Non-equilibrium statistical mechanics and turbulence
Workshop 15-21 July 2006, Warwick

Comparative experimental study of quasi-2D turbulence in fluids and plasma

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Motivation

Plasma physics perspective

- Understanding improved plasma confinement
- Physics behind modifications of turbulence during confinement transitions

2D fluid turbulence

- Conditions and thresholds for spectral condensation of 2D turbulence
- Modification to the 2D turbulence spectra in the presence of condensate

Content

- **Transitions to improved plasma confinement (self-organization in quasi-2D plasma turbulence as a paradigm for L-H transitions)**
- **Spectral transfer in plasma turbulence**

  (i) Single-field description; (ii) Power transfer analysis; (iii) Inverse energy cascade
- **Modifications in plasma turbulence consistent with spectral condensation in plasma turbulence**

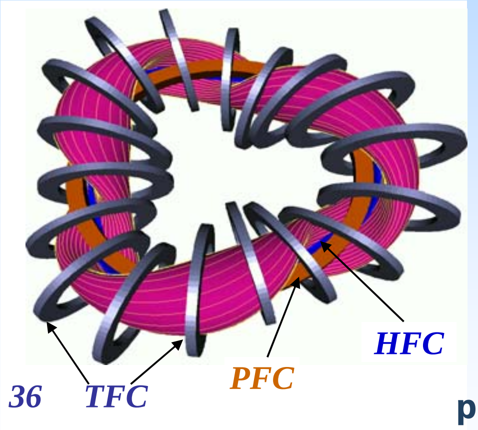
  (i) Transitions from random-phase cascade-type transfer to non-local coherent transfer; (ii) Zonal flows and transport barriers
- **Spectral condensation in 2D fluid turbulence**

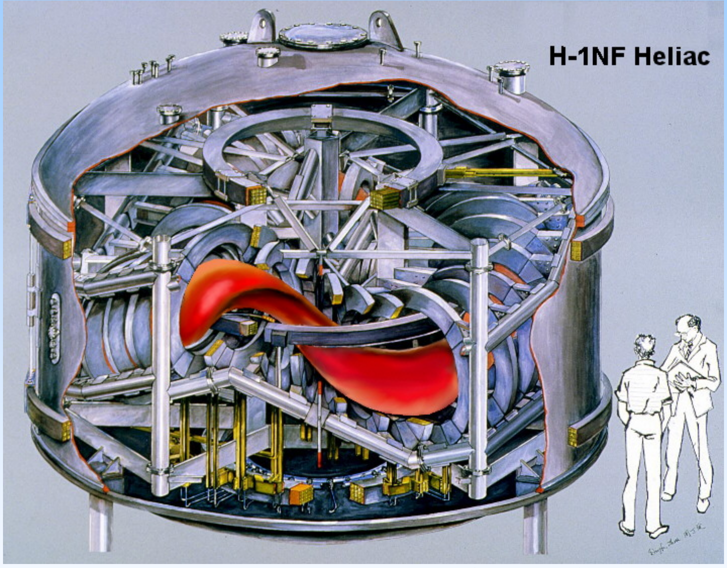
  (i) Introduction; (ii) Previous experiments, modified setup; (iii) evolution of turbulence during condensation; (iv) Approaches to threshold studies (reduced linear damping; role of induced anisotropy…)
- **Summary**

Plasma experiment : H-1 heliac – helical axis stellarator (ANU)

Plasma parameters:

$R = 1$ m, $\langle a \rangle \sim 0.2$ m, $N = 3$,
low magnetic shear, $B < 0.1$ T, Argon
$T_i \sim 30$ eV, $T_e \sim 10$ eV, $n_e \sim 1\text{x}10^{18}$ m^{-3},





Dimensionless plasma parameters similar to the edge region of other "fusion-relevant" machines:

$$\rho^* = \rho_i / (n_e / \nabla n) = 0.5 - 1$$

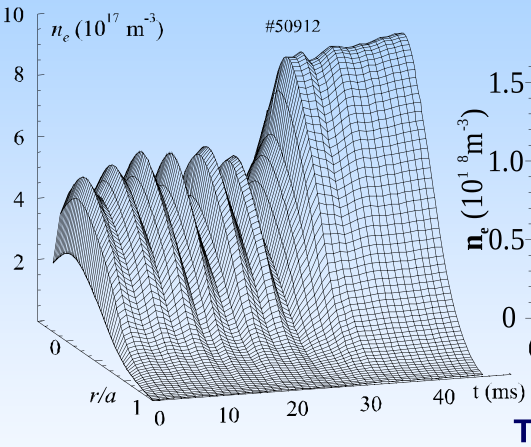
$$\upsilon^* = \upsilon_{ei}^{eff} / \omega_{be} \approx 1$$

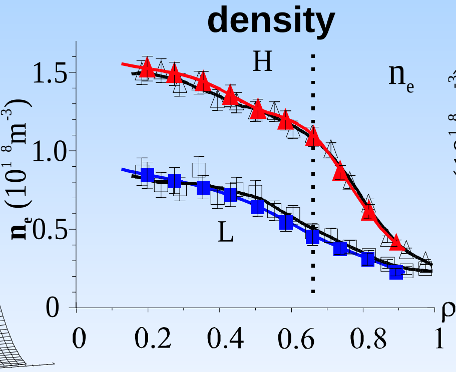
$$\beta \approx 0.01$$

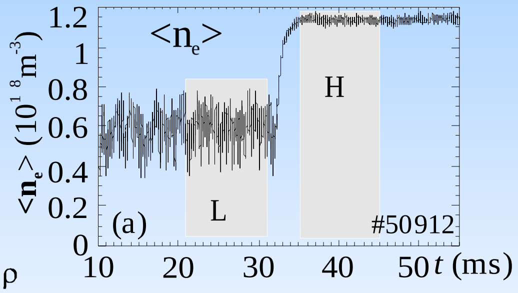
Low temperatures allow probes, visible spectroscopy ... to be used to characterize plasma and turbulence with better spatial / temporal resolution

Confinement transitions in H-1 plasma

Low-to-high transitions in the H-1 heliac







ASDEX Tokamak:

Wagner et al. Phys.Rev.Lett. (1982)

H-1 stellarator:

Shats et al. *Phys.Rev.Lett.* (1996)

Punzmann & Shats, *Phys.Rev.Lett* (2004)

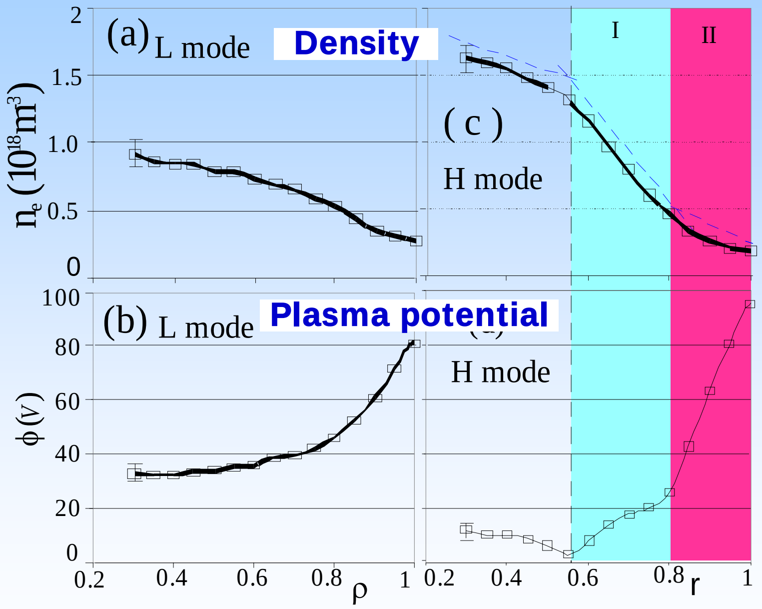
Transitions in H-1 similar to tokamak / stellarator L-H transitions:

- **Increased plasma pressure**
- **Reduction in turbulence**
- **Increase in the Er**
- **Decreased diffusion**
- **Pedestal formation**

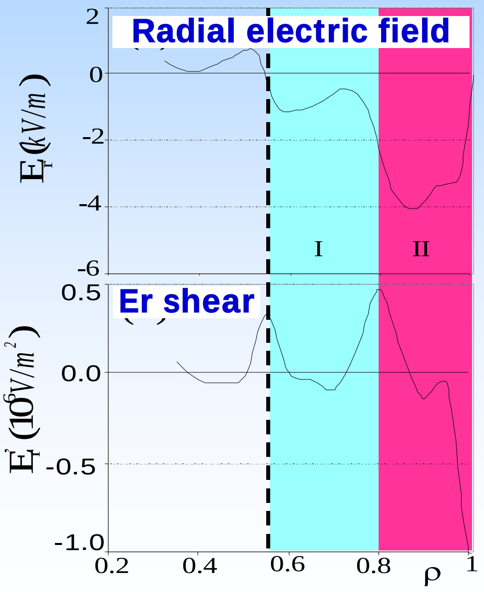
- **H-mode regime is the key for successful operation of ITER**
- **Pedestal control – crucial tool for shaping plasma (confinement + stability)**

Formation of n_e - pedestal in H-mode

Radial profiles in L and H modes



ExB flow velocity profile is "corrugated" in the region of the pedestal



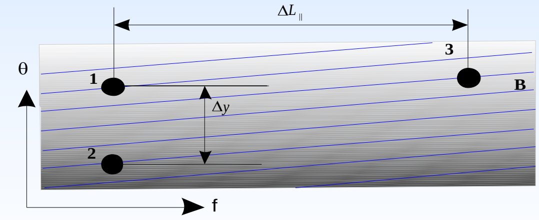
Strong E_r shear regions mark top / foot of the density pedestal

Turbulence spectrum in L-mode

- Broadband spectrum ($P \sim f^{-6}$)
- Stationary (low frequency <0.6 kHz) zonal flows
- Geodesic acoustic modes f = 4 – 5 kHz

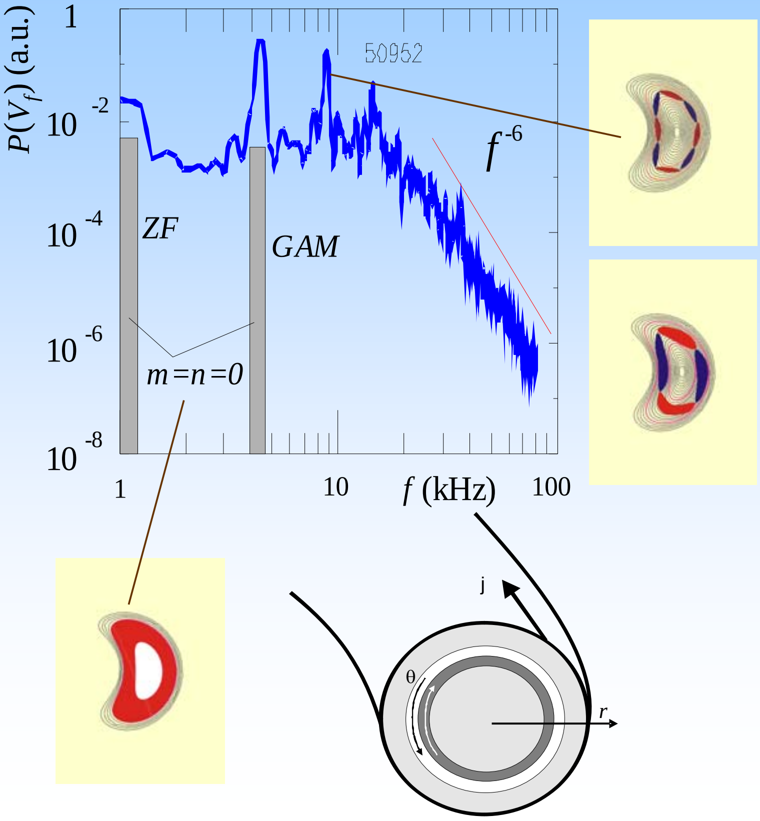
$$\omega_{GAM} \approx c_s / L_{\|}$$

Good agreement with GAM frequency corrected for stellarator geometry [Watari et al. PoP 2005]



$$m(f) = k_\theta(f) r = r(\Delta\varphi_{12}(f)/\Delta y)$$

$$\Delta\varphi_{13}(f) = k_{\|}(f)\Delta L_{\|} + k_\theta(f)\Delta y_{13}$$



Non-linear power transfer analysis

If single-field description of turbulence is valid,

mode coupling and the direction of energy cascading is described by the wave kinetic equation

[e.g Ritz et al. *Phys. Fluids* **B1** (1989)]

change of the spectrum is due to the linear growth, dispersion and wave-wave coupling coefficient Λ

$$\frac{\partial\phi_k(t)}{\partial t} = (\gamma_k + i\omega_k)\phi_k(t) + \tfrac{1}{2}\sum_{k=k_1+k_2}\Lambda_{k_1,k_2}\phi_{k_1}(t)\phi_{k_2}(t)$$

⇩

$$P_k = \phi_k(t)\phi_k^*(t)$$

⇩

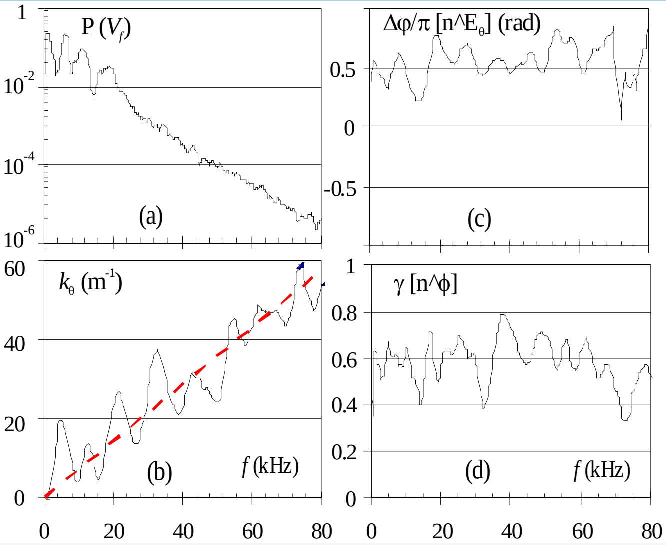
$$\frac{\partial P_k}{\partial t} = 2\gamma_k P_k + \sum_{k=k_1+k_2} T_k(k_1,k_2)$$

γ_k is the linear growth (damping) rate

T_k is the non-linear power transfer function - quantifies the energy exchanged between the different waves in the spectrum due to 3-wave interactions :

$$T_k(k_1,k_2) = \mathrm{Re}[\Lambda_k^Q(k_1,k_2)\langle\phi_k^*\phi_{k_1}\phi_{k_2}\rangle]$$

Justification of single field description of quasi-2D plasma turbulence



- ☐ "Linear" *k-f* dispersion justifies time-domain analysis
- ☐ Density and potential fluctuations are in-phase – adiabatic response $\tilde{E}_\theta = -\nabla_\theta \tilde{\phi}$
- ☐ Density and potential fluctuations are highly coherent (~60%)

$$\tilde{n}_e = \tilde{\phi} + \cancel{\delta n_e} \qquad \tilde{n}_e = n_0 \exp(e\tilde{\phi}/T)$$

$$\partial n_i/\partial t + \mathbf{V}_E \cdot \nabla n_i + \nabla \cdot (n_i \mathbf{V}_P) = 0 \qquad n_i = n_0 + \tilde{n}_i$$

Polarization drift nonlinearity ~ *ExB* drift nonlinearity when

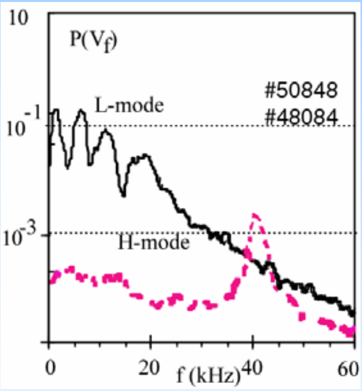
$$k_\perp \rho_s = \delta = c_s/(L_n \nu_e)$$

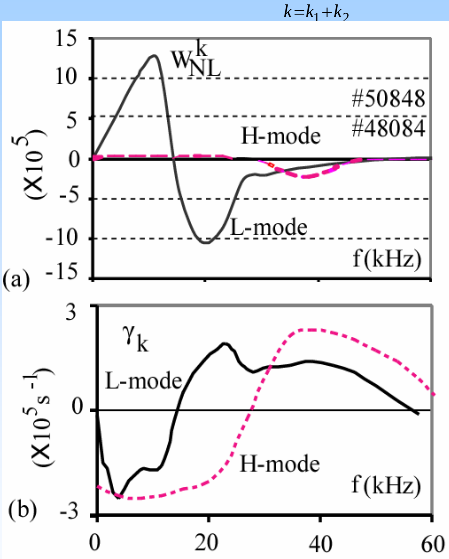
In our conditions $k_\perp \rho_s (= 0.5 - 2.5) >> \delta (\approx 0.1)$ **- polarization drift dominates**

Spectral energy transfer turbulence

$$W_{NL}^k \approx (1 + k_\perp^2) \sum_{k=k_1+k_2} T_k(k_1, k_2)$$

$$\frac{\partial P_k}{\partial t} = 2\gamma_k P_k + \sum_{k=k_1+k_2} T_k(k_1, k_2)$$





L-mode: Spectral power (energy) is transferred from high to low wave numbers

H-mode: linear growth rate is stronger, but the energy cascade is weaker

Linear instability at ~ 25 (40) kHz

Produces unstable range (20-40kHz)

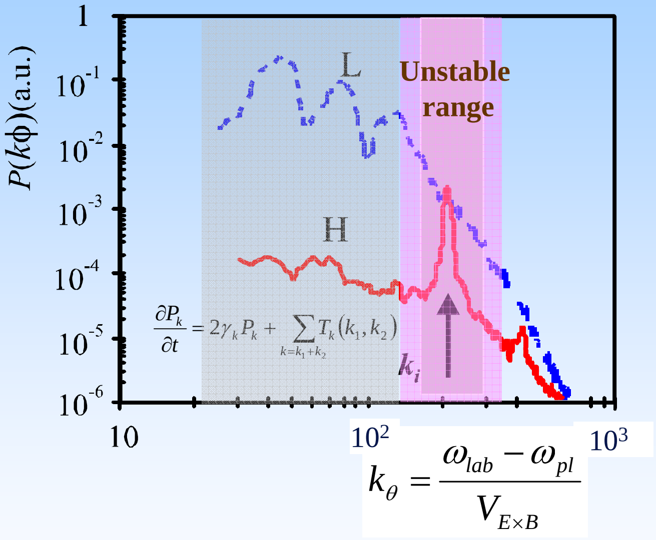
Inverse Energy Cascade

Energy transfer function does not show coherent spectral features

ETF computation requires heavy statistical averaging. Capable of accounting for random-phase 3-wave interactions

Spectral energy transfer in turbulence

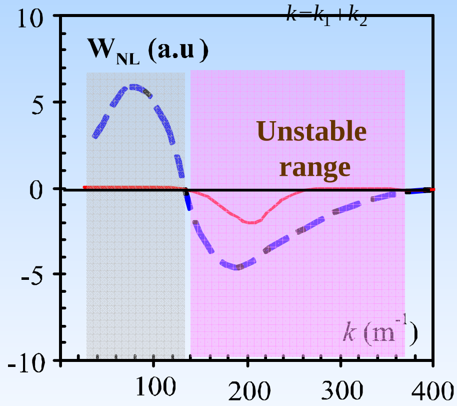
Turbulence spectrum



$$k_\theta = \frac{\omega_{lab} - \omega_{pl}}{V_{E\times B}}$$

Spectral energy transfer function (3-wave interactions)

$$W_{NL}^{k} \approx (1+k_\perp^2) \sum_{k=k_1+k_2} T_k(k_1,k_2)$$



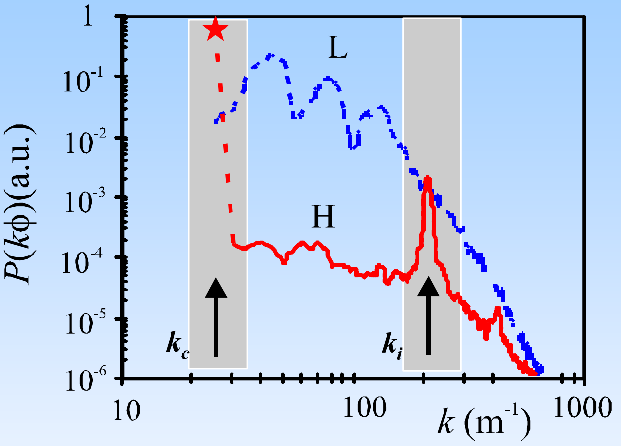
[H. Xia, M. Shats, *Phys. Rev.Lett.* (2003)

Phys. Plasmas (2004)

Applicable if single-field description is justified (n-ϕ fluctuations are in phase), e.g. Hasegawa-Mima-type turbulence

Broadband turbulence is generated via inverse energy cascade

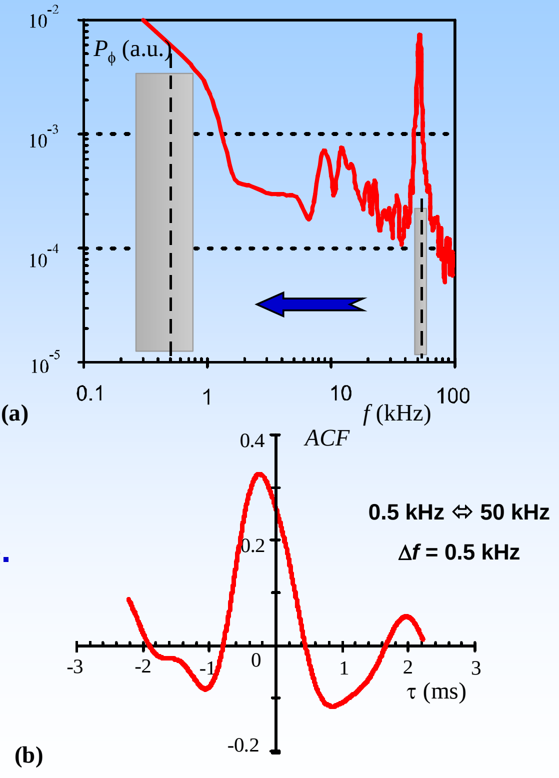
Spectral energy transfer in H-mode



In H-mode intermediate scales are reduced.
Stationary ZF receives energy from unstable range.

[Shats, Xia, Punzmann., *Phys. Rev.E* (2005)]

Hypothesis of redistribution of spectral energy of turbulence into large-scale zonal flow



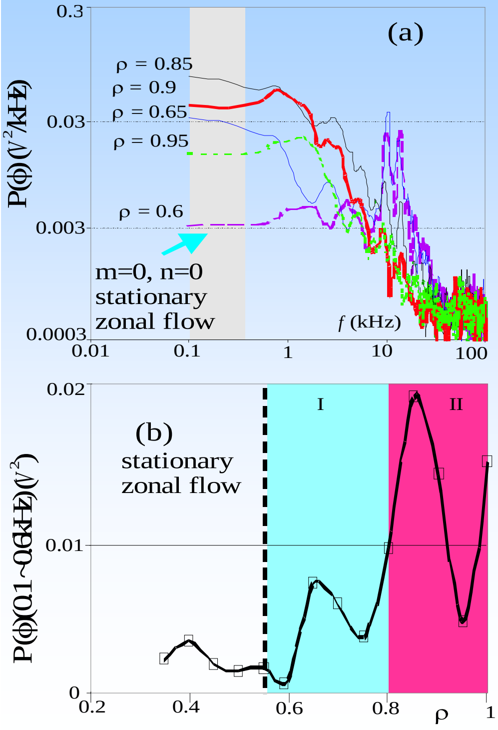
0.5 kHz ⇔ 50 kHz

Δf = 0.5 kHz

f =0.5 kHz lags the f =50 kHz ZF band

Formation of ZF in H-mode pedestal II

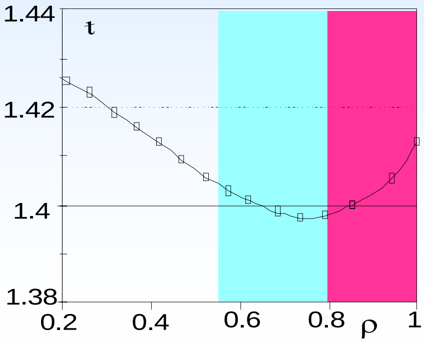
Strong stationary ZF found in the pedestal region

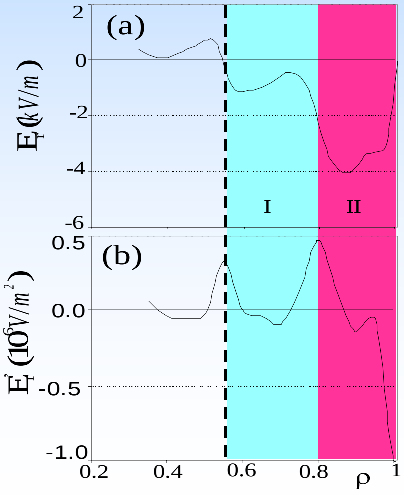


Increased E_r regions coincide with

- rational 7/5 surfaces
- maxima in stationary *f* ~ 0 ZF

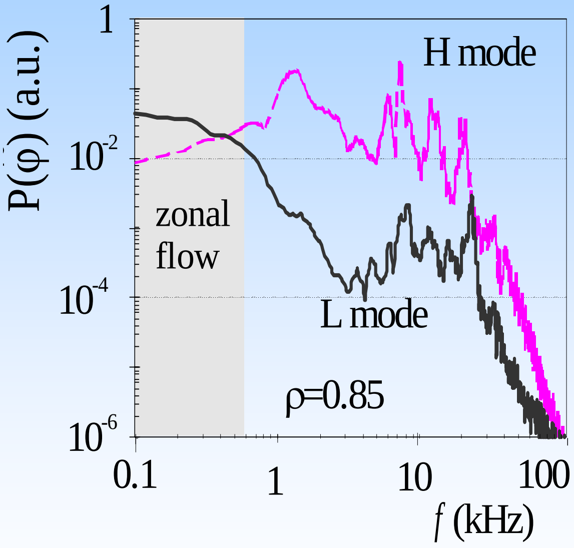
Rotational transform





Evidence in support of spectral condensation in plasma

Strong stationary ZF develop in the pedestal region



Both development of stationary ZF & the BB turbulence reduction might result from redistribution of spectral energy towards lower *k*

Non-local spectral transfer in plasma

[Balk, Zakharov, Nazarenko JETP (1990), Physics Letters A (1990)

Dyachenko, Nazarenko, Zakharov Phys. Letts. A (1992)]:

Hypothesis of a nonlocal nature of drift-wave turbulence =>

Inverse energy cascade enhances spectra at large scales;

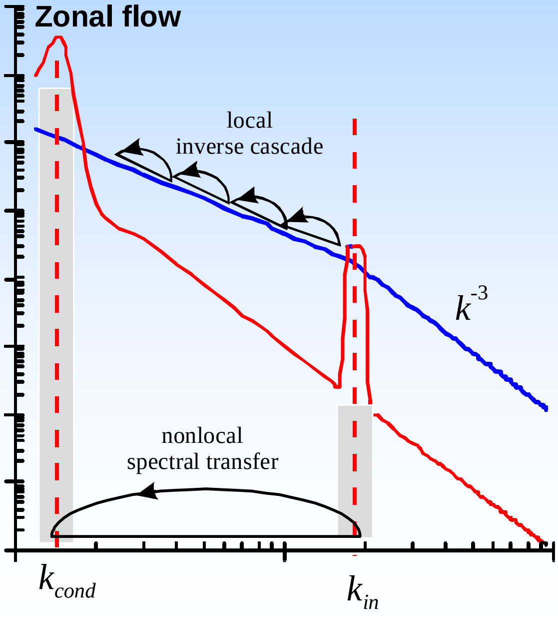
Turbulence becomes nonlocal;

Spectral energy is pumped from the small-scale jet into a zonal flow ;

Intermediate scales are reduced;

Two spectrally unconnected components:
ZF($\boldsymbol{k}_{\mathbf{ZF}}$) and jet ($\boldsymbol{k}_{\mathbf{inj}}$)

Spectral energy delivered from $\boldsymbol{k}_{\mathbf{inj}}$ **to** $\boldsymbol{k}_{\mathbf{ZF}}$ is sufficient to overcome damping of zonal flow



Condensation of turbulence in 2D fluids

The maximum of the energy spectrum lies in the low-*k* range, at k_E, and in the absence of the energy dissipation at large scales can not be constant in time since it accumulates spectral energy $k_E = f(\varepsilon, t)$

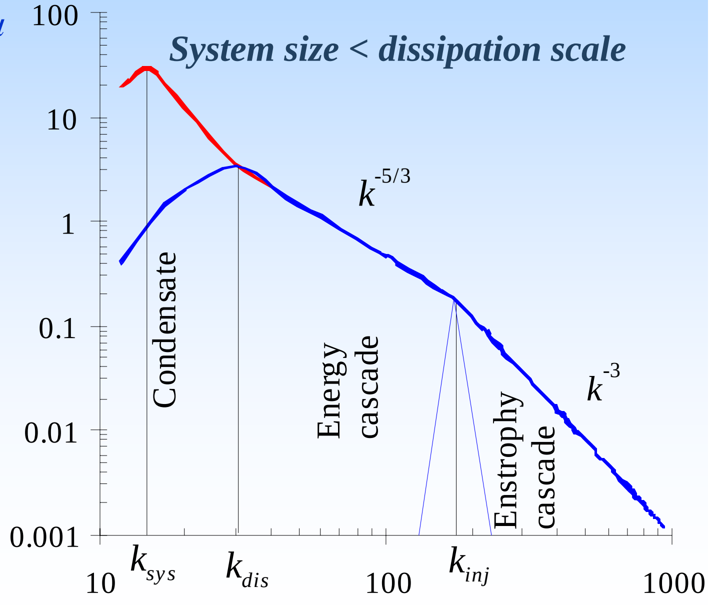
Damping for large scales (e.g.linear damping) μ stabilizes the maximum of the spectrum at the scale $k_{dis} \approx \left(\mu^3/\varepsilon\right)^{1/2}$

At low dissipation in a bound system,
at $k_{dis} << k_{sys}$
spectral condensation occurs into the largest vortex allowed by the system size

Theory: [Kraichnan, 1967- qualitative=> Bose condensate];

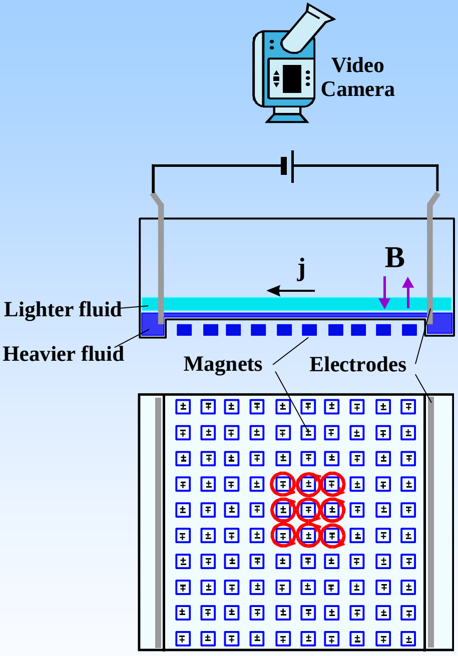
Experiments: [Sommeria (1986), Paret&Tabeling (1998)];

Modelling [Hossain et al.(1983), Smith&Yakhot (1993)…
van Heijst, Clercx, Molenaar (2004-2006)]



Condensation of turbulence in 2D fluid: experiment

Electromagnetically driven turbulence
Thin electrolyte layers. **10 x 10 J**x**B** driven vortices.
Stratified fluid (2 layers, NaCl solution of different density)
[Paret&Tabeling (1998)]



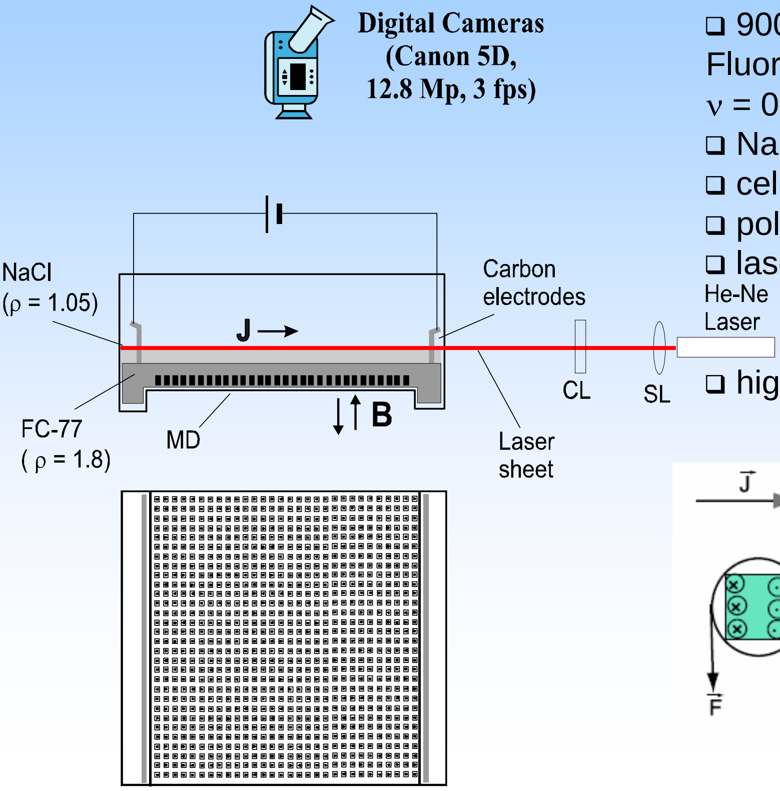
Constantly forced turbulence
(previous studies – decaying, or randomly forced)

Turbulence visualization – particle image velocimetry (PIV), latex particles (~0.1mm)

$$v_x = dx/(t_2 - t_1)$$

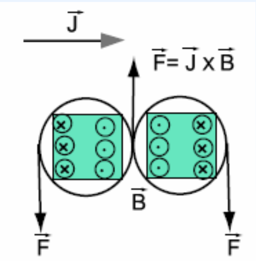
$$v_y = dy/(t_2 - t_1)$$

2D fluid turbulence: experimental setup



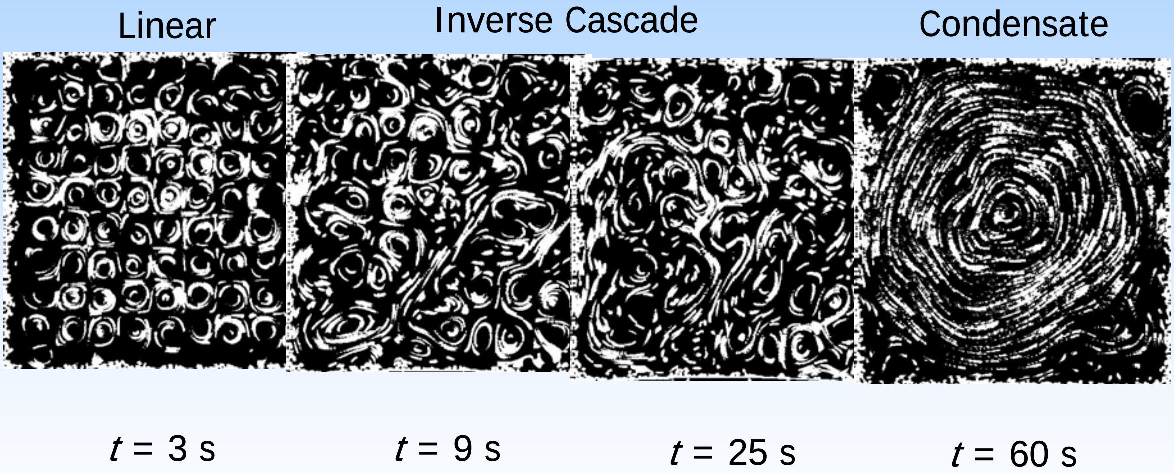
Modified setup, 30x30 vortices

- ❑ 900 rare-earth magnets submerged in Fluorinert FC-77 (resistivity = 2×10^{15} Ohm cm; ν = 0.7 centistokes; density = 1780 kg/m^3)
- ❑ NaCl solution conducting layer
- ❑ cell-size 0.3 x 0.3 m^2
- ❑ polyamid particles (1.04 sp. gravity, 50 μm)
- ❑ laser Particle Image Velocimetry
- ❑ high-resolution camera (4368x2912 pixels)



Flow evolution during spectral condensation

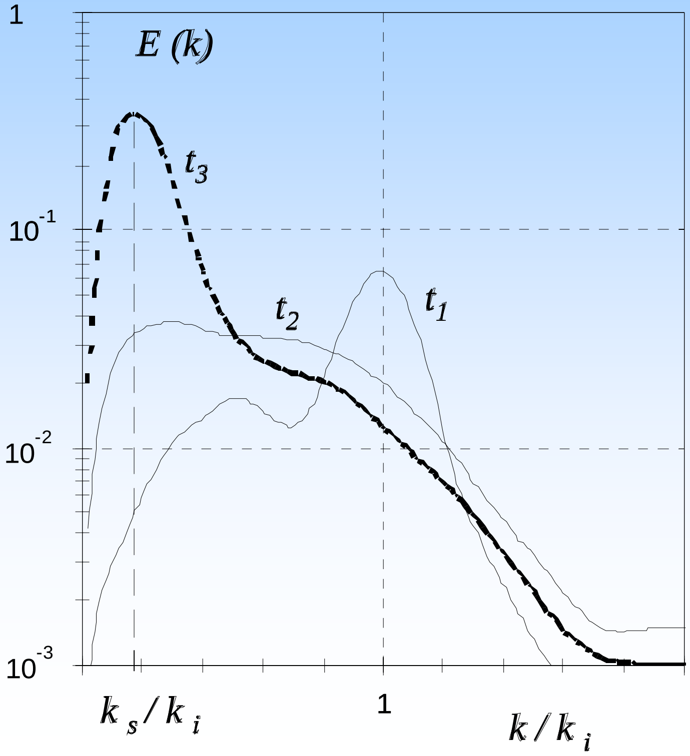
Turbulence evolution in 2D fluid **(10 x 10 vortices)**



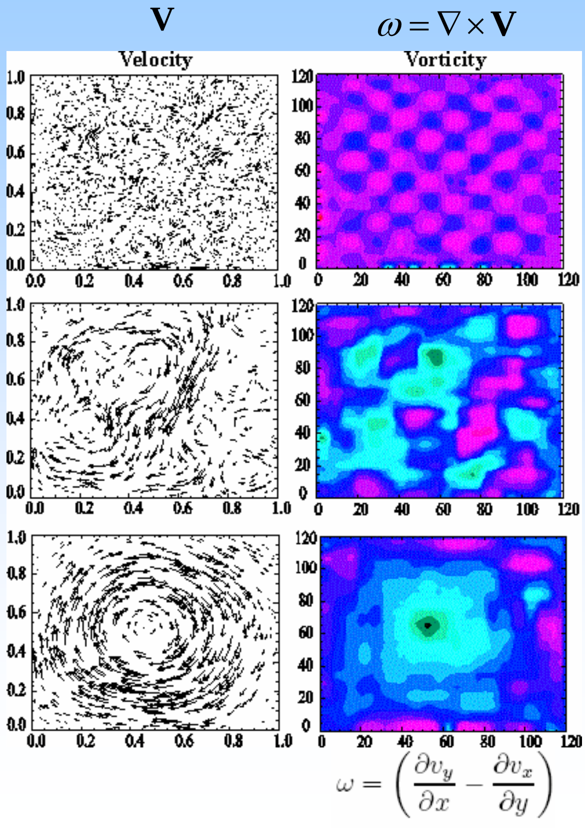
[M. Shats, H. Xia, H. Punzmann *Phys. Rev.E.* (2005)

Spectral condensation in 2D fluid

Spectra evolution in 2D fluid **(10 x 10 vortices)**

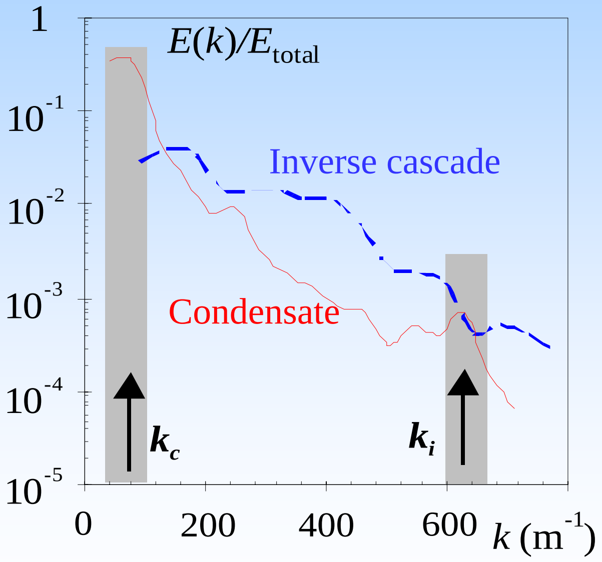


Spectral evolution of 2D turbulence

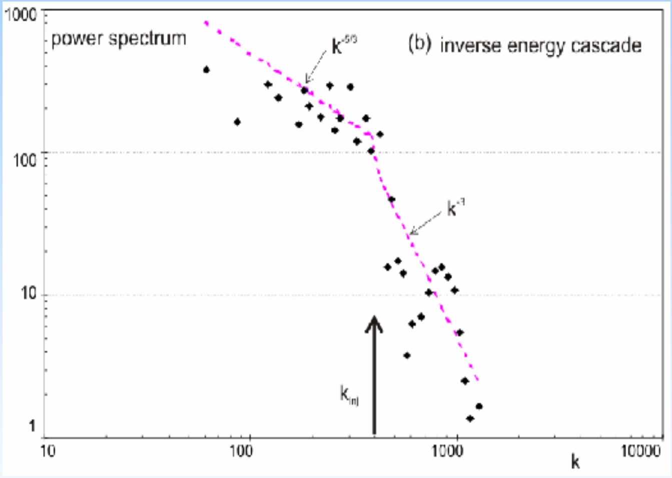


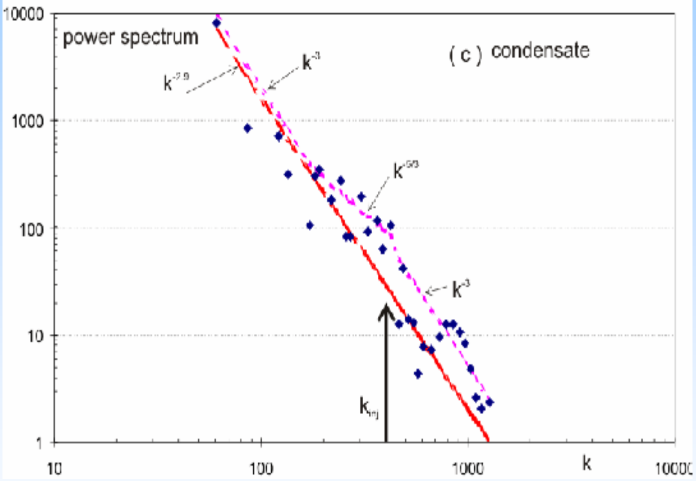
$$\omega = \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right)$$

Energy spectrum (10 x 10 vortices)



Spectra modifications during condensation





Change in the power law at $k < k_i$ from ~k^{-1} to k^{-3} is indicative of changed nature of the spectral transfer (e.g., local to nonlocal)

Spectral condensation as a paradigm for confinement transitions in plasma

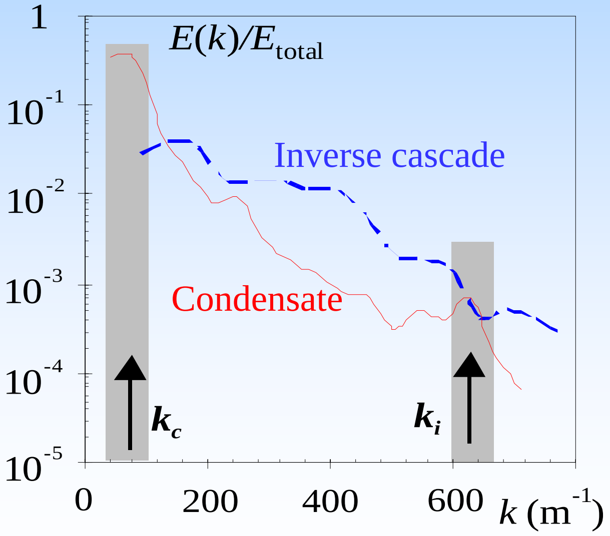
Spectral condensation

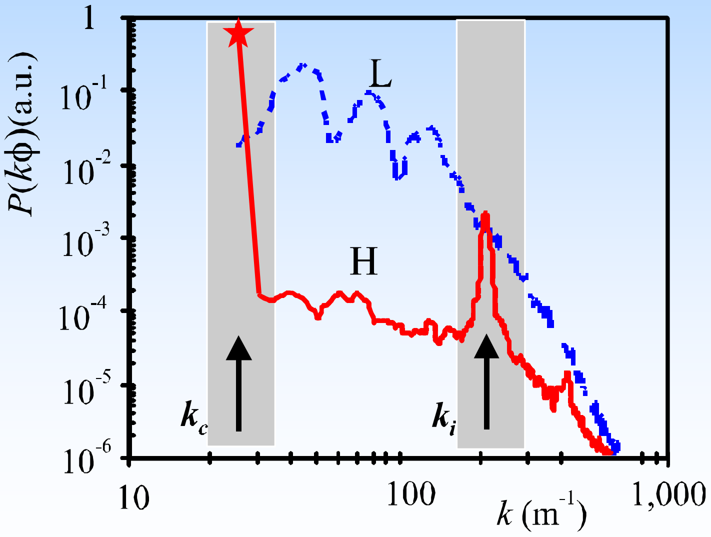
in 2D fluid

?

Spectra evolution

from L to H mode





Summary

1. Empirical similarity between self-organization in 2D fluid turbulence and in quasi-2D toroidal plasma demonstrated;
2. Both systems show onset of regular anisotropic flow and reduction in the intermediate-scale turbulent eddies;
3. Spectral condensation paradigm “confirmed” in the pedestal region during L-H transitions in plasma;
4. No simple threshold condition for the condensate formation has yet been found in our experiments in 2D fluid turbulence
5. Larger scale separation experiments are under way