Nonequilibrium dynamics of inertial particles in random flows

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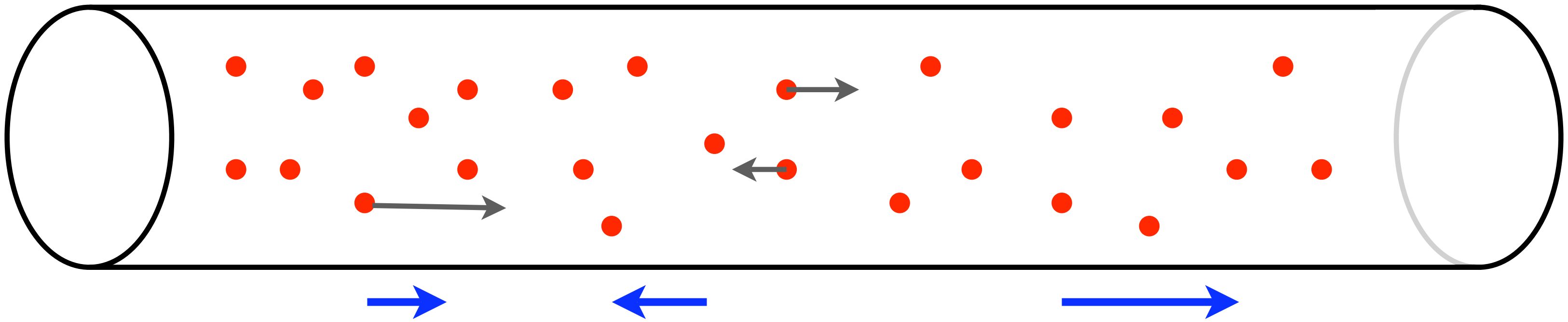
Outline

- Description of the model
- Qualitative picture
- Methods and results

Inertial particles in one dimension



Inertial particles are placed in one dimensional random fluid flow. Because of the inertia the velocity of particles is different from the velocity of the flow.

After some time the breakdowns happen, and particles with different velocities meet at the same point. The problem turns from hydrodynamic to kinetic.

Lagrangian vs Eulerian volumes

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Sometimes the markers pass through each other and a folding appears:

The Lagrangian volume grows, the Eulerian stays constant

Lagrangian concentration

The Lagrangian concentration is defined as an inverse distance between neighbour markers:

$$n = \frac{1}{|R|}$$

We are interested in the temporal behavior of the concentration moments:

$$\langle n^{-k} \rangle \propto e^{\tilde{\gamma}_k t}$$

Basic equations

The dynamics of a single particle (marker) is described by the Newton equation:

$$\frac{d^2 r_s}{dt^2} = -\tau^{-1}\left(u(t, r_s) - \frac{dr_s}{dt}\right)$$

For the distance between two markers we have

$$\frac{dR}{dt} = v$$

$$\frac{dv}{dt} = -\tau^{-1} v + \zeta R$$

$$R = r_{s+1} - r_s, \quad v = v(t, r_{s+1}) - v(t, r_s), \quad \zeta = \frac{\partial v}{\partial r}$$

Basic equations

The velocity gradient field is a random gaussian process:

$$\langle \zeta(t)\zeta(t')\rangle = 2D\delta(t-t')$$

Stokes number $St = D\tau$ is a degree of inertiality.

We will consider small particles with low Stokes numbers. This limit corresponds to the weakly compressible effective flows. However, the folding events can not be accounted in the framework of usual perturbative approach.

Properties of the solution

One can rewrite the equations in the following form

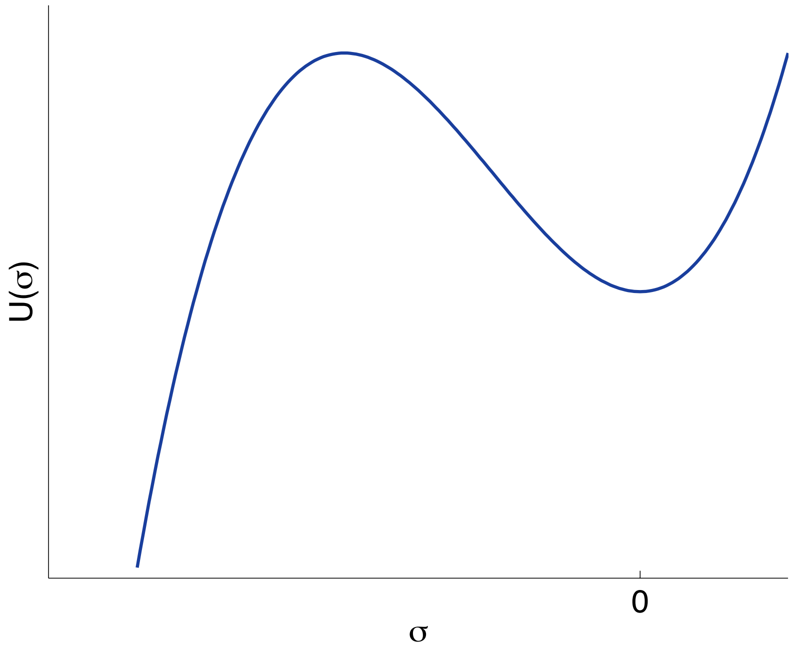
$$\dot{R} = \sigma R$$

$$\dot{\sigma} = -\sigma/\tau - \sigma^2 + \zeta$$

with $\sigma = R/v$

There is a finite probability of breakdown in a finite time. The probability distribution does not have a form of Gibbs distribution and corresponds to a finite flux.

Properties of the solution



There is formal analogy with the Langevin equation:

$$\dot{\sigma} = -U'(\sigma) + \zeta$$

However the usual Gibbs solution $P(\sigma) = C \exp\left[-U(\sigma)/D\right]$ can not be normalized

The real stationary solution corresponds to a finite flux:

$$P(\sigma) = \frac{F}{D} \exp\left[-\frac{U(\sigma)}{D}\right] \int\limits_{-\infty}^{\sigma} \exp\left[\frac{U(\sigma')}{D}\right] d\sigma'$$

Properties of the solution

It is possible to find the probabilities of having a breakdown at time *T*:

$$P(T) \propto \exp\left(-\frac{c\tau^2}{DT^3}\right), \quad T \ll \tau$$

$$P(T) \propto T \exp\left(-\frac{1}{6D\tau}\right), \quad \tau \ll T \ll \tau \exp\left(\frac{1}{6D\tau}\right)$$

Properties of the solution

One can derive the closed set of equations for the moments in the case of delta-correlated velocity field:

$$\dot{R}_{l,k} = -lR_{l,k}/\tau - (l-k)R_{l+1,k} + l(l-1)DR_{l-2,k}$$

$$R_{l,k} = \langle \sigma^l R_k \rangle = \langle v^l R^{k-l} \rangle$$

Substituting $R_{l,k} \propto \exp(\gamma_k t)$ one obtains the set of algebraic equations for γ_k

$$\gamma_1(\gamma_1 + \tau^{-1}) = 0$$

$$\gamma_2(\gamma_2 + \tau^{-1})(\gamma_2 + 2\tau^{-1}) = 4D$$

For even k these equations give the exponents of concentration moments growth rates.

Properties of the solution

For $k = 1$ we have:

$\langle R \rangle = const$ $\Rightarrow$ Conservation of Eulerian volume

$\langle n^{-1} \rangle = \langle |R| \rangle \propto e^{\tilde{\gamma}_1 t}$ $\Rightarrow$ Growth of Lagrangian volume

It is possible to find the exact expression $\tilde{\gamma}_1 = \frac{St}{\pi} \exp\left[-\frac{1}{6St}\right]$

The result is strongly non-perturbative in the Stokes number

Main results

- Stationary distribution for $P(\sigma)$
- Algebraic equations for $\gamma_{2k} = \tilde{\gamma}_{2k}$
- Exact expression for $\tilde{\gamma}_1$