Collision of inertial particles in turbulent flows.

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Introduction (1)

Particles advected by a fluid flow, with a mass that does not match the fluid density, have a nonuniform distribution in space.

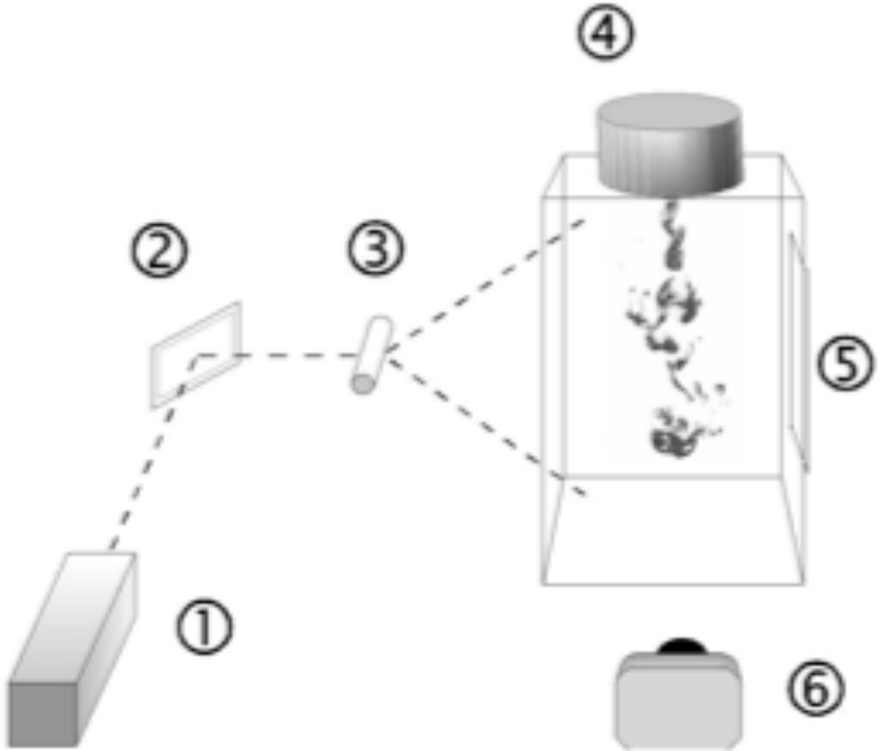
Here, focus on the case of **particles heavier than the fluid.**

Interesting questions:

- characterize the distribution of particles in the flow.
- estimate the collision rates between particles.

Applications coalescence of droplets in clouds (how long does it take before rain falls ?)

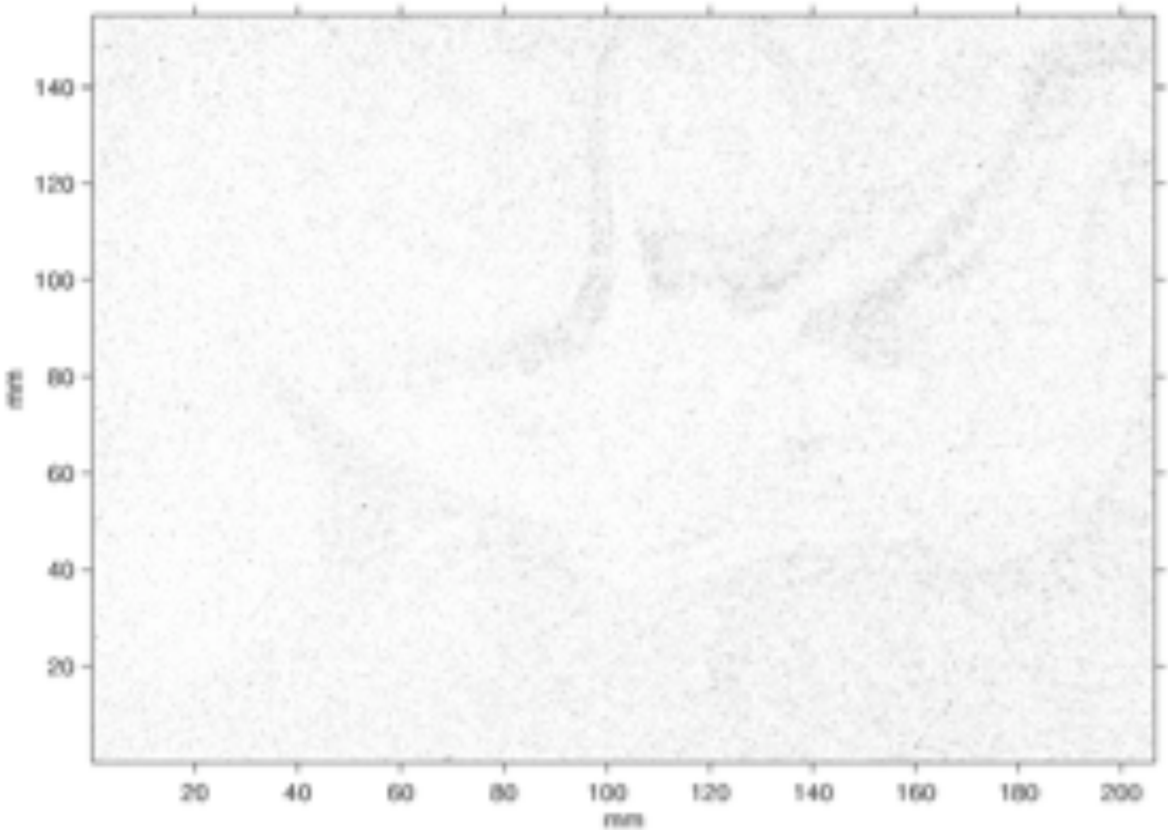
Preferential concentration in a cloud chamber



Experimental observation

(from A. Jaczewski and S. Malinowski, 2005)

Preferential concentration in a cloud chamber

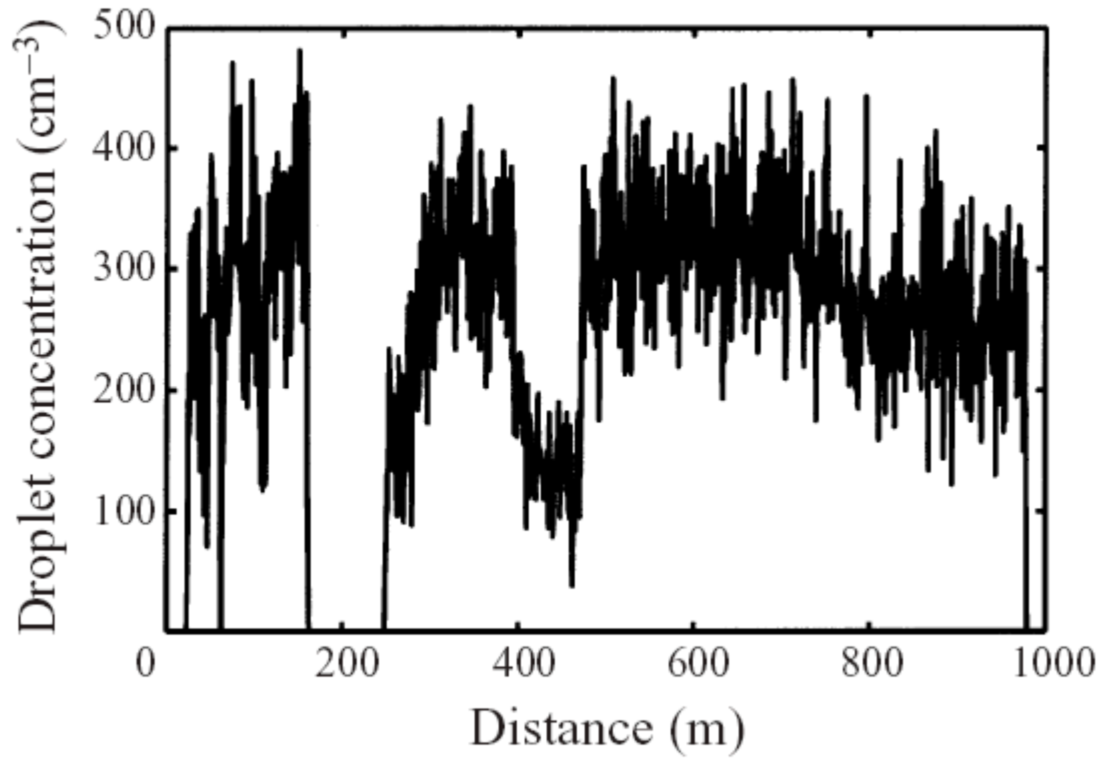


Droplet density in a laboratory flow (Re ~ 10^3; St ~ 0.02)

(from A. Jaczewski and S. Malinowski, 2005)

Preferential concentration in clouds

A. B. Kostinski and R. A. Shaw



Recorded density of clouds in a cumulus (Re ~ 10^7; St ~ 0.01)
From A. Kostkinski and R. Shaw, 2000.

Introduction (2)

Here, consider a ***dilute*** **suspension** :

the mean distance between particles is large enough, so the motion of one particle does not influence the motion of other particles.

The particles' density, n(x,t), evolves according to (v = particle's velocity) :

$$\partial_t n(x,t) + \nabla.(v(x,t)n(x,t)) = 0$$

Preferential concentration is a ***small scale effect***; it is most significant at scales *smaller* than the Kolmogorov scale.

Physical origin of preferential concentration.

Inertial particles do not exactly follow the flow !!

- u = flow velocity; ρ= fluid density;
- v = particle velocity; ρ_0= particle density;

$$v \neq u$$

$$\frac{dv}{dt} = \beta \frac{du}{dt} + \frac{(u - v)}{\tau_s} + g$$

$$\tau_s = (2/9)(\rho_0/\rho)(a^2/\upsilon) \qquad \beta = \frac{3\rho}{(\rho + 2\rho_0)}$$

For very heavy particles : $\rho << \rho_0$ ($\beta \sim 0$).

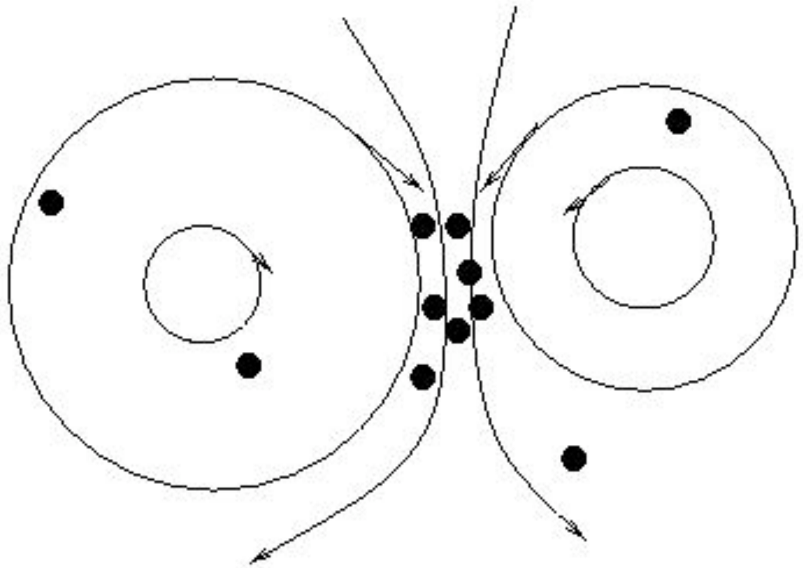
Preferential concentration : small Stokes numbers (1).

When τ_s is short : $$v = u - \tau_s(\partial_t u + (u.\nabla)u) + g\tau_s \approx u$$

So : $$\nabla.v = -\tau_s \nabla.\{(u.\nabla)u\} = -\tau_s(S^2 - \Omega^2)$$

Where S = rate of strain, Ω = vorticity.

Heavy particles are expelled from vortices.



Preferential concentration : small Stokes numbers (2).

At small values of $St = \tau_s/\tau_K$:

$$v = u - \tau_s(\partial_t u + (u.\nabla)u) + g\tau_s \approx u + O(St)$$

the particle and fluid velocities are close to one another.

Preferential concentration can be understood by studying lagrangian trajectories (dx/dt = u), and by studying the density n, that follows :

$$\frac{d}{dt}n(x,t) = -(\nabla.v)n(x,t)$$

Balkovsky et al, 2001, Falkovich et al., 2002,2004.

Preferential concentration : Stokes numbers ~ 1 (1)

At larger St, possibility of **sling effect** (Falkovich et al, 2002).

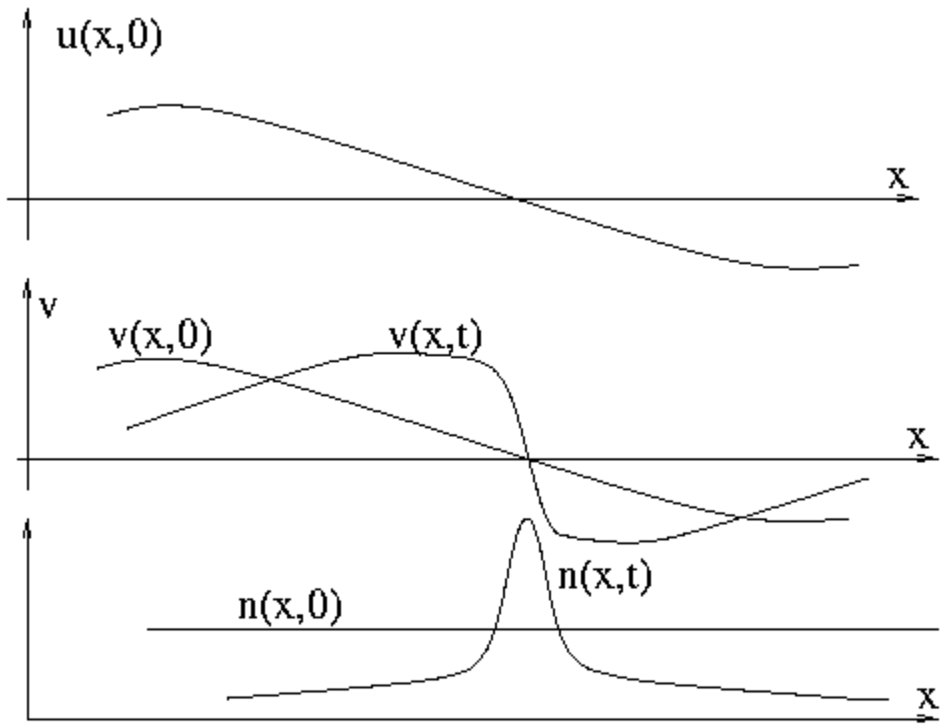
Consider again the equation for the particle velocities, v, and differentiate w.r.t . x ($s_{ij} = \partial_\iota u_j$; $\sigma_{ij} = \partial_i v_j$)

$$\frac{d\sigma}{dt} + \sigma^2 = \frac{(s - \sigma)}{\tau_s}$$

Burgers type equation !

Preferential concentration : Stokes numbers ~1 (2)

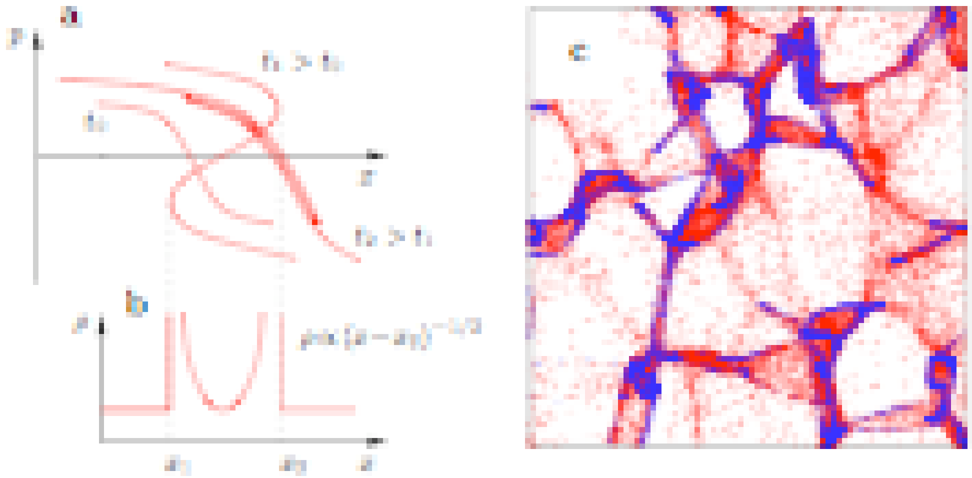
Burgers-like behavior : in 1-d, when σ_{ij} becomes too small ($\sigma_{ij} < -1/\tau_s$), the gradient blows-up, along with the density n : $\sigma_{ij} \sim (t_0 - t)^{-1} \sim n$.



Falkovich et al., 2002.

Preferential concentration : Stokes number ~ 1 (3)

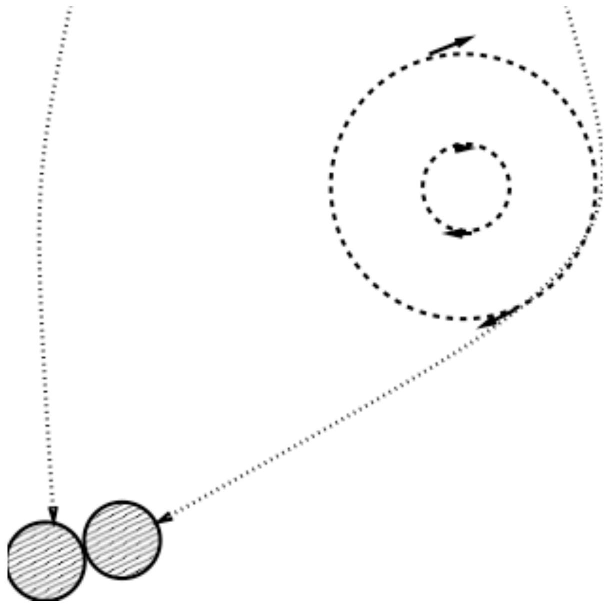
Geometrical point of view : appearance of caustics for the distribution of n.



Wilkinson and Mehlig, 2005

Preferential concentration : Stokes number ~ 1 (4)

The divergence of σ at (x,t) indicates that particles originating from different regions of space run into each other at (x,t).



Questions : frequency of these blow up events ? Contribution to the collision rate ??

Preferential concentration : large Stokes numbers (5)

Question : what happens **after a blow-up** ?

The particle velocity derivative tensor is naturally regularized after the explosion.

From a mathematical point of view :

So :

$$\frac{d\sigma}{dt} + \sigma^2 = 0 \Leftrightarrow \frac{d}{dt}\left(\frac{1}{\sigma}\right) = +1$$

$$\left(\frac{1}{\sigma}\right) = (t - t_0) \Rightarrow \sigma \approx \frac{1}{(t - t_0)}$$

Effectively, when σ becomes too large, the sign of σ flips !

nb : the density increases temporarily during the blow-up event, as it should.

Summary

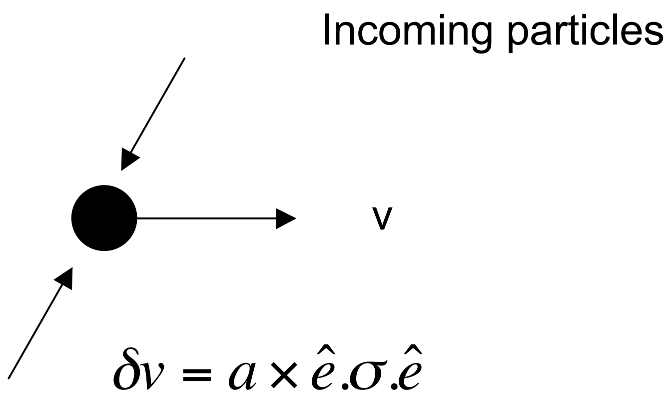
Two main physical effects :

- Particles do not exactly follow the flow, and their effective velocity is *compressible*. The effect exists for all Stokes numbers.

- A ‘sling effect’, due to the fact that particles originating from very different regions occasionally run into each other. The effect is important only when the Stokes number is high enough.

Collision rates (1)

Regular contribution :



$$\delta v = a \times \hat{e}.\sigma.\hat{e}$$

Flux of incoming particles on one given particle :

$$K_{reg} = \int_{\hat{e}.\sigma.\hat{e}<0} n \times a^2 \times a \times (\hat{e}.\sigma.\hat{e}) d\Omega \ \ (\propto a^3)$$

Collision rates (2)

Regular contribution :

Estimate the collision rate in the case of an homogeneous, isotropic turbulent flow.

Number of collision per particle of size a per unit time at small St (Saffman and Turner) :

$$N_{ST} = n \times (2a)^3 \times (8\pi\varepsilon / 15\nu)^{1/2}$$

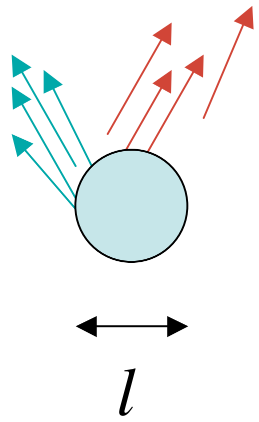
Empirical observation : the Saffman-Turner formula seriously understimates the collision rates. This is a manifestation both of the nonhomgeneous distribution of particles, and of the 'sling effect'.

Collision rates (3)

Sling contribution (phenomenological estimate; for St ~ 1) :

During the blow-up of σ, the gradient $s > 1/\tau_s$, in a region of size:

$$l \sim (\nu \times \tau_s)^{1/2} \sim \eta \times St^{1/2}$$



- collision lasts for $\sim \tau_s$
- size of the domain where collisions occur : $l \sim \eta x St^{1/2}$

- characteristic velocity difference of interacting particles :

$\delta v \sim l/\tau_s$

Estimate of the number of collisions during the sling event :

$$N_{sling} \approx 4\pi a^3 n(\tau)(l/a) \approx 4\pi a^2 \times l \times n(\tau) \ \ (\propto a^2)$$

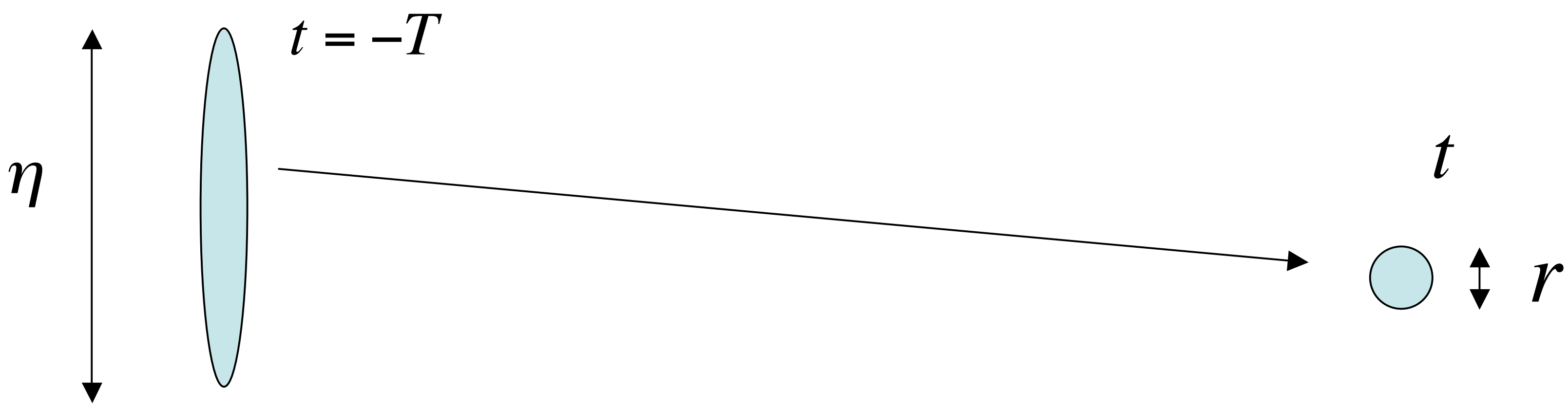
Estimation of the concentration of particles at a scale r (1)

Objective : estimate the coarse-grained distribution of the density at a scale r ($r < \eta$),

$$\langle n^2 \rangle_r$$

The general idea : as the fluid evolves, small scales are generated.

To estimate $\langle n^2 \rangle_r$, consider a parcel of fluid with a characteristic size r.
This parcel of fluid comes from a parcel at an earlier time (-T) which was at scale η.



Estimation of the concentration of particles at a scale r (2)

Method (Falkovich et al, 2002, 2004).

• Determine the particle trajectory that arrives at point r, at time t.

• Keep track of the deformation of volume induced by the fluid element. To this end, compute the tensor W, such that W(t) δl(0) = δl(t) :

$$\frac{dW}{dt} = (\sigma.W + W.\sigma^T)$$

• Find the time -T, such that W(-T) = η Id

and that the smallest eigenvalue of W(0) = r.

• The contribution to the coarse grained density at scale r is :

$$n = n_0 \exp(\int_{-T}^{0} \exp(-tr(\sigma(t')) dt')$$

• Statistical weight of each trajectory ~ 1/n

Numerical methods (1)

• Solve the Navier-Stokes equations in the simplest possible geometry (cube with periodic boundary conditions), using standard pseudo-spectral methods => generate the velocity field u(x,t)

• In the flow, follow the motion of inertial particles :

$$\frac{dv}{dt} = \frac{(u-v)}{\tau_s} + g$$

along with the equation of evolution for σ = dv/dx :

$$\frac{d\sigma}{dt} + \sigma^2 = \frac{(s-\sigma)}{\tau_s}$$

•In addition, keep track of the deformation matrix W, induced by σ :

$$\frac{dW^{-1}}{dt} = -(W^{-1}.\sigma + \sigma^T.W^{-1})$$

Numerical methods (2)

• Keep track of the density n of particles :

$$n(x,t) = n_0 \exp\left(-\int_0^t tr(\sigma(t'))dt' \right)$$

• When $|W^{-1}(t)|$ exceeds η/r, record n, compute the moments and the contributions to the collision terms.

Technically, work in the range of Reynolds numbers (fully resolved flows) :

$$21 < R_\lambda < 130$$

Numerical results

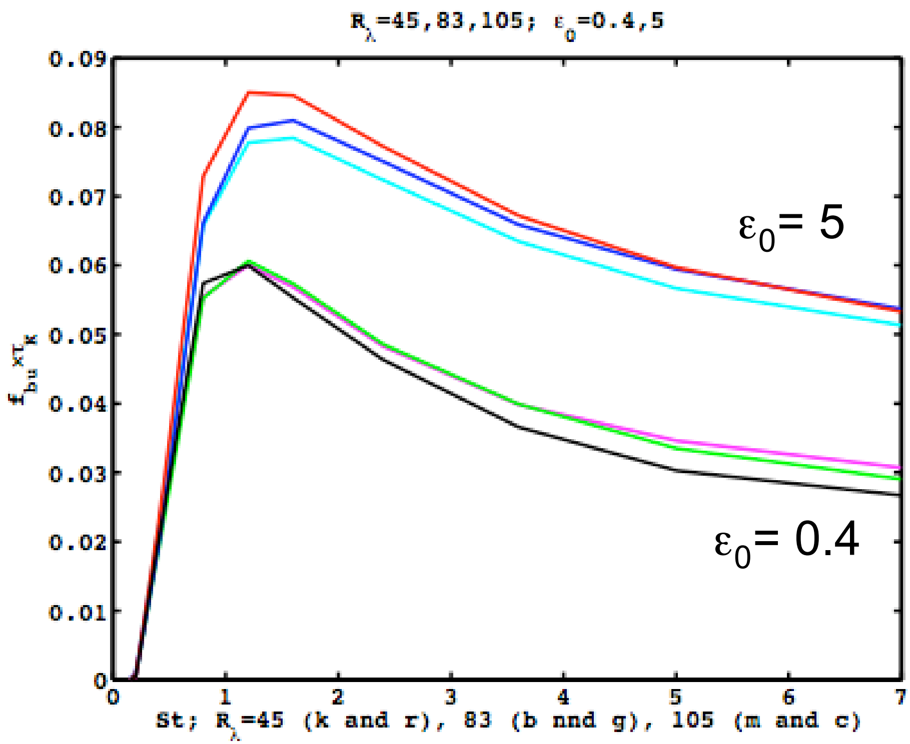
Blow-up frequency (1)

Observations :

• At very small Stokes numbers, the blow-up frequency is ~ 0.

• The probability of blow-up increases up to a maximum value of St ~ 1, then decreases again.

• The blow-up frequency is weaker when gravity is stronger.

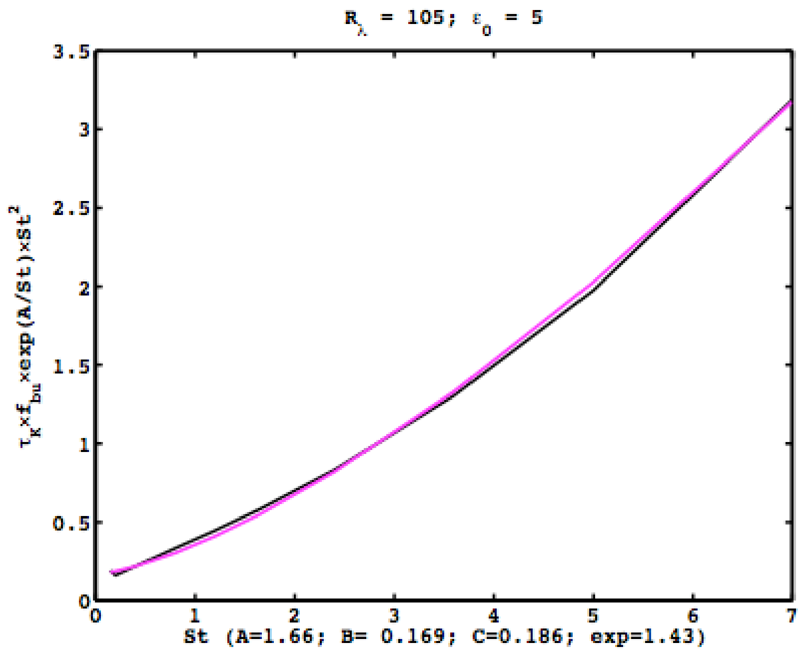


Blow-up frequency (2)

Fit of the blow up frequency :

$$f_{bu} = \exp(-A/St) \times St^{-2} \times (B + C St^{n})$$

n.b. : the exp(-A/St) -dependence can be fully justified in 1d; see Derevyanko et al., 2006; see also Wilkinson et al, 2006.



Black line : numerics;
Purple line : fit

Blow-up frequency (3)

• The value of A decreases slightly with R_λ

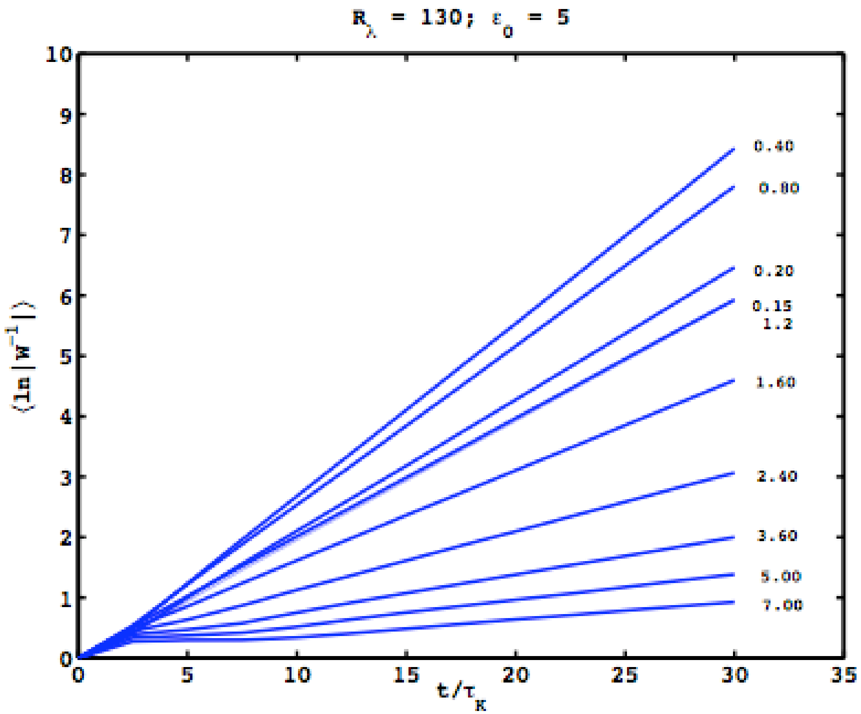
($A(R_\lambda=45) = 2.1$; $A(R_\lambda=83) = 1.85$ and $A(R_\lambda=105)=1.70$).

• Increasing ε_0 above 5 (decreasing gravity) does not change anything; $\varepsilon_0 = 5$ corresponds effectively to the very low gravity case.

• The main difference between low gravity ($\varepsilon_0=5$) and higher gravity ($\varepsilon_0=0.4$) is mostly in the coefficient C.

Rate of compression along trajectories (1)

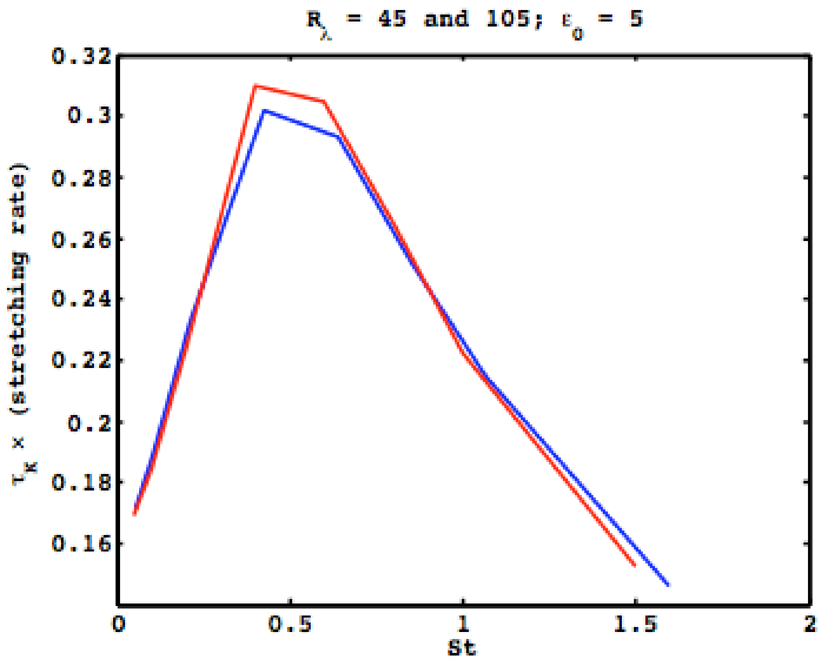
The rate at which a parcel of particles gets compressed plays a crucial role in this work.

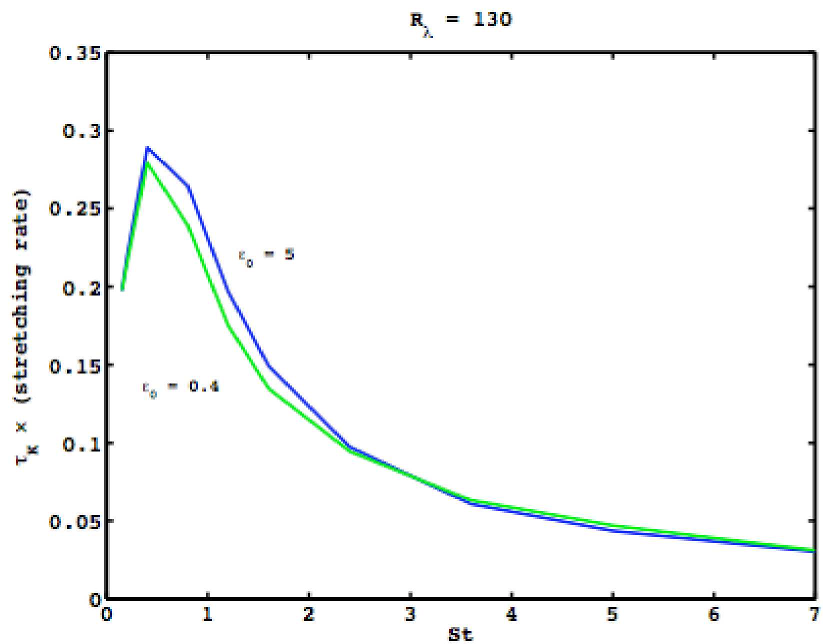


W^{-1} grows exponentially in time => exponential contraction along trajectories.

Rate of compression along trajectories (2)

The rate of growth has a nonmonotonic dependence as a function of the Stokes number.





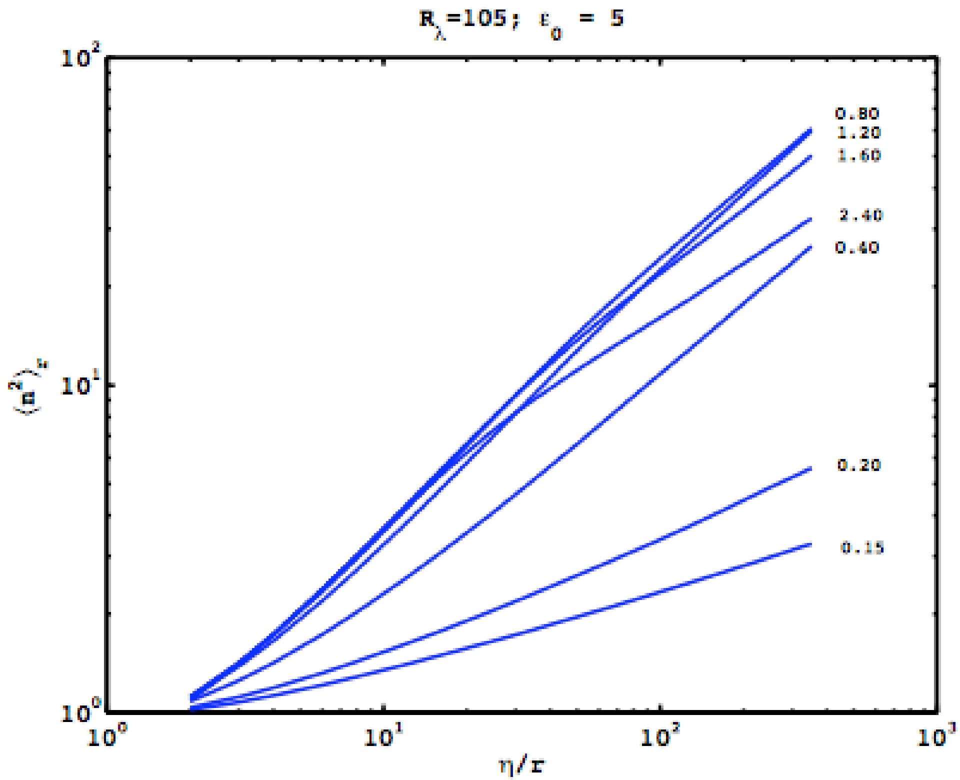
Remarks : for St = 0, the compression rate ~ $0.166/\tau_K$.

limited dependence as a function Reynolds and of gravity.

Spatial dependence of $\langle n^2 \rangle_r$ (1)

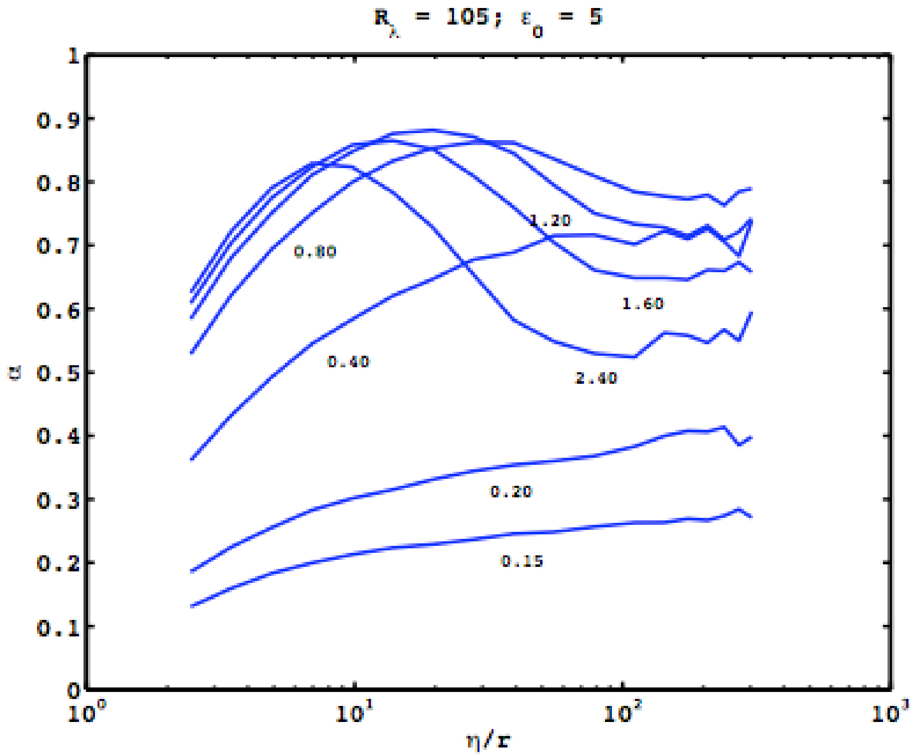
Approximate power-law dependence of $\langle n^2 \rangle_r$:

$$\langle n^2 \rangle_r \sim r^{\alpha}$$



Spatial dependence of $\langle n^2 \rangle_r$ (2)

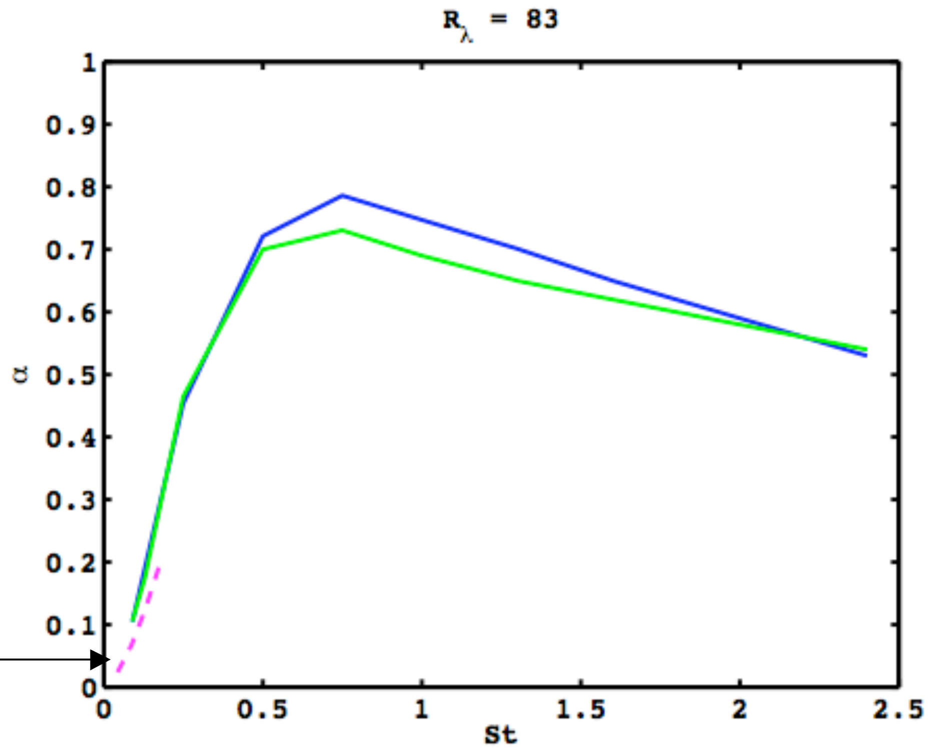
(approximate) power law dependence of $\langle n^2 \rangle_r$ as a function of r :
plot $\alpha = d\ln\langle n^2 \rangle_r / d\ln r$.



Plausible power law at low Stokes; more complicated behavior at higher Stokes numbers.

Spatial dependence of $<n^2>_r$ (3)

Compare with the zero Stokes limit (Falkovich and Pumir, 2004).



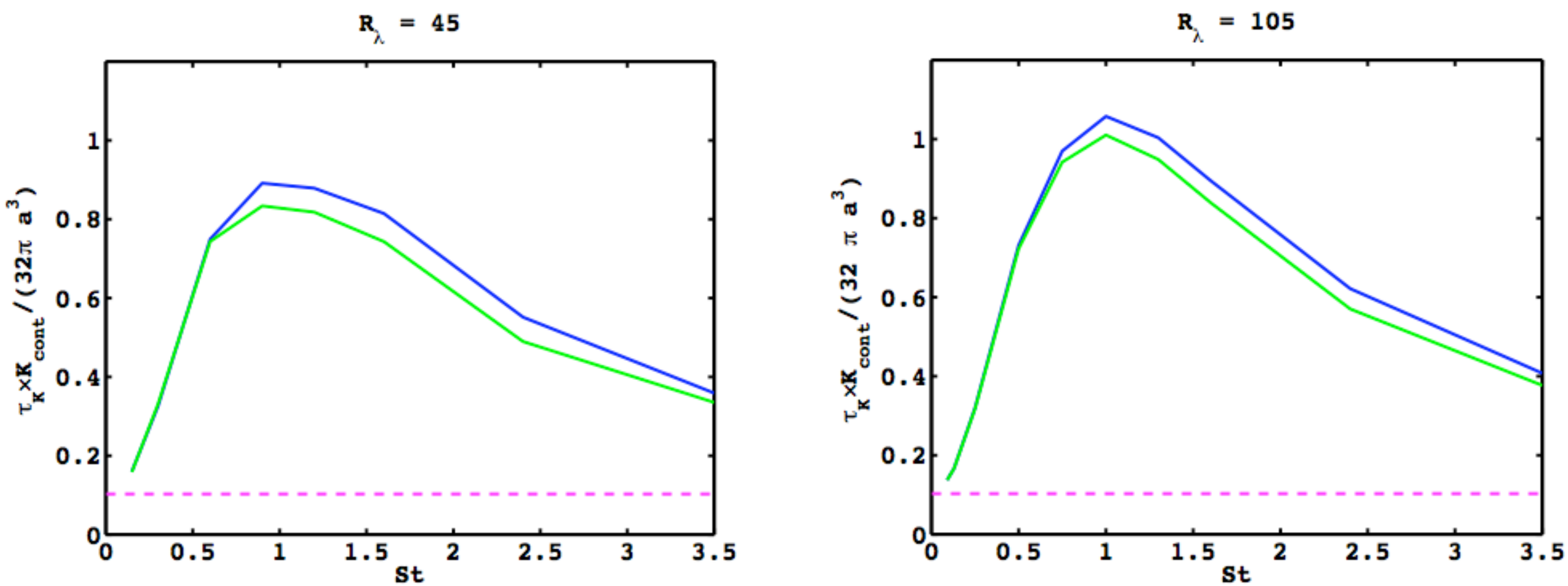
green : ε_0=5
blue : ε_0=0.4

Purple line :
St -> 0 limit

The 'St -> 0' limit slightly underestimates the exponents.

Collision rate (1)

Continuous part of the collision rate.



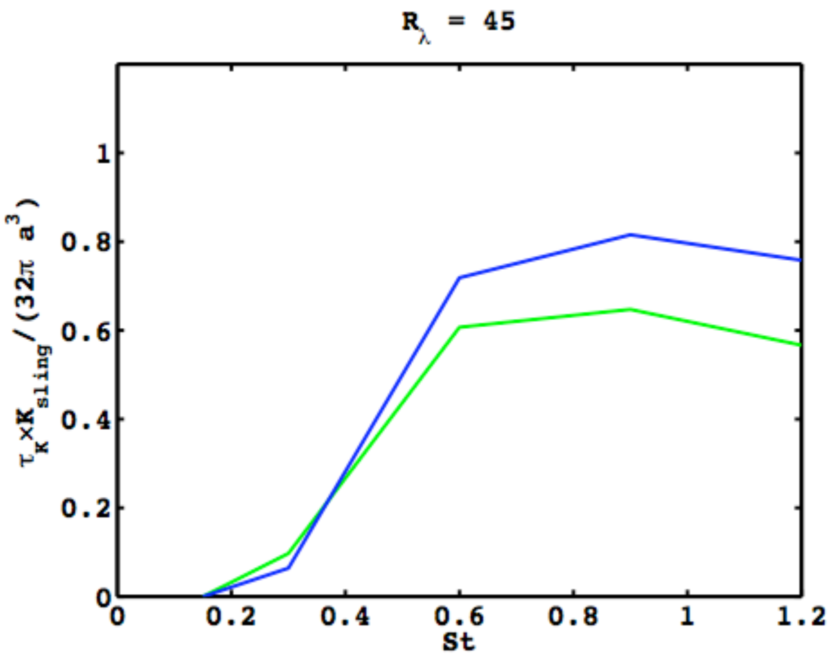
Enhancement of the collision rate with respect to the Saffman-Turner formula (purple line).

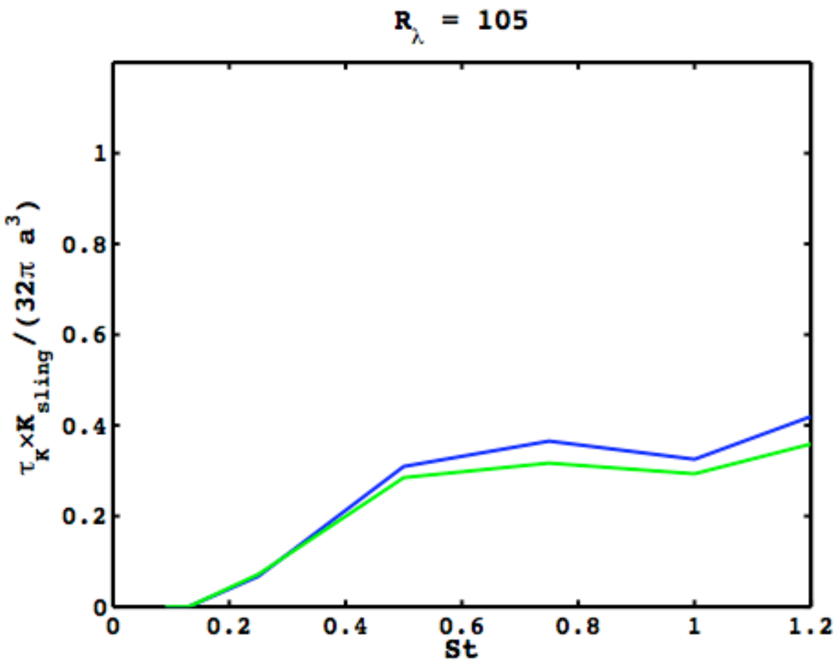
The collision rate has a maximum for St ~ 1.

Slight growth of the effect when R_λ increases.

Collision rate (2)

Sling contribution to the collision rate.





Essentially NO effect for St less than 0.15 or so.

The effect Is maximum for St ~ 1.

Dependence as a function of the Reynolds number ??

Collision rate (3)

Comparison with direct numerical simulations (Franklin et al, 2005).

- Numerical work at lowish Reynolds numbers ($R_\lambda \sim 50$); where collision rates in a monodisperse solution of bubbles of size a = 10μm and 20μm at low density are computed.

=> Compare the results of the 'lagrangian approach' of the full DNS/kinetic results.

Collision rate (R_λ=48; a = 10μm; St = 0.08) : 1.0×10^{-6} cm^3/s

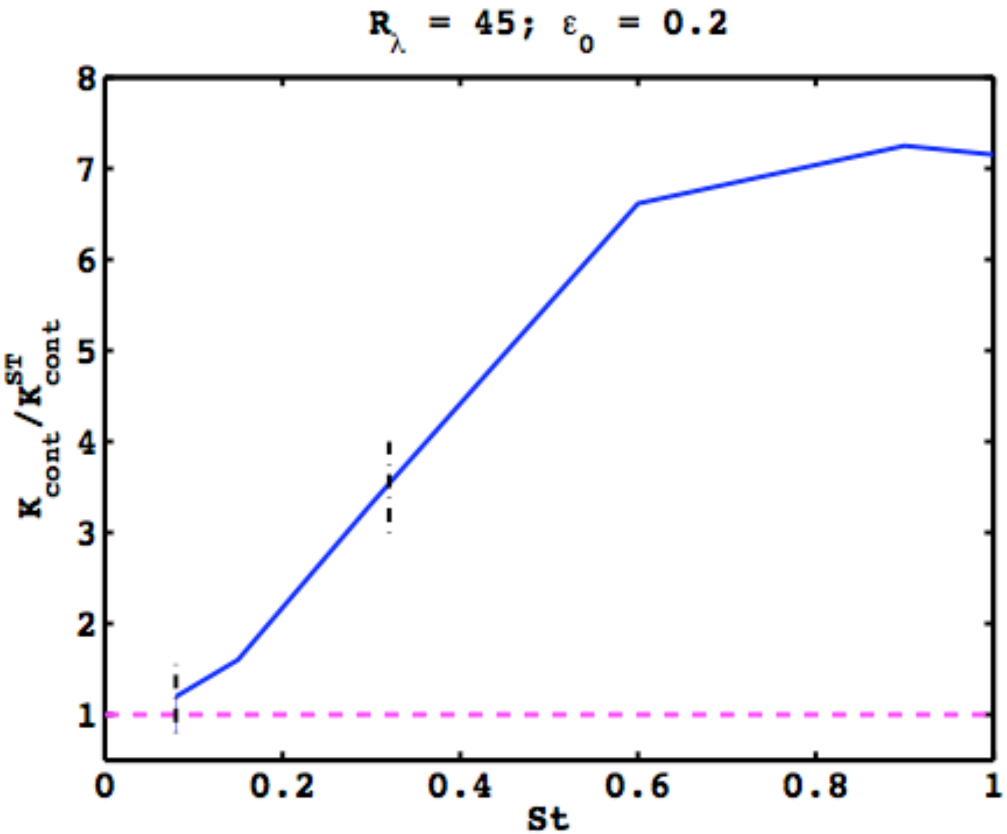
0.9×10^{-6} cm^3/s (+ 0 sling term)

Collision rate (R_λ=48; a = 20μm; St = 0.32) : 5.9×10^{-5} cm^3/s

2.2×10^{-5} cm^3/s

(+ ~1cm^3/s sling contribution)

Collision rate (4)



The enhancement wrt. the Saffman-Turner formula becomes more siginificant as the Stokes number increase.

Conclusion

- The problem of advection of inertial particles by a turbulent flows offers a number of interesting challenges, ranging from fundamental questions to very practical (meteorological applications, among others).

- Much can be learned by stydying particle trajectories (lagrangian approach).

- Two different physical regimes : St << 1 and St ~ 1, the latter being characterized by the spontaneous formation of **caustics**, which are responsible for '**sling** events'.

- Lagrangian methods can be used to estimate rather reliably the collision rates of particles.

- Intriguing questions remain.

Acknowledgement

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