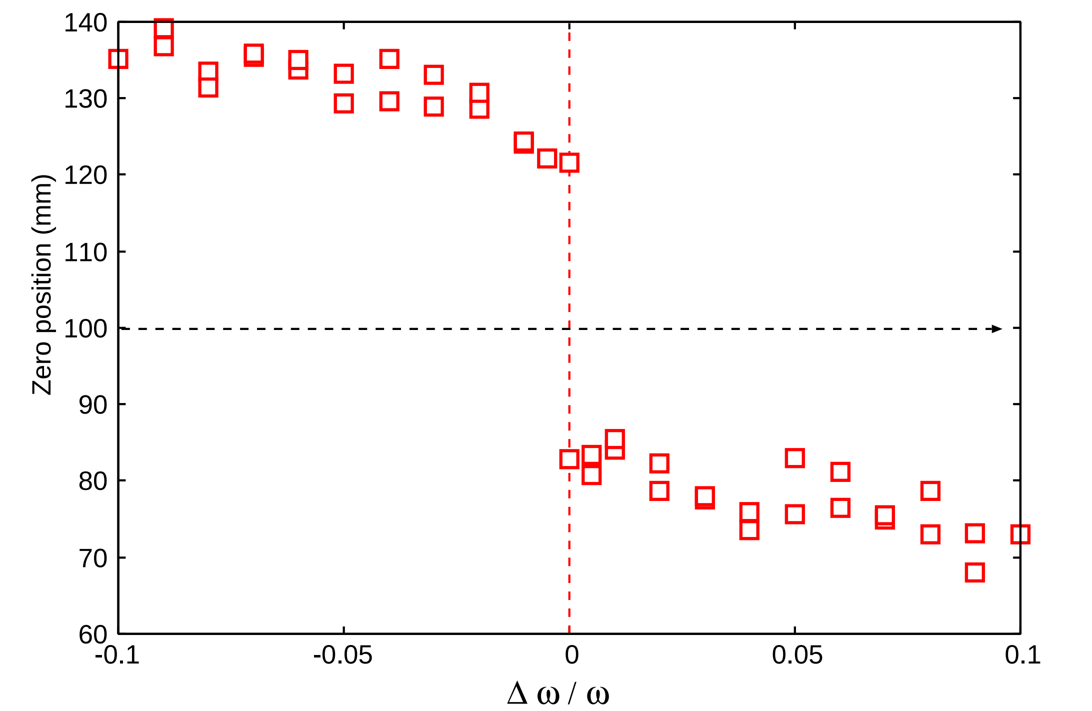
Test: $f_L = const, f_R = f_L \pm \Delta f$

$$f_L > f_R$$



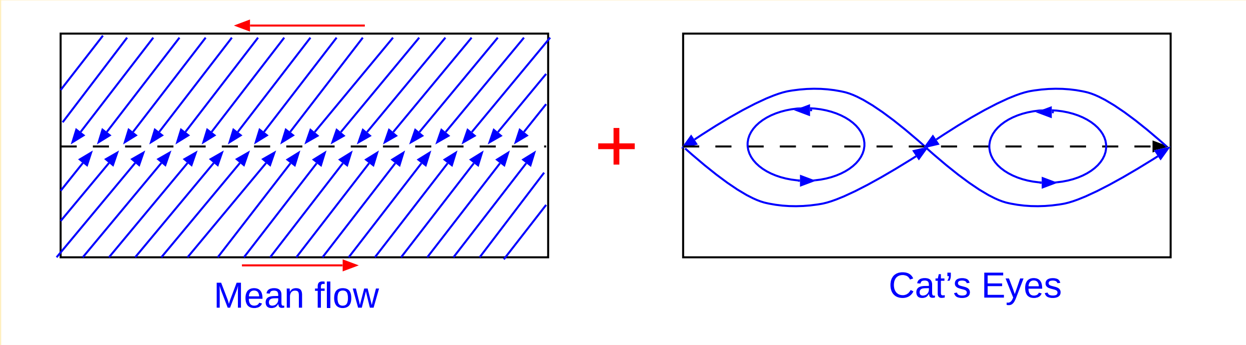
Dissymmetry

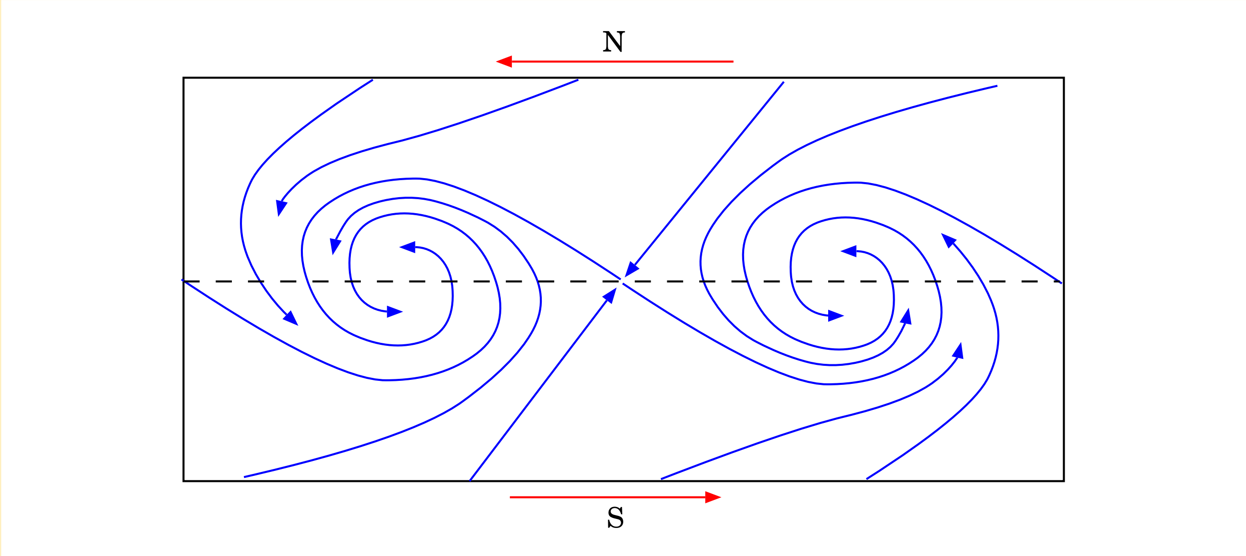
When $\Delta f \longrightarrow 0$:



Vortex velocity

In a symmetric flow ($[\theta, z]$ plane at $r = R$):

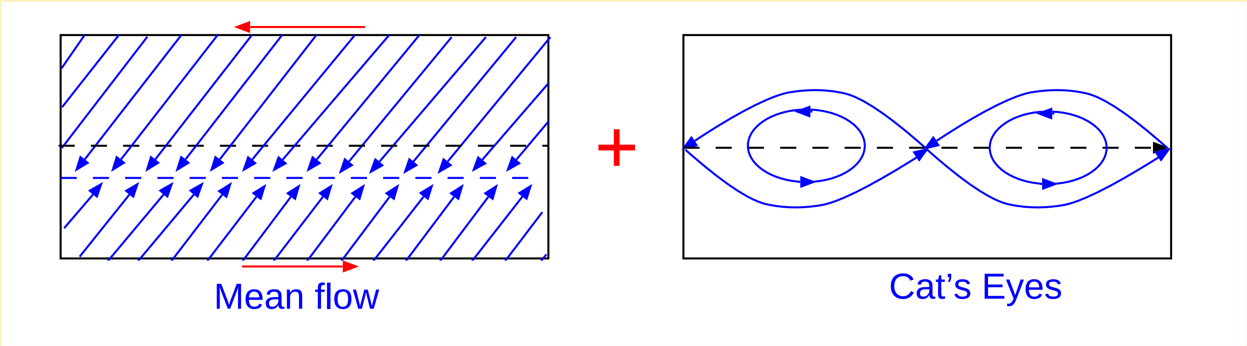


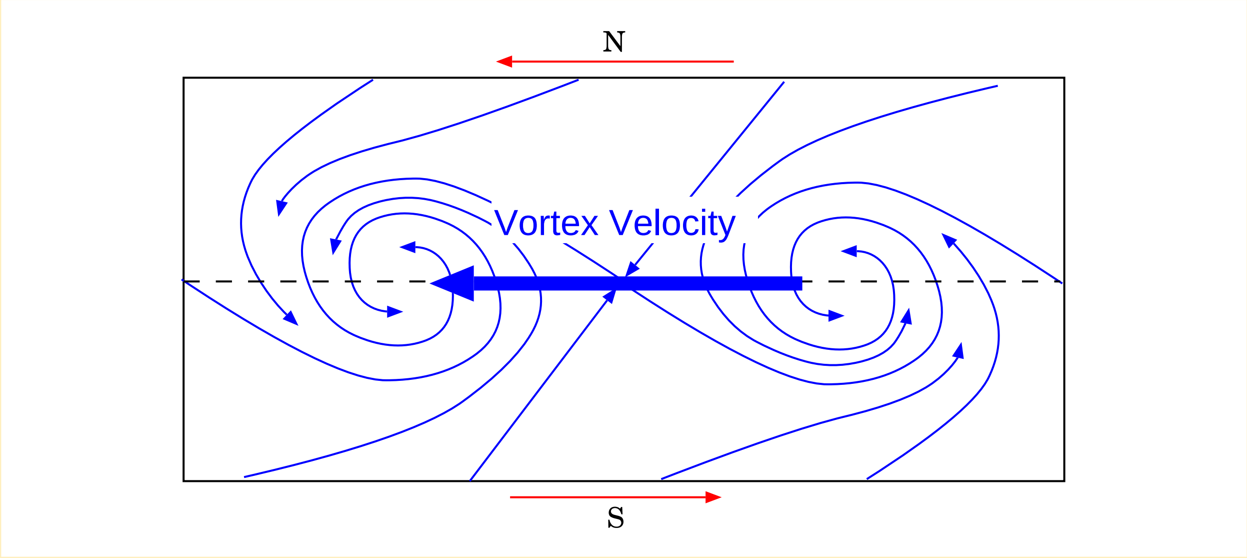


Steady vortices

Vortex velocity

In a non-symetric flow ($[\theta, z]$ plane at $r = R$):

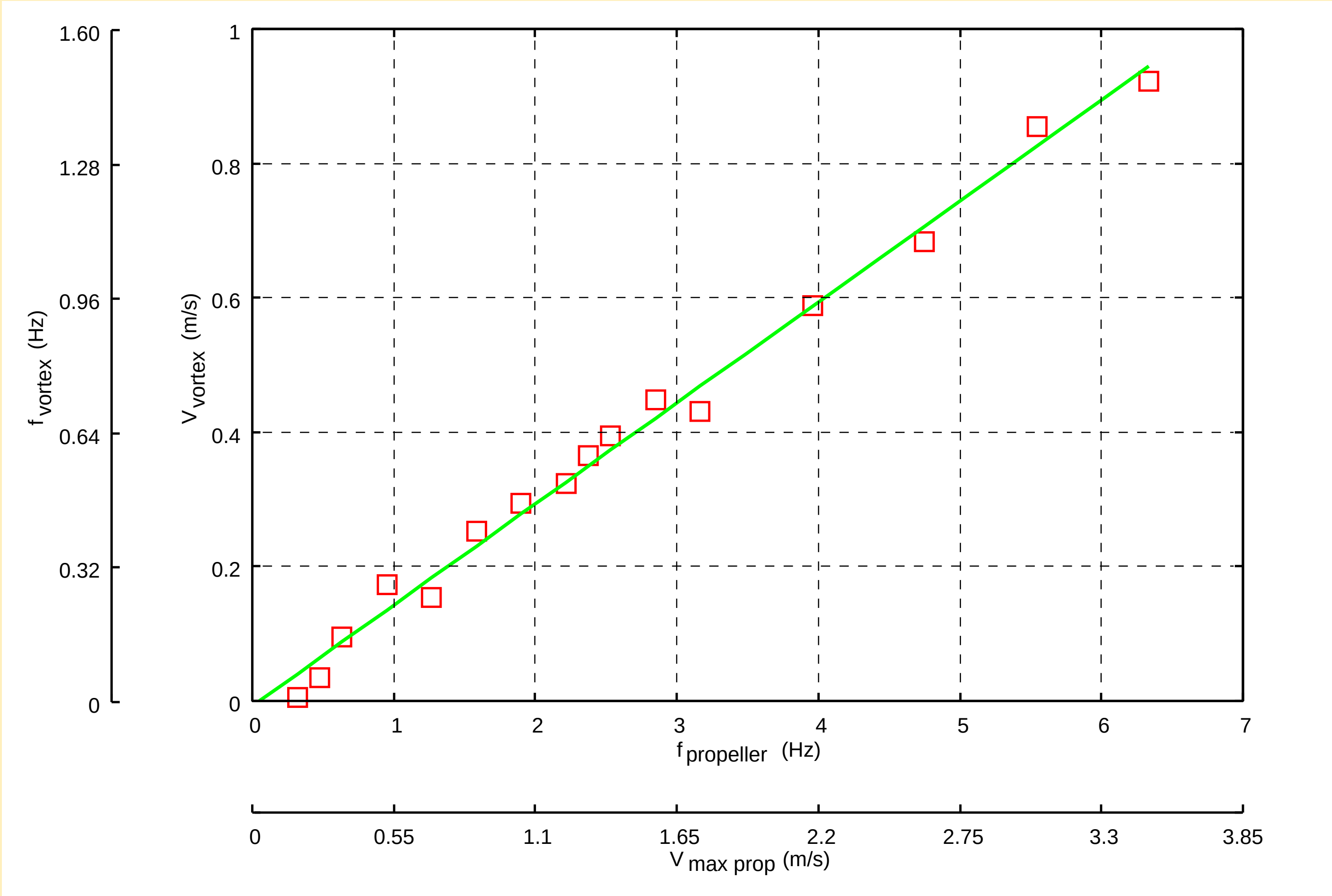




The vortex must move!

Vortex velocity

Absolute velocity:



Vortex velocity

Consequences:

- Relationship between f_{vortex} and $f_{propeller}$ is around 1/3.
- A peak in the power spectrum appears around $f_{propeller}/3$
- Two solutions allowed for $\Delta f_{prop} = 0$
- No steady vortices have been found.

Kinematic dynamo

Main characteristics:

- Pseudo-spectral code:
  - Finite differences in r
  - Periodic (Fourier) in θ, z
- 5^{th} order in space
- Single-step mixed Adams-Bashforth/Adams-Moulton scheme (2^{nd} order)

$$\vec{B}(\vec{s},t) = \sum_{n,m} \vec{b}_{n,m}(r) \exp\left[i(m\theta + n2\pi z/H)\right]$$

Kinematic dynamo

TIme-dependent velocity fields:

- Rotating vortices? $\rightarrow$ PIV
- Slowly evolving axisymmetric flows:

$$u(t) = \left(\frac{u_1 + u_2}{2}\right) + \left(\frac{u_1 - u_2}{2}\right)\sin(\omega t)$$

where u_1 and u_2 are dynamo-producing velocity fields.

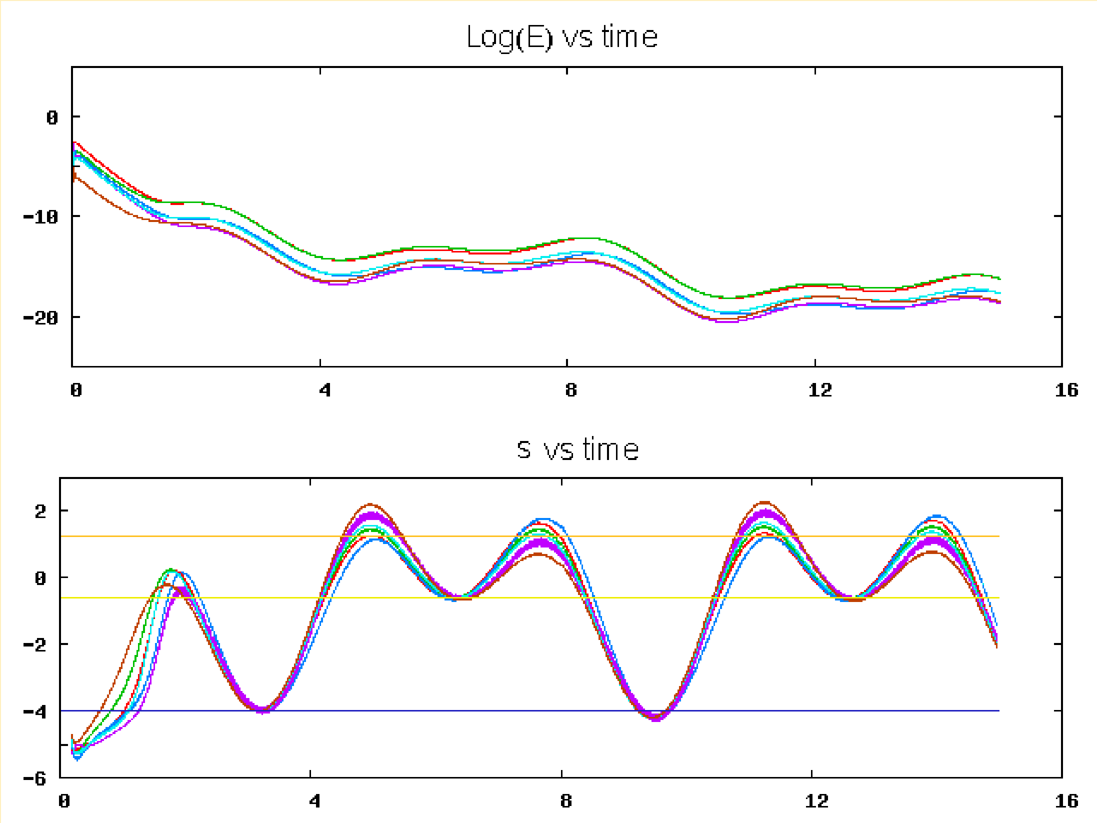
Output:

- Magnetic energy growthrates:

$$E_{m,n} = e^{\sigma_{n,m} t}$$

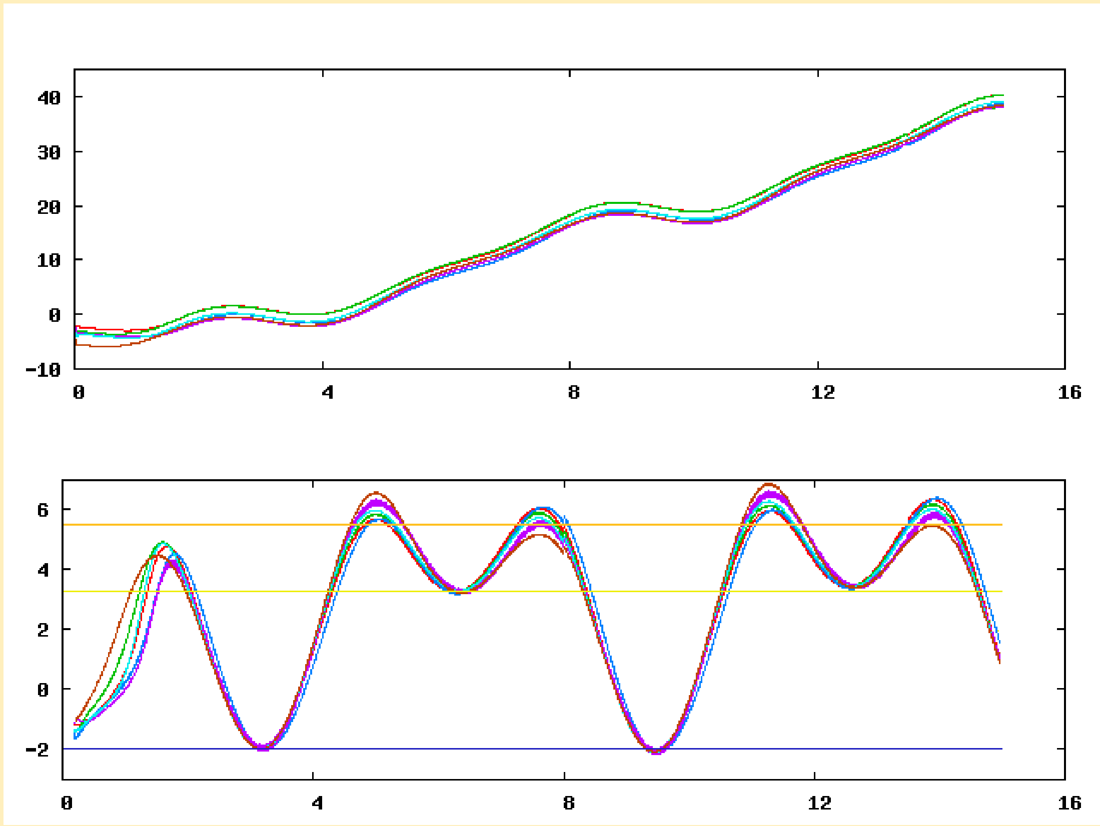
Magnetic energy evolution

$$f = 1 \qquad Rm = 80$$



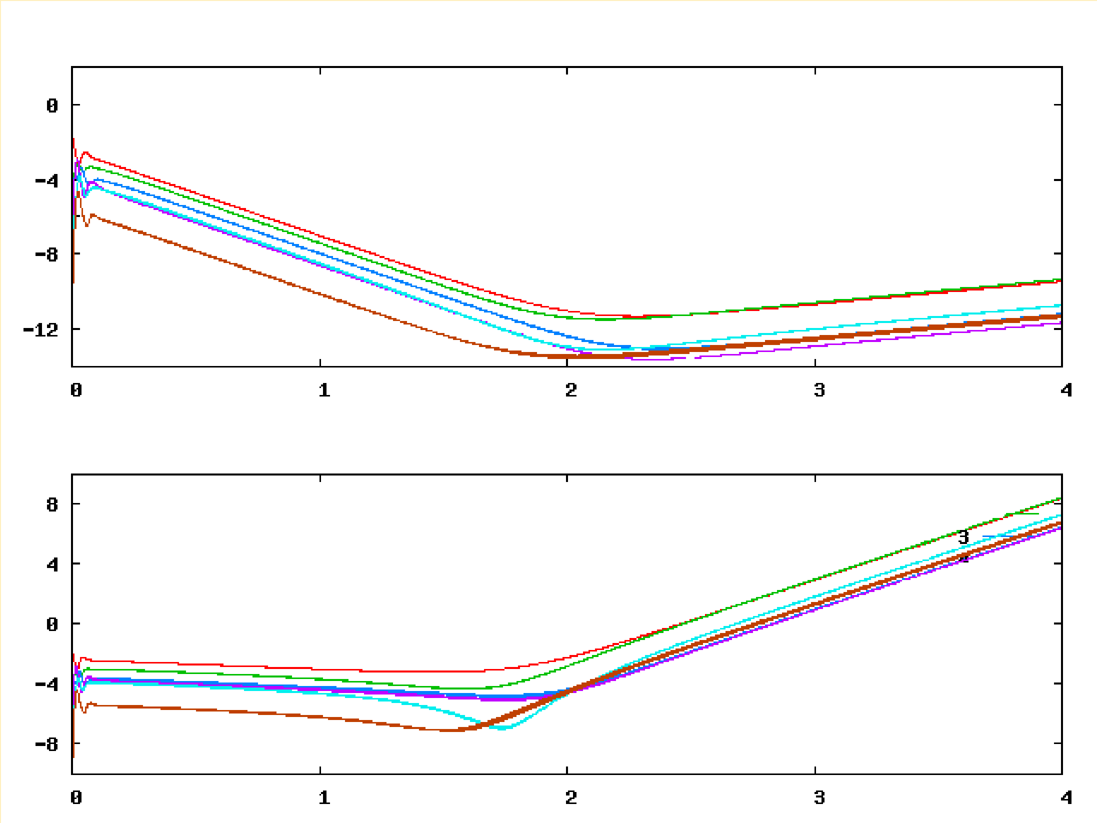
Magnetic energy evolution

$f = 1$ $\qquad$ $Rm = 100$



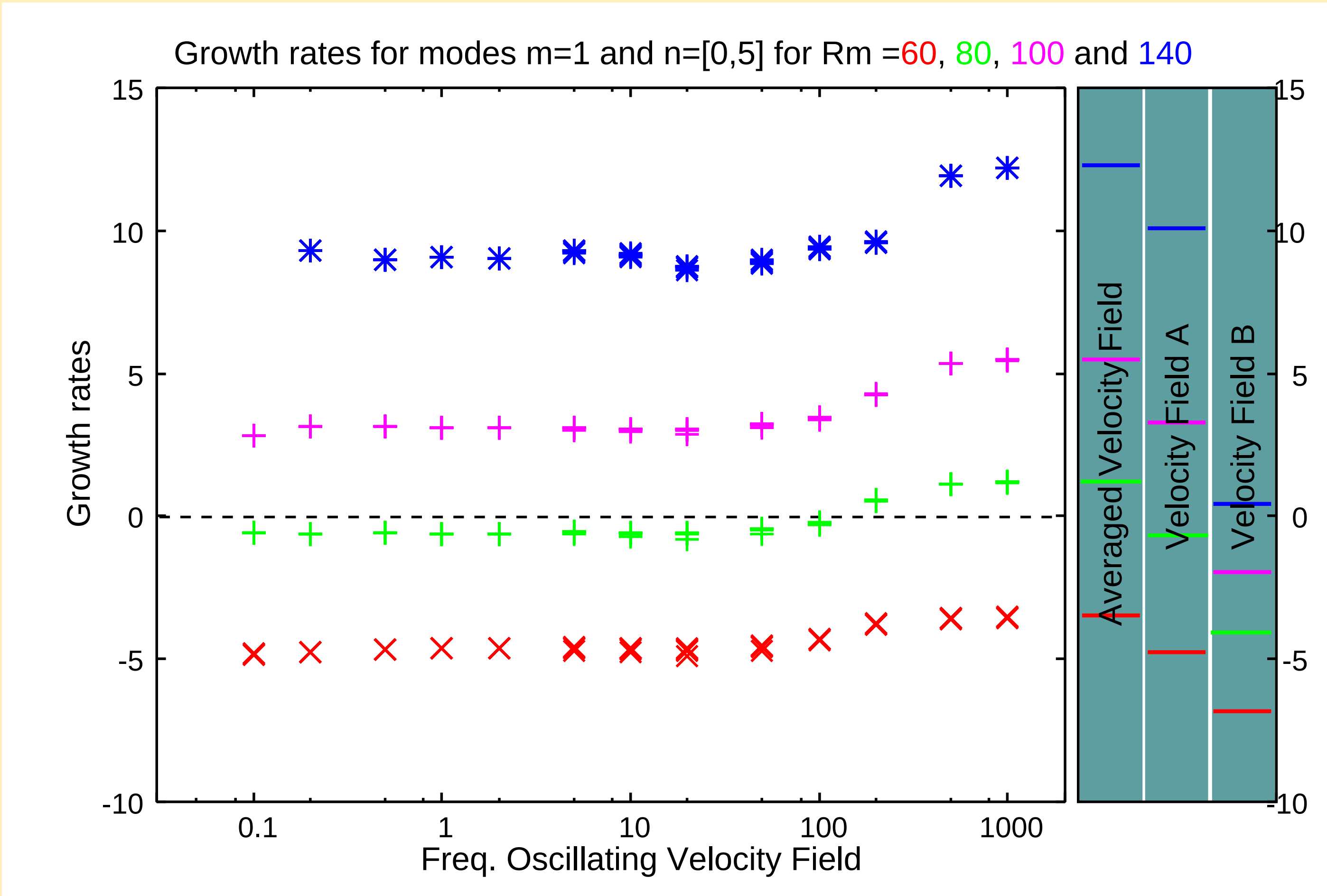
Magnetic energy evolution

$f = 1000 \qquad Rm = 80, 100$



Growth rates

Growth rates *vs.* the frequency



Conclusions

Hydrodynamics analysis:

- Slow scales can be very important $\rightarrow$ vortices
- Vortices $\rightarrow$ bistable (hysteresis?)
- $f_{vort} \sim f_{prop}/\kappa$, where $\kappa \in [3, 5]$

MHD analysis:

- Using axisymmetric time-evolving flows
  dynamo threshold is increased
- This effect is more important for low frequencies
- It dissapears for large frequencies