Roughness-induced criticality in turbulence

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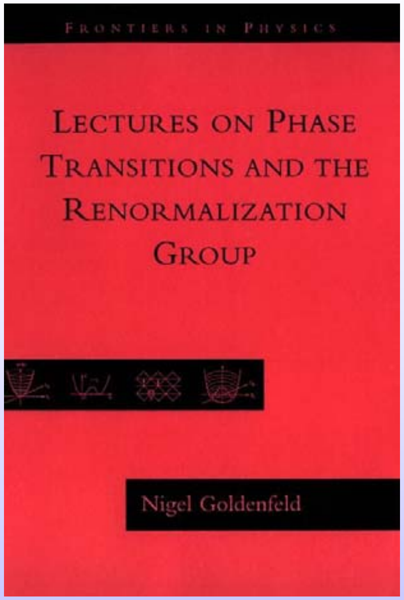
Reference

- G. Gioia and P. Chakraborty, Turbulent Friction in Rough Pipes and the Energy Spectrum of the Phenomenological Theory. *Phys. Rev. Lett*. **96**, 044502 (2006).

- N. Goldenfeld, Roughness-induced criticality in turbulence. *Phys. Rev. Lett*. **96**, 044503 (2006).

Some questions

Is turbulence a critical phenomenon?



- **Common features**
  - Strong fluctuations
  - Power law correlations
- **Critical phenomena now solved**
  - Widom discovered "data collapse" (1963)
  - Kadanoff explained data collapse from coarse-graining (1966)
  - Wilson systemised and extended Kadanoff's theory (1971)
- **Turbulence still unsolved**
  - Can we repeat the pattern of discovery exemplified by critical phenomena?

Experimental signatures of criticality?

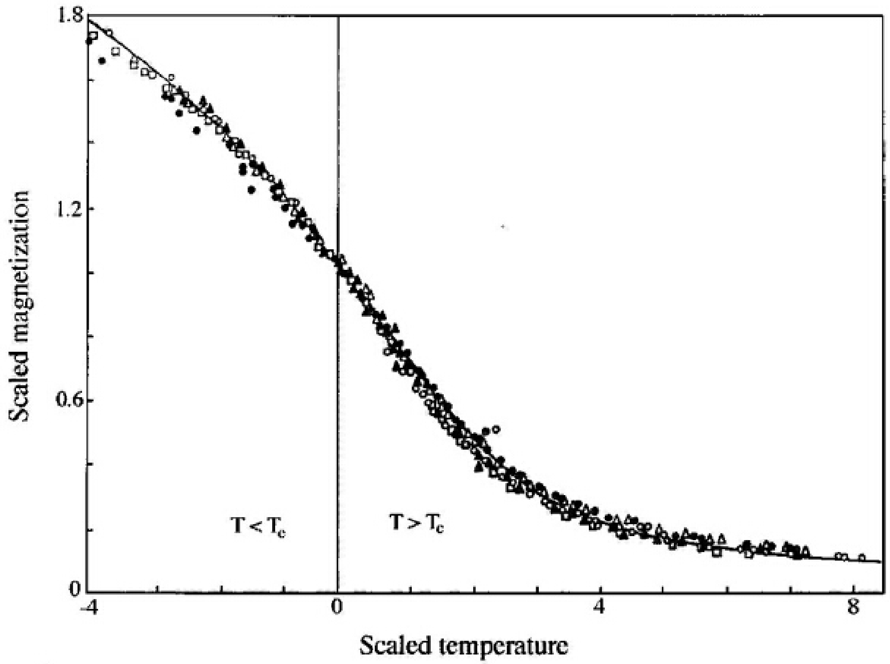


FIG. 1. Experimental *MHT* data on five different magnetic materials plotted in scaled form. The five materials are $CrBr_3$, EuO, Ni, YIG, and Pd_3Fe. None of these materials is an idealized ferromagnet: $CrBr_3$ has considerable lattice anisotropy, EuO has significant second-neighbor interactions. Ni is an itinerant-electron ferromagnet, YIG is a ferrimagnet, and Pd_3Fe is a ferromagnetic alloy. Nonetheless, the data for all materials collapse onto a single scaling function, which is that calculated for the $d = 3$ Heisenberg model [after Milošević and Stanley (1976)].

- **M(H,T) ostensibly a function of two variables**
- **Plotted in appropriate scaling variables get ONE universal curve**
- **Scaling variables involve critical exponents**

Stanley (1999)

Experimental signatures of criticality?

- **Amorphous magnet in a field hysteresis loop**
- **Crackling noise as a result of avalanches**
- **Jump distribution depends on size *s* and aspect ratio *k***
- **Nonequilibrium critical point data collapse**

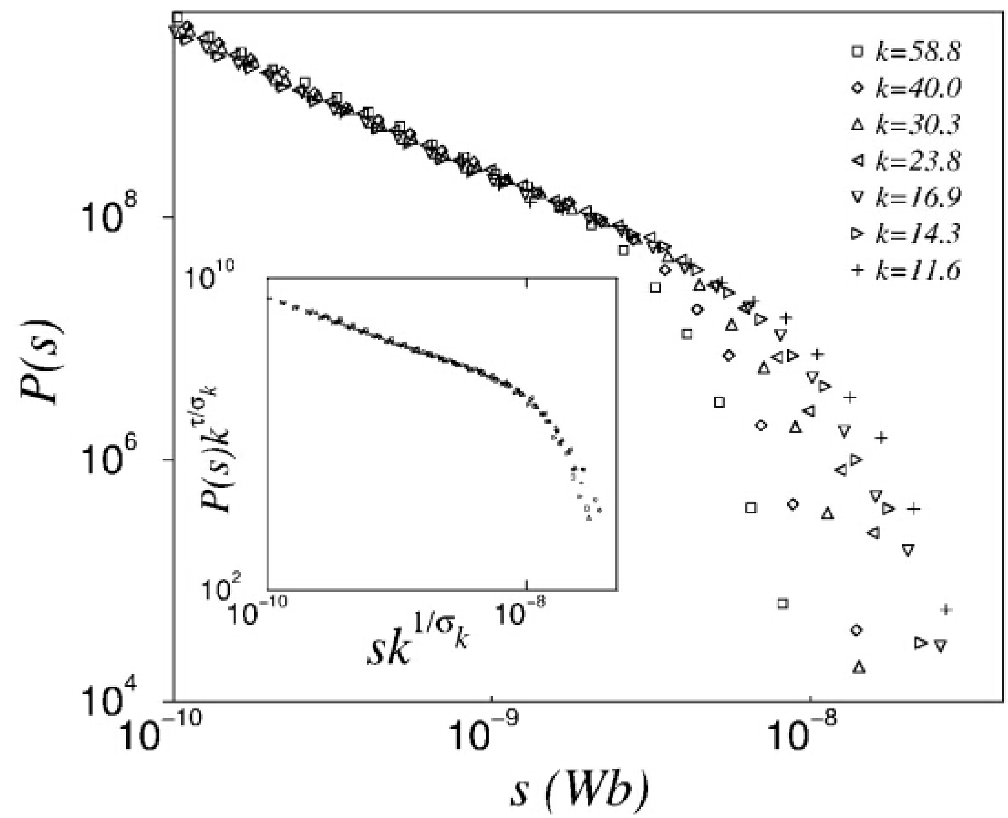


FIG. 2. The Barkhausen jump size distribution for different values of the demagnetizing factor k in the $Fe_{21}Co_{64}B_{15}$ amorphous alloy under tensile stress The data collapse reported in the inset is done using $\tau = 1.27$ and $1/\sigma_k = 0.79$.

Durin and Zapperi (2000)

Experimental signatures of criticality?

- **Criticality is a concept usually associated with equilibrium phase transitions.**
  - Phenomenology well-established
- **Non-equilibrium critical points also studied**
  - Distinct from concept of "self-organized criticality" because there is a phase diagram with critical points only at certain parameter values
  - Rich phenomenology with predictive power
    - Noise spectra, scaling functions, exponent relations
    - Comparison with experiment possible (e.g. disordered magnets)
- **Power laws are not enough: frequently hard to get enough decades**
  - Must seek universal scaling functions, crossover phenomena

What are the analogues of data collapse for turbulence?

Power law scaling is not enough!

Outline of talk

1. Review analogy between turbulence and critical phenomena
2. Experimental data on friction factor of turbulent pipe flow
3. Argument for data collapse
4. Physical model for friction factor
5. Predictions and tests for 2D turbulence

Critical phenomena and turbulence

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Analogies between scaling in turbulence, field theory, and critical phenomena

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We discuss two distinct analogies between turbulence and field theory. In one analog, the field theory has an infrared attractive renormalization-group fixed point and corresponds to critical phenomena. In the other analog, the field theory has an ultraviolet attractive fixed point, as in quantum chromodynamics.

$E(k) \sim k^{-5/3}$

Energy spectrum

$G(k) \sim k^{-2}$

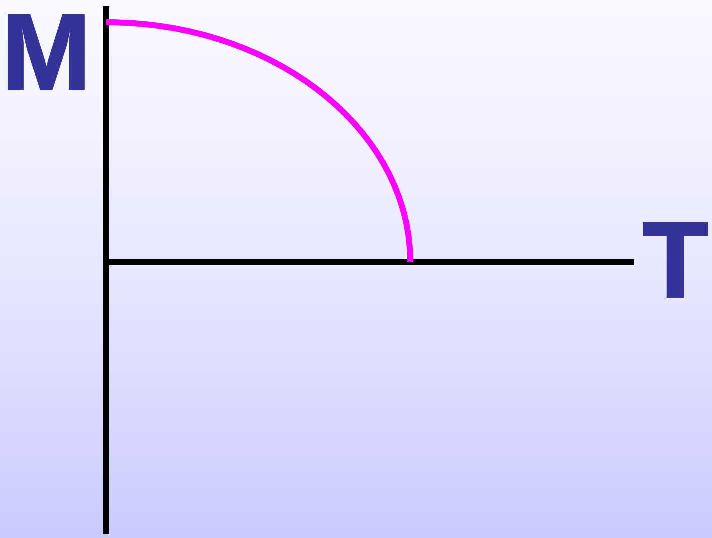
Spin correlations

Turbulence	Critical phenomena
space separation r	wave number k
viscosity ν	temperature variable $\tau - \tau_c$
energetic length scale L	uv cutoff Λ (or inverse lattice spacing a^{-1})
mean dissipation $\bar{\varepsilon}$	stiffness constant K
dissipation wave number $k_d \equiv \eta_d^{-1}$	correlation length ξ
velocity correlation function $S_2(\mathbf{r}) = \langle [\mathbf{v}(\mathbf{r}'+\mathbf{r}) - \mathbf{v}(\mathbf{r}')]^2 \rangle$	spin correlation function $C(\mathbf{k}) = \sum_{\mathbf{r} \in a\mathbb{Z}^d} a^d e^{i\mathbf{k}\mathbf{r}} \langle \sigma(\mathbf{r}) \sigma(0) \rangle$
intermittency exponent μ	correlation exponent η_c

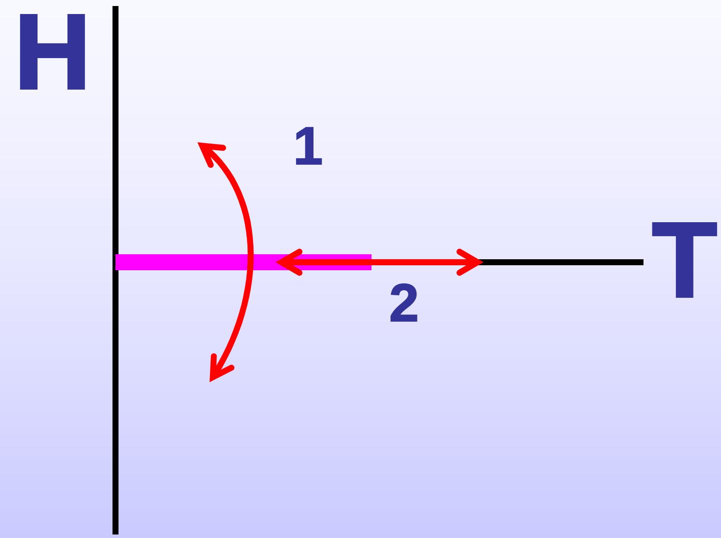
Critical phenomena and turbulence

	Critical phenomena	Turbulence
Correlations	$G(k) \sim k^{-2}$	$E(k) \sim k^{-5/3}$
Large scale thermodynamics	$M(h,t) = \vert t\vert^{\beta} f_M(ht^{-\beta\delta})$	?

Critical phenomena and turbulence



$M \sim M_0[|T - T_c|/T_c]^\beta$ for $H = 0$ as $T \to T_c$



Critical isotherm: $M \sim H^{1/\delta}$ for $T = T_c$

- Widom (1963) pointed out that both these results followed from a *similarity formula*:

$$M(t, h) = |t|^\beta f_M(h/t^\Delta)$$

where $t \equiv (T - T_c)/T_c$ for some choice of exponent Δ and scaling function $f_M(x)$

Critical phenomena and turbulence

$$M(t,h) = |t|^{\beta} f_M(h/t^{\Delta})$$

where $t \equiv (T - T_c)/T_c$ for some choice of exponent Δ and scaling function $f_M(x)$

- **To determine the properties of the scaling function and unknown exponent, we require:**
  - **$f_M(z)$ = const. for z = 0**
    - **This gives the correct behaviour of the magnetization at zero field, for $T < T_c$**
  - **For large values of z, i.e. non-zero h, and t →0, we need the t dependence to cancel out.**

    Thus $f_M(z) \sim z^{1/\delta}, z \to \infty$.

    Calculate Δ: t dependence will only cancel out if $\beta - \Delta/\delta = 0$

$$M = |t|^{\beta} f_M(h/|t|^{\beta\delta})$$

- **This data collapse formula connects the scaling of correlations with the thermodynamics of the critical point**

What is analogue of critical point data in turbulence?

- **Need analogues of the *two* scaling limits $T \rightarrow T_c$ and $H \rightarrow 0$**
- **Experimental data on a real flow**
  - Systematic in same geometry over many decades of Re
  - Systematic variation over the other parameter
- **Classic experiments on pipe flow …**
  - Pipes with radius D and roughness r

Nikuradse's pipe experiment (1933)

Pipe diameter is 25-100 mm
Pipe length is ~ 70 diameters

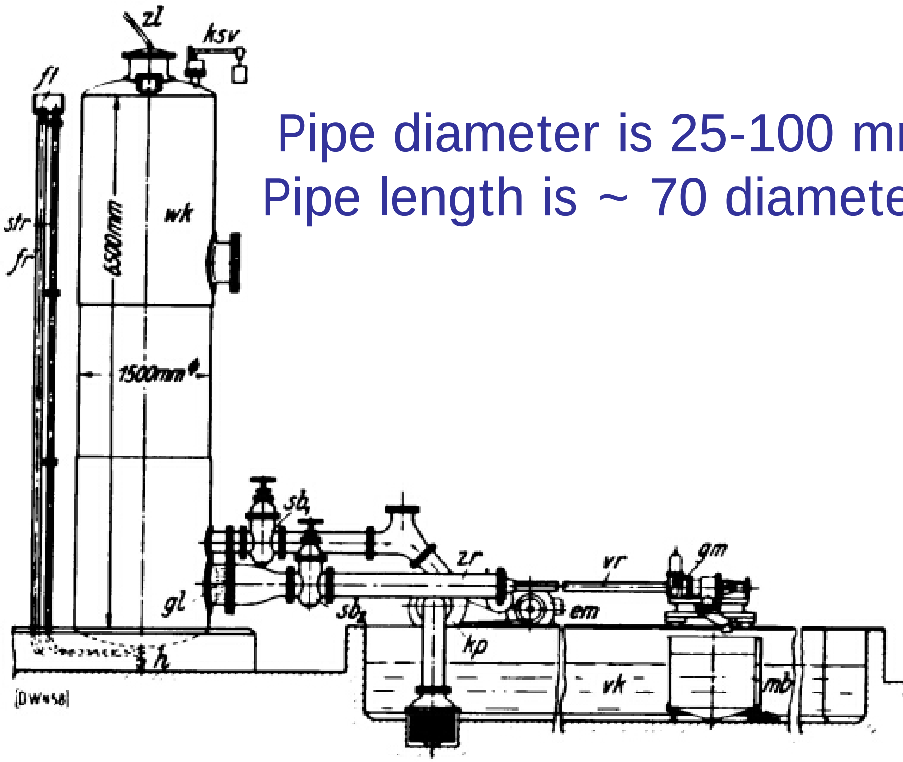


Figure 3.- Test apparatus.

em = electric motor	h = outlet valve
kp = centrifugal pump	zr = feed line
vk = supply canal	mb = measuring tank
wk = water tank	gm = velocity measuring device
vr = test pipe	ksv = safety valve on water tank
zl = supply line	sb_1 = gate valve between wk and kp
str = vertical pipe	sb_2 = gate valve between wk and zr
fr = overflow pipe	gl = baffles for equalizing flow
ft = trap	

Monodisperse sand grains
0.8mm glued to sides of pipe

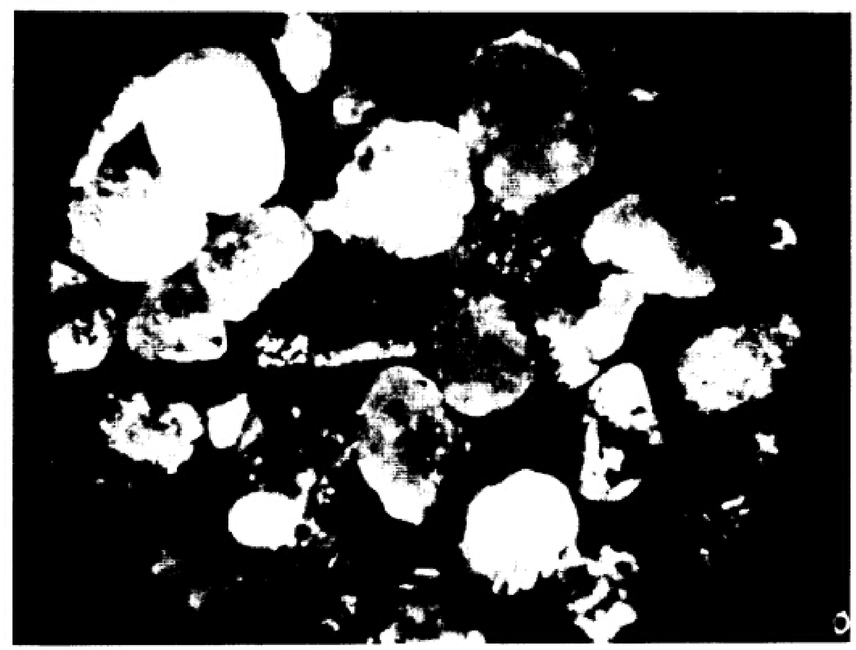


Figure 4.- Microphotograph of sand grains which produce uniform roughness.
(Magnified about 20 times.)

Friction factor in turbulent rough pipes

Laminar

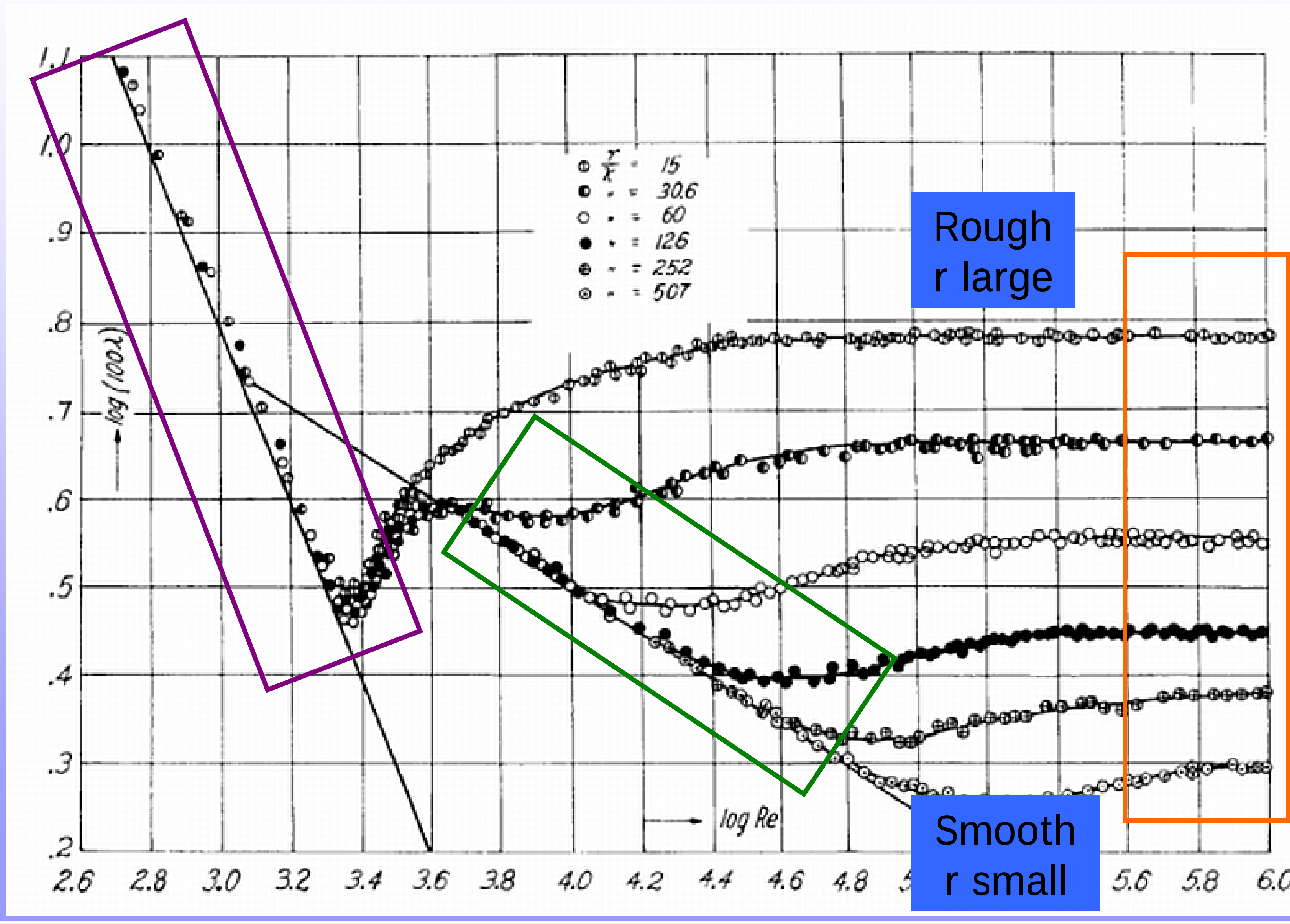
f ~ 12/Re

Blasius

f ~ $Re^{-1/4}$

Strickler

f ~ $(r/D)^{1/3}$



Strickler scaling

Gioia and Chakraborty (2006)

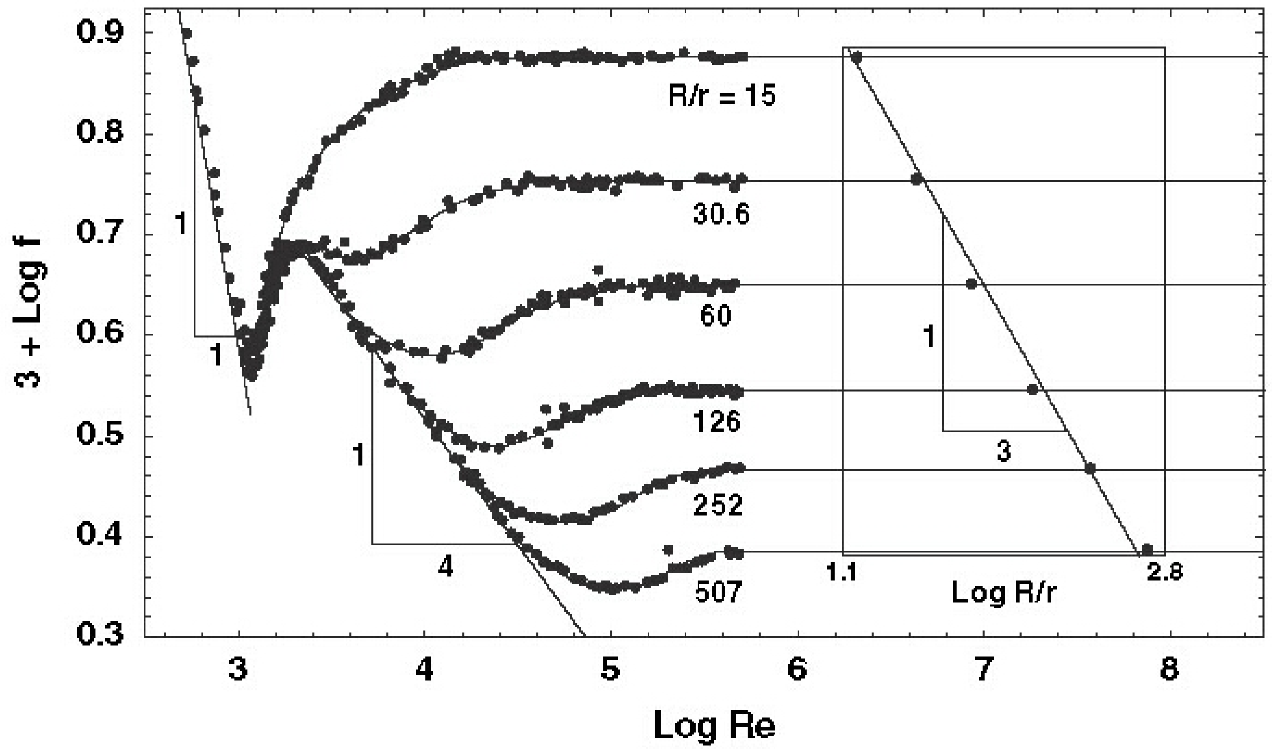


FIG. 1. Nikuradse's data. Up to a Re of about 3000 the flow is streamlined (free from turbulence) and $f \sim 1/\mathrm{Re}$. Note that for very rough pipes (small R/r) the curves do not form a belly at intermediate values of Re. Inset: verification of Strickler's empirical scaling for f at high Re, $f \sim (r/R)^{1/3}$.

Scaling of Nikuradse's data

	Critical phenomena	Turbulence
Temperature control	$t \to 0$	$1/Re \to 0$
Field control	$h \to 0$	$r/D \to 0$

Scaling of Nikuradse's Data

- In the turbulent regime, the extent of the Blasius regime is apparently roughness dependent.
  - $f \sim Re^{-1/4}$ as $r/D \rightarrow 0$

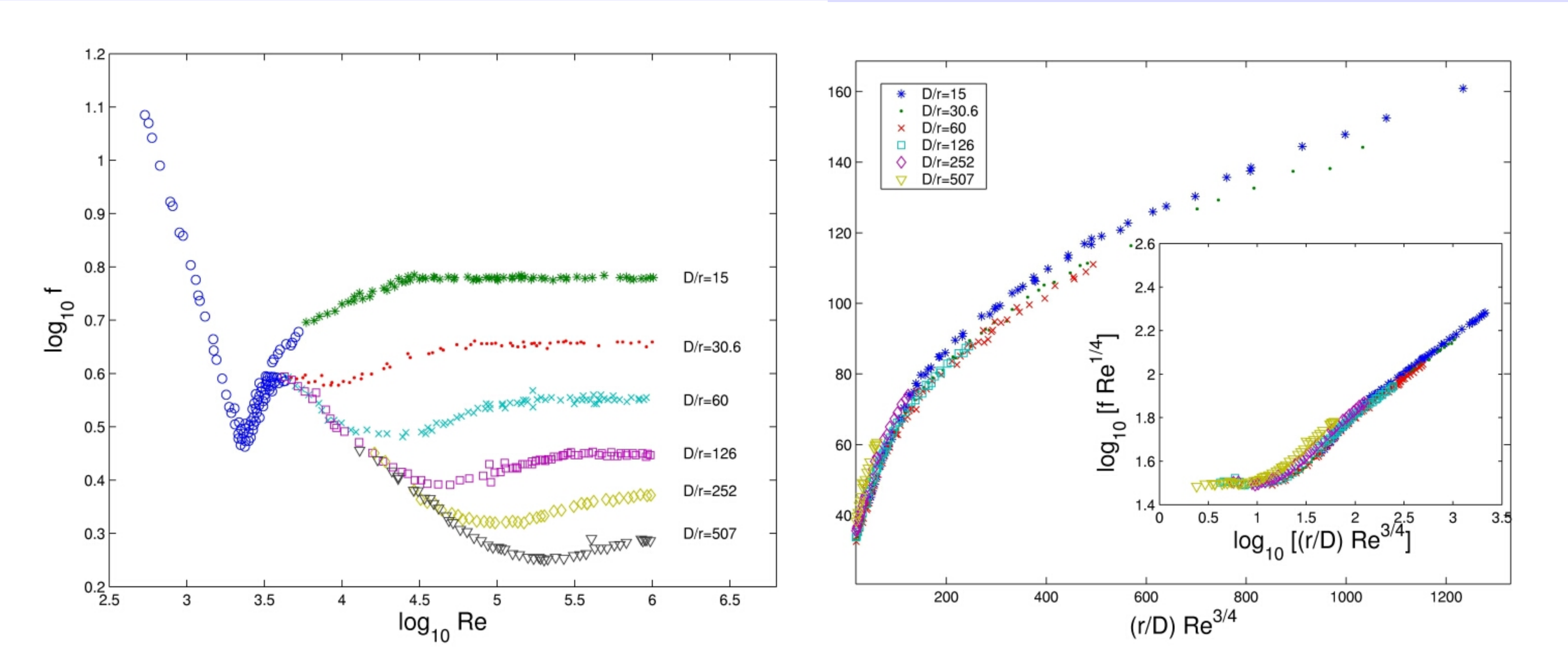
- At large Re, f is independent of roughness.
  - $f \sim (r/D)^{1/3}$ for $Re \rightarrow \infty$

- Combine into unified scaling form
  - $f = Re^{-1/4}\, g([r/D]\, Re^{\alpha})$
    - Determine α by scaling argument: Re dependence must cancel out at large Re to give Strickler scaling
  - Exponent $\alpha = 3/4$ and the scaling function $g(z) \sim z^{1/3}$ for $z \rightarrow \infty$

- $f = Re^{-1/4}\, g([r/D]\, Re^{3/4})$

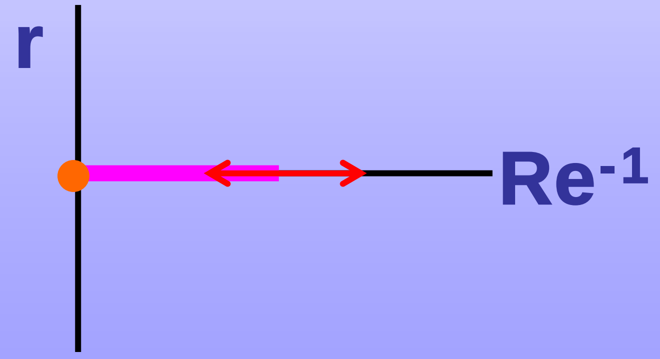
Scaling of Nikuradse's Data

- Is it true that $f = Re^{-1/4} g([r/D]\, Re^{3/4})$?
  - Check by plotting $f\, Re^{1/4}$ vs. $[r/D]\, Re^{3/4}$
    - Do data as a function of two variables collapse onto a single universal curve?

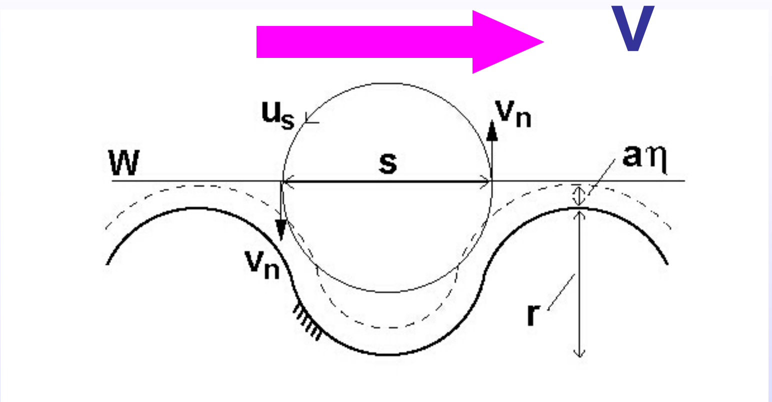


Scaling of Nikuradse's Data

- Scaling function is unlikely to be universal, independent of the nature of the roughness
  - Grains, riblets, mesh, …
  - May account for slight departures from data collapse
- Roughness = 0 is a critical point.
  - In order to understand the turbulent state, important to consider flows with boundary roughness
  - Singular behaviour in roughness as well as Re



Scaling argument for Blasius and Strickler regimes (Gioia and Chakraborty 2006)



- **$f \sim \rho V u_s / \rho V^2 \sim u_s / V$**
  - Contribution to friction factor from dominant eddy on scale of roughness element, $s = r + 5\eta$
  - Sum over scales smaller than "cove" to get

$$f \propto \left[\int_0^{s/R} \frac{E(\sigma)}{\sigma^2} d\sigma \right]^{1/2}$$

- **K41**
  - $s/D = (r/D + \text{const.}\ Re^{-3/4})$ and $E \sim \sigma^{5/3}$
  - Large Re: $f \sim (r/D)^{1/3}$ Strickler law predicted!
  - Small r/D: $f \sim Re^{-1/4}$ Blasius law predicted!

- **Friction factor formula satisfies our scaling relation**

Evaluation of friction factor

- **Now include the dissipation range and integral scale**

$f = \kappa_\tau u_s / V$ or

$$f = K\left(\int_0^{s/R} x^{-1/3} c_d(b\,\mathrm{Re}^{-3/4}/x)\, c_e(x)\, dx\right)^{1/2}, \qquad (1)$$

where $K \equiv \kappa$ [illegible] $\sqrt{2/3}$, $s/R = r/R + ab\,$R[illegible]$^{-3/4}$, and $b \equiv$ $(\kappa_\varepsilon \kappa^3$[illegible]$)^{-1}$[illegible]. Equation (1) gives f as an expli[illegible]it function of th[illegible] Reynolds number Re and the roughness [illegible]$/R$.

Dissipation range

Integral scale

Evaluation of friction factor

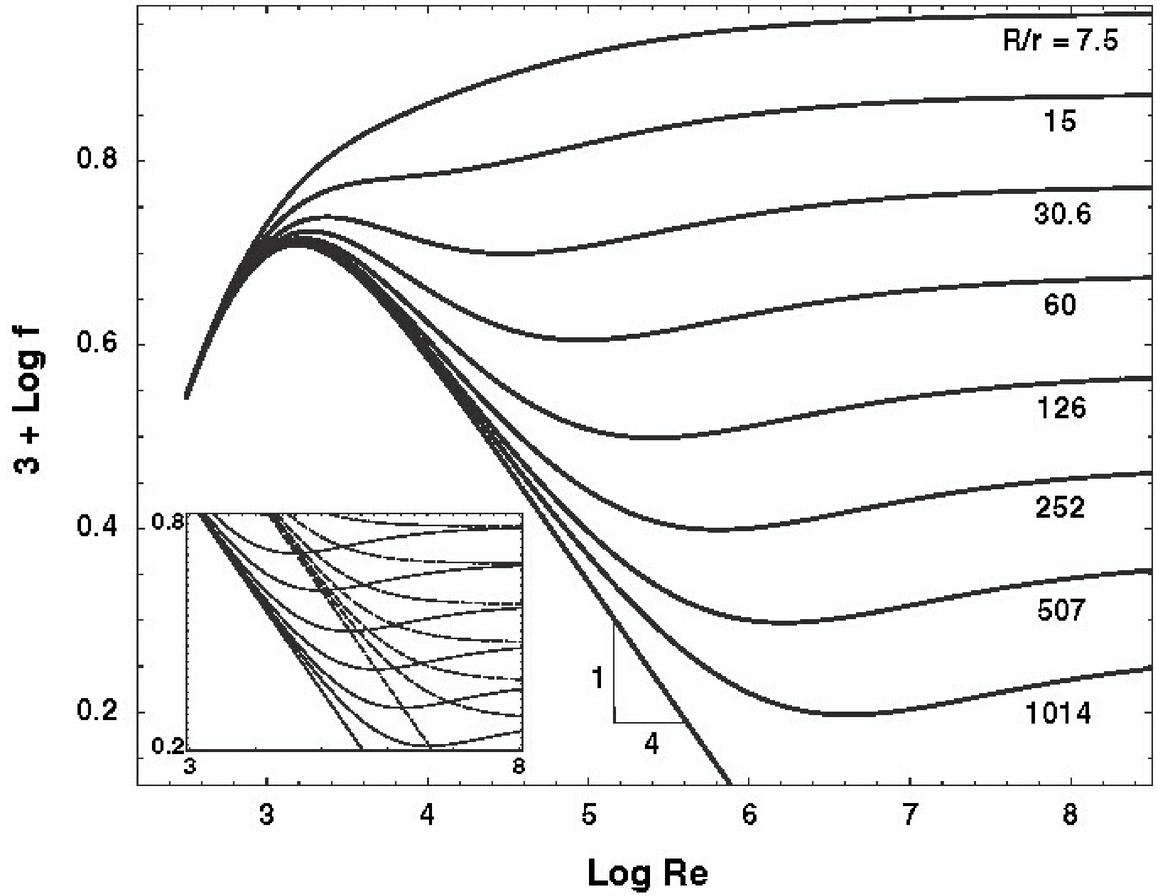
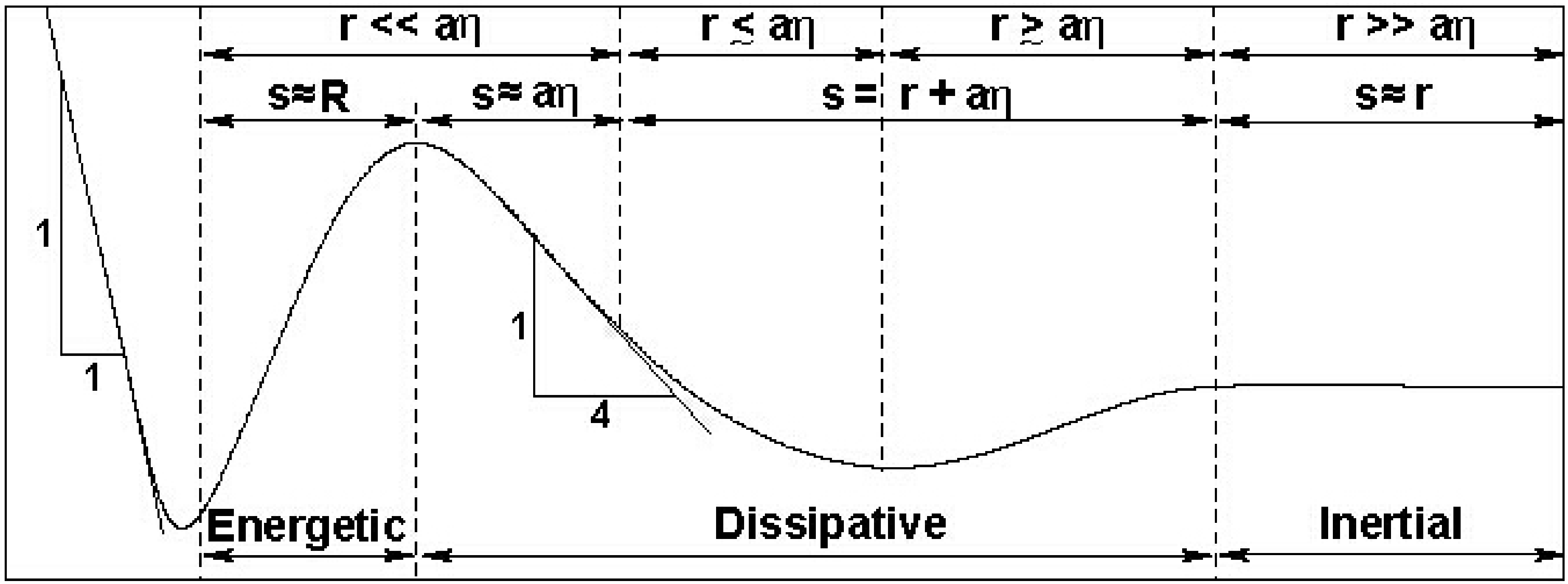


FIG. 3. Plot of (1). Inset: Plot of (2) (no correction for the energetic range: solid lines) and plot of (2) with $\gamma = 0$ (no correction for the energetic range and the dissipative range: dashed lines).

- **Functional form of friction factor reproduces all main features of data**
- **By removing parts of the spectrum, can identify origin of specific features**

Gioia and Chakraborty (2006)

Friction factor contributions



Gioia and Chakraborty (2006)

Boundary layer structure

- How many adjustable parameters in Gioia-Chakraborty model?
  - a = 5, so that thickness of viscous layer ~ 5 η
  - b measured to be 11.4 (Antonia and Pearson (2000)

- Model essentially completely determined.
- But: scale of curves do not match data!
  - Something missing!
  - Guess: need better model for structure of viscous layer

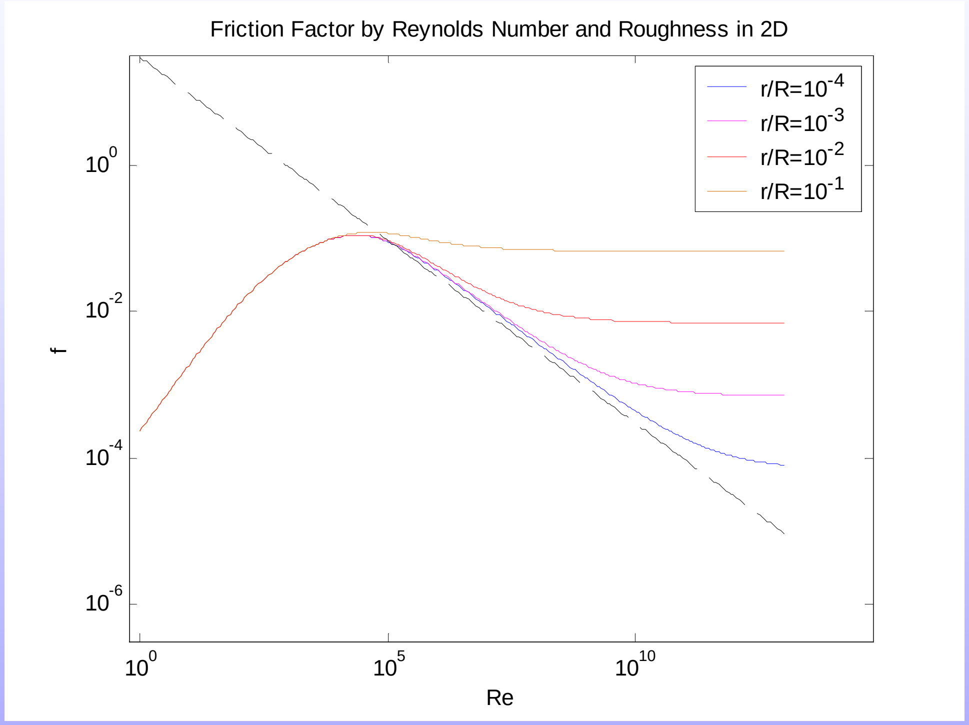
Friction factor scaling in two dimensions

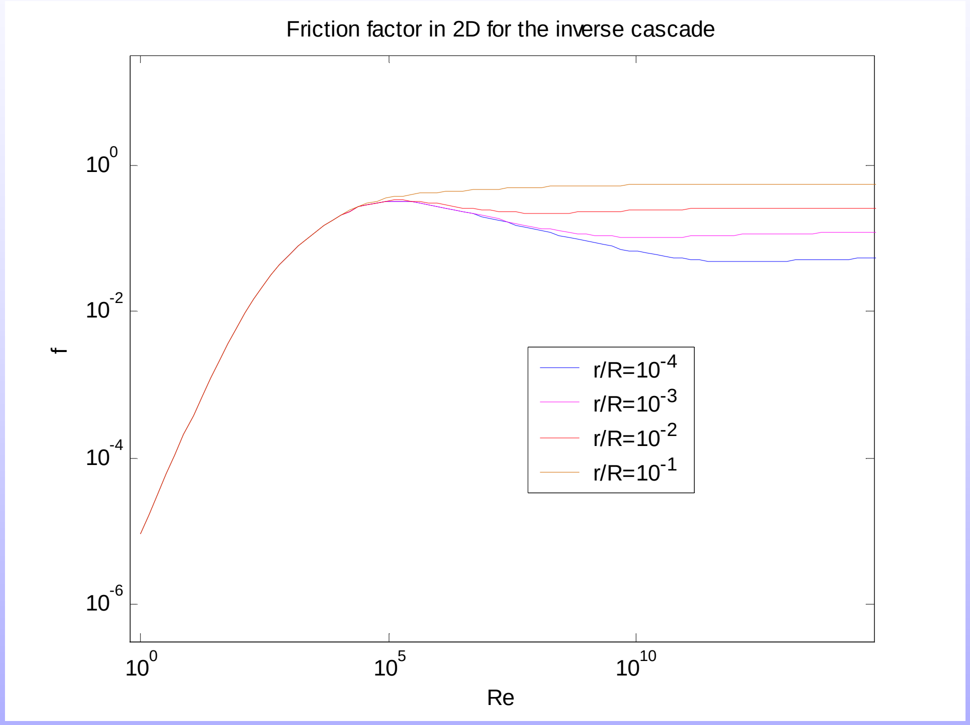
- **Test scaling argument: predict friction factor for a turbulent soap film suspended between rough wires: a 2D rough turbulent pipe**
  - **Now we use enstrophy cascade: E (σ) ~ σ^3**
  - **2D number of degrees of freedom scales as $Re^{2/4}$**
    - **$s/D = r/D + \text{const.}\ Re^{-1/2}$**

$$f = \left[\int_0^{s/R} \sigma^3 \frac{d\sigma}{\sigma^2} \right]^{1/2}$$

- **Blasius scaling: f ~ $Re^{-1/2}$**
- **Strickler scaling: f ~ (r/D)**

Evaluation of friction factor integral in 2D





Enstrophy cascade

$f \sim Re^{-1/2}$ (Blasius)

$f \sim (r/D)$ (Strickler)

Inverse cascade

$f \sim Re^{-1/6}$ (Blasius)

$f \sim (r/D)^{1/3}$ (Strickler)

Ongoing work

- Experiments on 2D soap films to test predictions for the friction factor (W. Goldburg)
- DNS of 2D rough pipes
- Block spin derivation of friction factor: improved calculation of momentum transfer
- Nikuradse experiment needs to be repeated.
  - Role of different forms of roughness?
  - Influence of correlations in roughness?
  - Probe mechanisms of momentum transfer
  - Verify identification of the features in the friction factor

Conclusion

- **Collapse of turbulent rough pipe flow data indicative of a governing non-equilibrium critical point**
- **Anatomy of friction factor curves obtained**
  - **But overall quantitative scale needs to be understood**
- **Friction factor in 2D predicted, and computational and experimental tests underway**