Multifractal PDF Analysis of Turbulence

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Outlines

- We will introduce the formulation of Multifractal Probability Density Function Analysis (MPDFA)
  - that provide us with a unified formulation of PDFs for variables representing intermittent large deviations due to the invariance of basic equations under a scale transformation.
- As we have succeeded to analyze PDFs of velocity fluctuations, of velocity derivatives and of fluid particle accelerations, we proceed, in this talk,
  - to the analyses of actual PDFs of energy transfer rates and of energy dissipation rates with the help of PDFs within MPDFA, and
  - to a new route to obtain the generalized dimension from actual PDF data by separating those contributions violating the scaling invariance in the analyses of negative moments, and by complementing a rack of extremely rare events data in actual PDF by theoretical PDF in the analyses of positive higher moments.

Historical survey

era	major evolution of researches
1513	sketch of turbulence (Leonardo da Vinci)
1883	start of systematic experiments (Reynolds)
1941	start of theoretical works (Kolmogorov)
44	criticism of K41, Landau equation (Landau)
48	turbulent viscosity (Heisenberg; 55 Chandrasekhar)
55	vortex tangle, quantum turbulence (Feynman)
57	closure problem (e.g.,Tatsumi)
59	DIA (Kraichnan; 64 Edwards, 65 Novikov)
62	log-normal model (Oboukhov, Kolmogorov; 66 Yaglom)
65	Lagrangian picture (Kraichnan)
77	concept of fractals (Mandelbrot)
	Renormalization Group (Forster-Nelson; 86 Yakhot)
78	β model (Frisch-Nelkin)
81	Lagrangian renormalized approximation (Kaneda)
84	random β model (Benzi)
	fractal objects and strange sets (Procaccia)
85	distribution of singularities (Frisch-Parisi)
87	p model (Meneveau-Sreenivasan)
91	generalized class of Cantor-set models (Hosokawa)
94	log-Poisson model (She-Leveque)
	re-summation (Procaccia)
	Lagrangian method (Pope; 99 Reynolds, 00 Sawford-Yeung)
2000	A&A model; MPDFA (Arimitsu-Arimitsu)
01	direct observation of singularities (Bodenschatz)

Table 1.1. Historical survey

Scale invariance

The Navier-Stokes equation for an incompressible fluid

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} = -\vec{\nabla}p + \nu\nabla^2\vec{u}$$

ρ : mass density
p : pressure / ρ
ν : kimenatical viscosity

is invariant under the scale transformation

$$\vec{x} \to \vec{x}' = \lambda\vec{x}, \quad \vec{u} \to \vec{u}' = \lambda^{\alpha/3}\vec{u}, \quad t \to t' = \lambda^{1-\alpha/3}t, \quad p \to p' = \lambda^{2\alpha/3}p$$
$$\nu \to \nu' = \lambda^{1+\alpha/3}\nu$$

for an arbitrary real number α.
This leads to the scalings

$$\left|\frac{\delta u_n}{\delta u_0}\right| = \delta_n^{\alpha/3}, \quad \left|\frac{\delta p_n}{\delta p_0}\right| = \delta_n^{2\alpha/3} \qquad \frac{|\delta \epsilon_n|}{\epsilon} = \delta_n^{\alpha-1}.$$

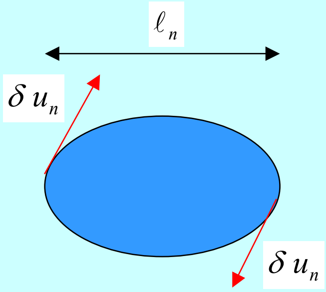
$$\delta u_n = |u(\bullet + \ell_n) - u(\bullet)|, \qquad \delta p_n = |(p/\rho)(\bullet + \ell_n) - (p/\rho)(\bullet)|$$

$$\ell_n = \ell_{\text{in}}\delta_{n-n_{\text{in}}}, \quad \delta_n = \delta^{-n} \ (n = 0, 1, 2, \cdots), \quad \delta > 1$$

$\ell_{n=n_{\text{in}}} = \ell_{\text{in}}$ **: Integral scale**

$\ell_n = \delta_n \ell_0 \quad (\delta_n = 2^{-n})$

A schematic interpretation



The eddy of the *n*th generation in the length cascade

$$\ell_n = \delta_n \ell_0 \quad (\delta_n = 2^{-n})$$

$$E_n = \tfrac{1}{2}\delta u_n^{\ 2} = \int_{k_n}^{k_{n+1}} dk\, E(k) : \text{ kinetic energy per unit mass in scale } \ell_n = k_n^{-1}$$

$$t_n = \ell_n / \delta u_n : \text{ eddy turnover time}$$

$$\delta\varepsilon_n = E_n / t_n \sim \delta u_n^{\ 3} / \ell_n : \text{ energy transfer rate that represents}$$

the rate of transfer of energy per unit mass from eddies of size ℓ_n to those of size ℓ_{n+1}.

$$\delta\varepsilon_n \in (-\infty, \infty)$$

"Singularities" 1/3

With the length $\ell_n = \delta_n \ell_0 \quad (\delta_n = 2^{-n})$, these quntities are given as follows.

$$|\vec{u}'| = \lim_{n\to\infty} u'_n = \lim_{\ell_n \to 0} \frac{\delta u_n}{\ell_n} \sim \lim_{\ell_n \to 0} \ell_n^{\frac{1}{3}\alpha - 1}, \qquad \delta u_n = |\, u(\bullet + \ell_n) - u(\bullet)\,|,$$

$$|\vec{a}| = \lim_{n\to\infty} a_n = \lim_{\ell_n \to 0} \frac{\delta p_n}{\ell_n} \sim \lim_{\ell_n \to 0} \ell_n^{\frac{2}{3}\alpha - 1}, \qquad \delta p_n = |(p/\rho)(\bullet + \ell_n) - (p/\rho)(\bullet)|,$$

$$\lim_{n\to\infty} \delta\varepsilon_n = \lim_{\ell_n \to 0} \left(\frac{\ell_n}{\ell_0} \right)^{\alpha - 1} \sim \lim_{\ell_n \to 0} \ell_n^{\ \alpha - 1}$$

where $\vec{a} = \dfrac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\, \vec{u}$ is the acceleration of fluid particle.

The velocity derivative, the acceleration and the energy transfer rate become, respectively, singular for $\alpha < 3$, $\alpha < 1.5$ and $\alpha < 1$ in the limit $\ell_n \to 0$.

When α <1, all these three quantities become large for each ℓ_n.

“Singularities” 2/3

The energy dissipation rate ε_n within the inertial range is introduced through

$$|\delta\epsilon_n| \sim \varepsilon_n \equiv \nu_n \left(\frac{\delta u_n}{\ell_n}\right)^2$$

with an effective viscosity (turbulent viscosity) ν_n satisfying the scale transformation, i.e.,

$$\frac{\nu_n}{\nu_0} = \delta_n^{1+\alpha/3},$$

The energy dissipation rates behaves as

$$\frac{\varepsilon_n}{\epsilon} = \delta_n^{\alpha-1}. \qquad \varepsilon_n \in [0, \infty)$$

having the same exponent as the energy transfer rates.

“Singularities” 3/3

The substitution for ε_n is usually given by the average of the microscopic dissipation rate $\breve{\varepsilon}(\mathbf{r})$ per unit mass, whose origin is due to the dissipation term in N-S equation, over the space in a volume element ΔV_n , i.e.,

$$\varepsilon_n \sim \frac{\int_{\Delta V_n} d^d r \; \breve{\varepsilon}(\vec{r})}{\ell_n^d}$$

Here, the volume of the element ΔV_n is chosen to be $\ell_n^{\,d}$ with *d* being the dimension of real space.

Definitions 1/2

Following Meneveau and Sreenivansan (1987), we introduce the mass exponent $\tau(\bar{q})$ through

$$Z_d^{(n)} \equiv \sum_{\#\text{ of boxes}} \left(\frac{\varepsilon_n \ell_n^d}{\epsilon\ \ell_0^d} \right)^{\bar{q}} = \sum_{\#\text{ of boxes}} \delta_n^{(\alpha-1+d)\bar{q}} \propto \delta_n^{-\tau_d(\bar{q})}$$

The summation can be translated into the integration with respect to α with the help of the maltifractal spectrum *f*(α) as

$$Z_d^{(n)} = \int d\alpha \rho(\alpha)\ \delta_n^{(\alpha-1+d)\bar{q}-f_d(\alpha)}$$

which provides us with the Legendre transformation

$$f_d(\alpha) - (\alpha - 1 + d)\bar{q} = \tau_d(\bar{q}), \quad \bar{q} = \frac{df_d(\alpha)}{d\alpha} \qquad \alpha - 1 + d = -\frac{d\tau_d(\bar{q})}{d\bar{q}}$$

The generalized dimension $D_{\bar{q}}$ is introduced by the relation

$$\tau_d(\bar{q}) = (1-\bar{q})D_{\bar{q}}$$

Definitions 2/2

The $\bar{q}\text{th}$ moment of the energy dissipation rate can be expressed by means of the mass exponent as

$$\left\langle \left(\frac{\varepsilon_n}{\epsilon} \right)^{\bar{q}} \right\rangle \equiv \int_0^\infty d\left(\frac{\varepsilon_n}{\epsilon}\right) \left(\frac{\varepsilon_n}{\epsilon}\right)^{\bar{q}} P_\varepsilon^{(n)} \left(\frac{\varepsilon_n}{\epsilon}\right) \sim \delta_n^{-\tau_d(\bar{q})+D_0-\bar{q}d}$$

where,

$$P_\varepsilon^{(n)} \left(\frac{\varepsilon_n}{\epsilon}\right) d\left(\frac{\varepsilon_n}{\epsilon}\right) P_{\varepsilon_n \neq 0}^{(n)} = P^{(n)}(\alpha) d\alpha$$

with

$$P_{\varepsilon_n \neq 0}^{(n)} \propto \frac{\#\text{of boxes satisfying } \varepsilon_n \neq 0 \text{ that covers the space with dim. } D_0}{\#\text{of boxes covering whole the space with dim. } d} = \frac{\delta_n^{-D}}{\delta_n^{-d}}$$

$$P^{(n)}(\alpha)d\alpha \propto \frac{\#\text{of boxes covering whole the space with dim. } f(\alpha)}{\#\text{of boxes covering whole the space with dim. } d} = \frac{\delta_n^{-f(\alpha)}d\alpha}{\delta_n^{-d}}$$

Note that we need δ_n^{-d} boxes (the vol. of each box is ℓ_n^d) to cover whole the space of vol. ℓ_0^d in *d* dim. space without vacancy.

$$\delta_n = \ell_n / \ell_0$$

$P^{(n)}(\alpha)$ plays the central role in the following formulation.

Mpdfa 1/5

MPDFA starts with the assignment of the probability, to find a singularity specified by the strength α within the range α~α+$d\alpha$, in the form

$$P^{(n)}(\alpha)d\alpha = \sqrt{\frac{|f''(\alpha_0)||\ln\delta_n|}{2\pi}}\ \delta_n^{1-f(\alpha)}\ d\alpha$$

Here, $f(\alpha)$ represents an appropriate multifractal spectrum defined in the range $\alpha_{\min} \le \alpha \le \alpha_{\max}$.

Then,

$$\left\langle \left(\frac{\epsilon_n}{\epsilon}\right)^{\bar q} \right\rangle = a_{3\bar q}\ \delta_n^{-\tau(\bar q)+1-\bar q}$$

with

$$a_{3\bar q} = \sqrt{\frac{|f''(\alpha_0)|}{|f''(\alpha_{\bar q})|}} \qquad \alpha_{\bar q} = -\frac{d\tau(\bar q)}{d\bar q}$$

Mpdfa 2/5

Let us now derive expressions of PDFs for observables

$$\delta x_n = x(\bullet + \ell_n) - x(\bullet)$$

of a physical quantity related to α by the relation

$$|x_n| \equiv \left|\frac{\delta x_n}{\delta x_0}\right| = \delta_n^{\phi\alpha/3}$$

Then, the spatial derivative defined by

$$|x'| = \lim_{\ell_n \to 0} \frac{\delta x_n}{\ell_n} \propto \lim_{n\to\infty} \ell_n^{\phi\alpha/3-1}$$

diverges when $\alpha < 3/\phi$.

The quantity *x'* reduces to the velocity derivative and fluid particle acceleration for $\phi = 1$ and $\phi = 2$, respectively, and formally to the energy transfer rate or the energy dissipation rate for $\phi = 3$.

Mpdfa 3/5

Now, we assume that the probability to find the physical quantity x_n taking a value in the domain $x_n \sim x_n + dx_n$ can be, generally, divided into two parts as

$$\Pi_{\phi}^{(n)}(x_n)dx_n = \Pi_{\phi,\mathrm{S}}^{(n)}(x_n)dx_n + \Delta\Pi_{\phi}^{(n)}(x_n)dx_n$$

Here, the first term describes the contribution from the abnormal part of the physical quantity x_n due to the fact that its singularities distribute themselves multifractal way in real space. This is the part given by

$$\Pi_{\phi,\mathrm{S}}^{(n)}(|x_n|)d|x_n| \propto P^{(n)}(\alpha)d\alpha \qquad |x_n| \equiv \left|\frac{\delta x_n}{\delta x_0}\right| = \delta_n^{\phi\alpha/3}$$

On the other hand, the second term represents the contributions from the dissipation term that violates the invariance based on the scale transformation.

Mpdfa 4/5

For those PDFs with variables whose domain is $(-\infty, \infty)$, we symmetrize the PDFs before we start analyses when they are not symmetric, under the assumption that *the intermittency manifests itself in the deviations from the mean value of the quantity under consideration*.

Mpdfa 5/5

The formula for the *mth order structure function* (moments) of the variable $|x_n|$ is given by

$$\langle\langle |x_n|^m \rangle\rangle_\phi \equiv \kappa \int_0^\infty dx_n |x_n|^m \Pi_\phi^{(n)}(x_n) = \kappa\gamma_{\phi,m}^{(n)} + \left(1-\kappa\gamma_{\phi,0}^{(n)}\right) a_{\phi m}\ \delta_n^{\zeta_{\phi m}}$$

with

$$\gamma_{\phi,m}^{(n)} = \int_0^\infty dx_n |x_n|^m \Delta\Pi_\phi^{(n)}(x_n), \qquad a_{\phi m} = \sqrt{\frac{|f''(\alpha_0)|}{|f''(\alpha_{\phi m/3})|}}$$

$$\zeta_{\phi m} = 1 - \tau\left(\frac{\phi m}{3}\right).$$

$\kappa = 1$: $[0, \infty)$; $\kappa = 2$: $(-\infty, \infty)$ **Normalization:** $\kappa \int_0^\infty dx_n \Pi_\phi^{(n)}(x_n) = 1$

The generalized dim. is given by $\zeta_{3\bar{q}} = 1 - (1-\bar{q})D_{\bar{q}}, \quad (-\infty < \bar{q} < \infty)$

A&A model

A&A model assumes that the distribution of α is given by the Tsallis-type distribution function

$$P^{(n)}(\alpha) = \frac{1}{Z^{(n)}}\left[1 - \frac{(\alpha-\alpha_0)^2}{(\Delta\alpha)^2}\right]^{n/(1-q)}$$

with $(\Delta\alpha)^2 = 2X/(1-q)\ln 2$ **. Then, the multifractal spectrum has the form**

$$f(\alpha) = 1 - \frac{1}{1-q}\log_2\left[1 - \frac{(\alpha-\alpha_0)^2}{(\Delta\alpha)^2}\right]$$

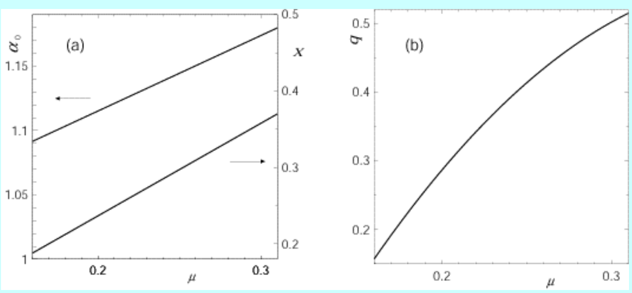
Three parameters α_0, X, q are determined by the conditons

$$\langle \varepsilon_n/\varepsilon \rangle = 1, \quad \langle (\varepsilon_n/\varepsilon)^2 \rangle = \delta_n^{-\mu}, \quad \frac{1}{1-q} = \frac{1}{\alpha_-} - \frac{1}{\alpha_+}.$$

$f(\alpha_\pm) = 0$

Note that $\langle \cdots \rangle$ is taken with $P^{(n)}(\alpha)$.

A generalization of the scaling relation proposed by Lyra & Tsallis (1998).



Choice of $P^{(n)}(\alpha)$
P model: Binomial dist.
Log-normal model: Gaussian dist.

A&A model 2/3

The **mass exponents**, the **scaling exponents** and **other important quantities**

$$\tau(\bar{q}) = 1 - \alpha_0 \bar{q} + \frac{2X\bar{q}^2}{1+\sqrt{C_{\bar{q}}}} + \frac{1}{1-q}\left[1 - \log_2\left(1+\sqrt{C_{\bar{q}}}\right)\right]$$

$$\zeta_m = \frac{\alpha_0 m}{3} - \frac{2Xm^2}{9\left(1+\sqrt{C_{m/3}}\right)} - \frac{1}{1-q}\left[1 - \log_2\left(1+\sqrt{C_{m/3}}\right)\right]$$

$$f''(\alpha_{\bar{q}}) = -\frac{\sqrt{C_{\bar{q}}}\left(1+\sqrt{C_{\bar{q}}}\right)}{2X} \qquad a_{3\bar{q}} = \sqrt{\frac{2}{\sqrt{C_{\bar{q}}}\left(1+\sqrt{C_{\bar{q}}}\right)}}$$

$$\alpha_{\bar{q}} = \alpha_0 - \frac{2\bar{q}X}{1+\sqrt{C_{\bar{q}}}}$$

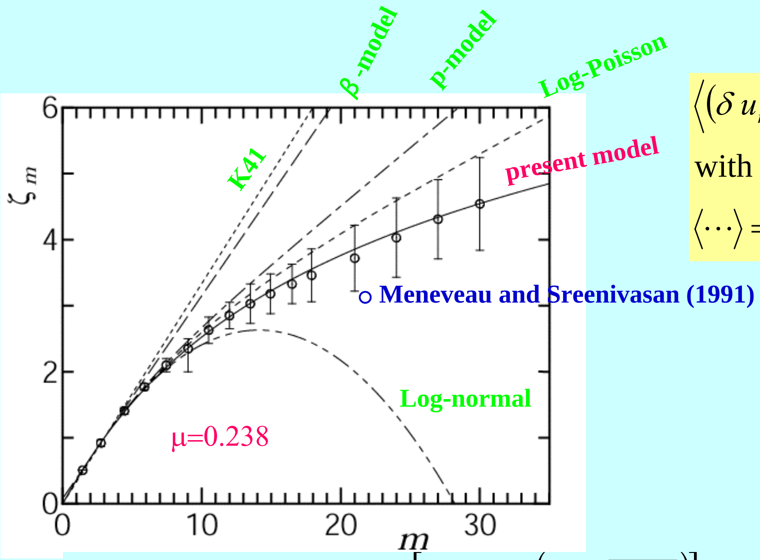
are expressed in terms of

$$C_{\bar{q}} = 1 + 2\bar{q}^2(1-q)X\ln 2$$

Note that q < 1 in the following analyses, hence C_q > 1.

Comparison

Scaling exponents ζ_m of velocity structure function



$$\left\langle (\delta u_n)^m \right\rangle \propto \ell_n^{\zeta_m}$$

with $\delta u_n = \left| u(\bullet + \ell_n) - u(\bullet) \right|,$

$$\langle \cdots \rangle = \int d\alpha \cdots P(\alpha).$$

$$\zeta_m = \frac{\alpha_0 m}{3} - \frac{2Xm^2}{9(1+\sqrt{C_{m/3}})} - \frac{1}{1-q}\left[1 - \log_2\left(1+\sqrt{C_{m/3}}\right)\right]$$

with **present model (2000)**

$$C_{\bar{q}} = 1 + 2X\bar{q}^2(1-q)\ln 2$$

$$\sqrt{2X} = \frac{\sqrt{\alpha_0^2 + (1-q)^2} - (1-q)}{\sqrt{b}}, \quad b = \frac{1-2^{q-1}}{(1-q)\ln 2}.$$

K41 (1941)

$\zeta_m = m / 3$

Log-normal (1962)

$\zeta_m = m / 3 - \mu\, m(m-3) / 18$

β -model (1978)

$\zeta_m = m / 3 - \mu\, (m-3) / 3$

p-model (1987)

$\zeta_m = 1 - \log_2 [p^{m/3} + (1-p)^{m/3}]$

$p = (1 + (2^{\mu} - 1)^{1/2})/2$

Log-Poisson (1994)

$\zeta_m = m / 9 + 2(1 - (2/3)^{m/3})$

Note

- $P^{(n)}(\alpha)$: distribution of α
  $\rightarrow f(\alpha)$: multifractal spectrum
- $\tau(\bar{q})\left(=f(\alpha)-\alpha\bar{q}\right)$: mass exponents
  $\rightarrow \varsigma_m \left(=1-\tau(m/3)\right)$: scaling exponents of VSF
- The argument up to here on the scaling exponents is the usual one referring only to the tail part of PDF representing the characteristics of the scaling invariance.
- In treating actual data, the contributions to the central part of PDF should be taken into account.

A&A model 3/3

Tail part for variables ξ_n with the ranges both $(-\infty,\infty)$ and $[0,\infty)$

$$\hat{\Pi}^{(n)}_{\phi,\mathrm{tl}}(\xi_n) = \bar{\Pi}^{(n)}_{\phi}\frac{\bar{\xi}_n}{|\xi_n|}\left[1-\frac{1-q}{n}\frac{(3\ln|\xi_n/\xi_{n,0}|)^2}{2\phi^2 X|\ln\delta_n|}\right]^{n/(1-q)}$$

Center part for variables ξ_n with the range $(-\infty,\infty)$

$$\hat{\Pi}^{(n)}_{\phi,\mathrm{cr}}(\xi_n) = \bar{\Pi}^{(n)}_{\phi}\left\{1-(1-q')\frac{\phi+3f'(\alpha^*)}{2\phi}\left[\left(\frac{\xi_n}{\xi_n^*}\right)^2-1\right]\right\}^{1/(1-q')}$$

Center part for variables ξ_n with the range $[0,\infty)$

$$\hat{\Pi}^{(n)}_{\phi,\mathrm{cr}}(\xi_n) = \bar{\Pi}^{(n)}_{\phi}\left(\frac{\xi_n}{\xi_n^*}\right)^{\theta-1}\left\{1-(1-q')\frac{\phi\theta+3f'(\alpha^*)}{2\phi}\left[\left(\frac{\xi_n}{\xi_n^*}\right)^2-1\right]\right\}^{1/(1-q')}$$

$$\xi_n = \frac{x_n}{\sqrt{\langle\langle (x_n)^2\rangle\rangle}}$$

$$|x_n| \equiv \frac{\delta x_n}{\delta x_0} = \delta_n^{\alpha\phi/3}$$

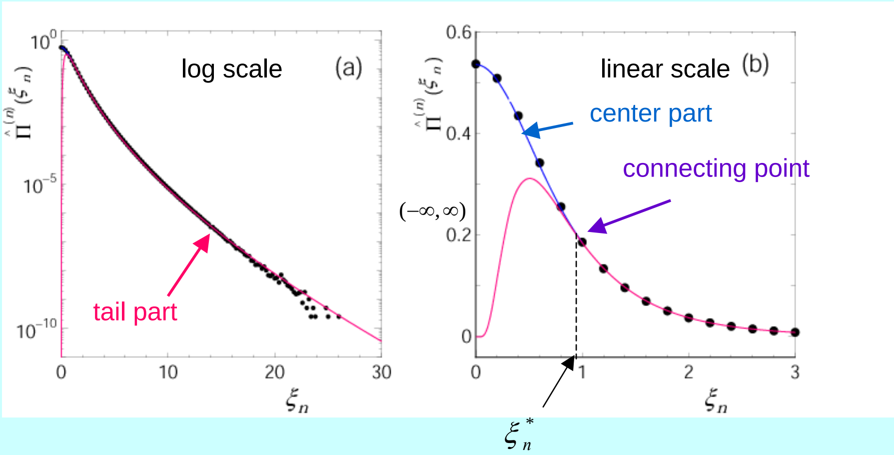
$$\delta_n = \frac{\ell_n}{\ell_0} = \delta^{-n}$$

$$\delta x_n = |x(\bullet+\ell_n)-x(\bullet)|$$

$\phi = 1$: veloocity fluctuations and derivatives
$\phi = 2$: pressure fluctuations and fluid particle accelerations
$\phi = 3$: energy transfer rates and dissipation rates

Structure of PDFs with the range $(-\infty, \infty)$

We are assuming that the contribution to PDF from intermittent large deviations is symmetric for the variables with the range $(-\infty, \infty)$.



It is revealed through the analyses of actual data that the minimal structure of PDF having fat tail should have at least two parts, i.e,

- **one is the tail part, and**
- **the other the center part.**

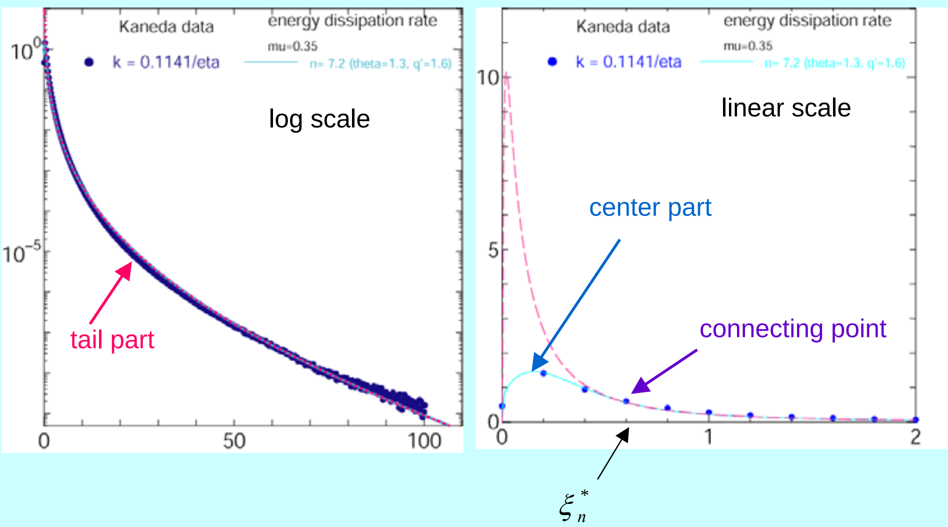
$$\frac{\zeta_{2\phi}}{2} - \frac{\phi\alpha}{3} + 1 - f(\alpha) = 0.$$

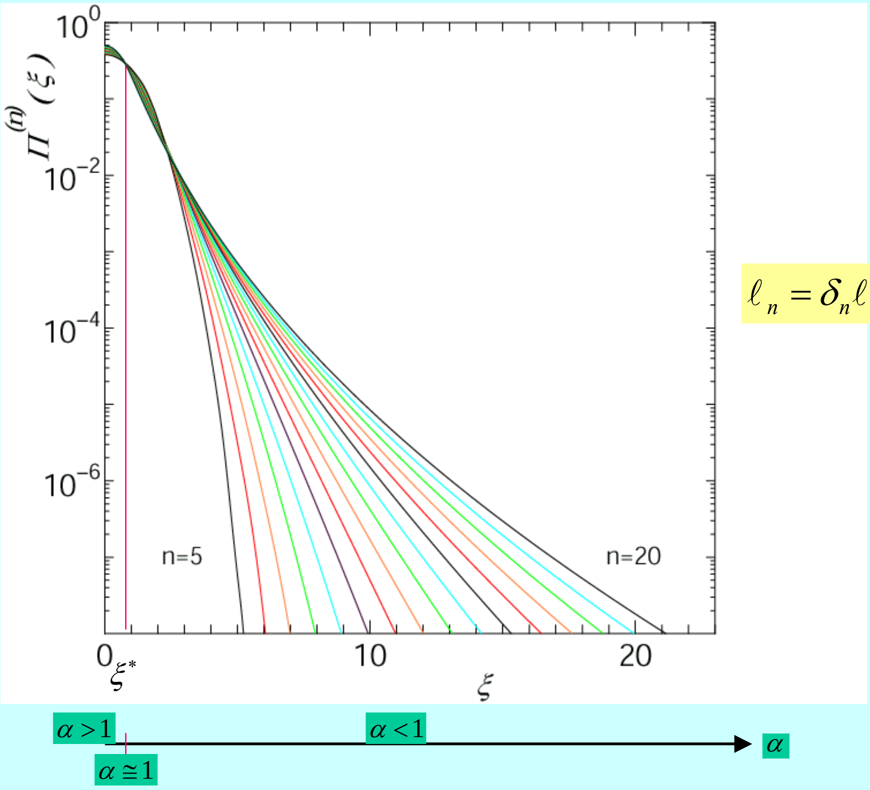
Structure of PDFs with the range $[0, \infty)$

It is also shown that the minimal structure of PDF for energy dissipation rates should have at least two parts, i.e,

- **one is the tail part, and**
- **the other the center part.**

$$\frac{\zeta_{2\phi}}{2} - \frac{\phi\alpha}{3} + 1 - f(\alpha) = 0.$$





$\ell_n = \delta_n \ell_0 \quad (\delta_n = 2^{-n})$

Parameters

The parameters are determined in order that the *zooming increment* $\Delta n = n' - n$ coincides with the increment $r - r'$ corresponding to the process how one extracted the series of PDFs by changing the consecutive distances $r = l_n$ and $r' = l_{n'}$ between two observing points, i.e.,

$$n = -\log_\delta(r/\eta) + \log_\delta(\ell_0/\eta)$$

The tail part of PDF is determined mainly by the intermittency exponent μ and the multifractal depth n (or, equivalently, the distance l_n), and the central part by the entropy index q' and θ.

Note that the values of parameters a_0, X and q are determined as functions of μ, self-consistently.

Wind tunnel 1/3

The graph shows the time dependence of the quantity closely related to the energy dissipation rates ε_n, obtained from the time-series data of rough-wall turbulence measured at boundary-layer in a wind tunnel.
(Mouri et al., Phys.Rev.E, 2004)

The Reynolds number for this experiment is Re_λ = 1258.

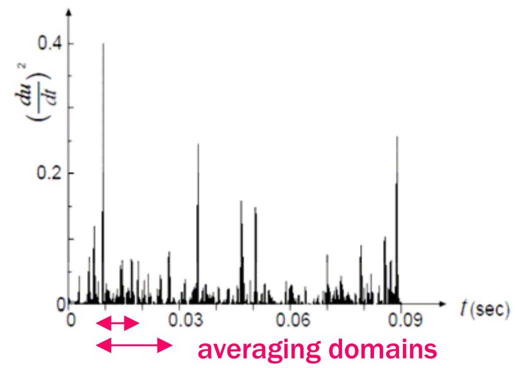
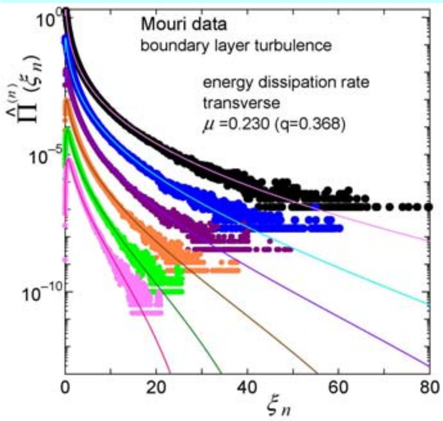
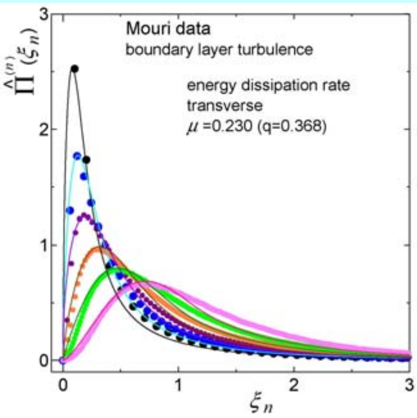


Figure 4: The time dependence of the quantity proportional to $\nu u'^2$ (cm²/sec⁴).

Wind tunnel 2/3

PDF of energy dissipation rates





$$n = -0.994 \log_2 r/\eta + 15.1$$

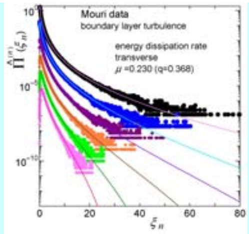
Closed circles
Experimental PDFs by Mouri
r/η from top to bottom:
32.6, 65.1, 130, 260, 521, 1040
with η = 0.106 mm
Inertial range: 67.3< r/η <642

Lines
Theoretical PDF with q = 0.368
(μ = 0.230) by A&A model
(n, q', θ) from top to bottom:
(10.1, 1.88, 1.50), (9.00, 1.81, 1.60), (8.18, 1.76, 1.45), (7.20, 1.54, 1.65), (6.20, 1.35, 1.90), (5.20, 1.10, 2.40)

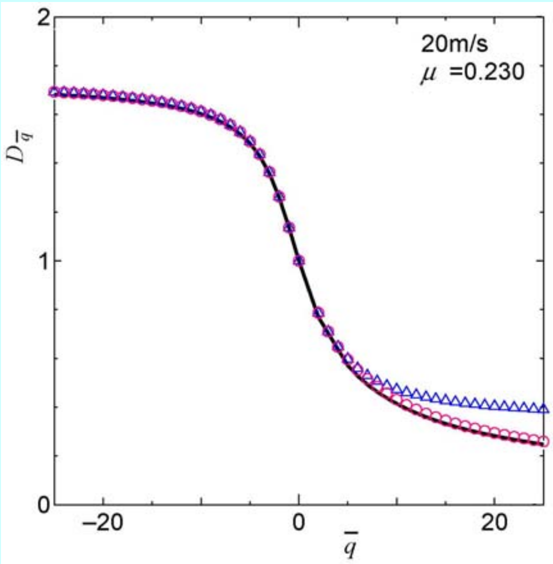
Connection pts. ξ_n^* from top to bottom:
0.835, 0.844, 0.856, 0.899, 0.992, 1.12 (α^*= 0.948)
ξ_n^{max} from top to bottom:
571, 283, 169, 94.3, 54.5, 32.2

For better visibility, the left PDF is shifted by −1 unit along the vertical axis.

Wind tunnel 3/3



Generalized dimension



generalized dimension (theory)

generalized dimension (data) by separating those contributions violating the scaling invariance, and by complementing a rack of extremely rare events data in actual PDF by theoretical PDF in the analysis of positive higher moments.

generalized dimension (data) by separating those contributions violating the scaling invariance, but without the complement.

$$\langle\langle |x_n|^m \rangle\rangle_\phi \equiv \kappa \int_0^\infty dx_n |x_n|^m \Pi_\phi^{(n)}(x_n) = \kappa \gamma_{\phi,m}^{(n)} + \left(1 - \kappa \gamma_{\phi,0}^{(n)}\right) a_{\phi m}\ \delta_n^{\zeta_{\phi m}}$$

$$\zeta_{3\bar{q}} = 1 - (1-\bar{q}) D_{\bar{q}}, \quad (-\infty < \bar{q} < \infty)$$

DNS 4096^3 1/3

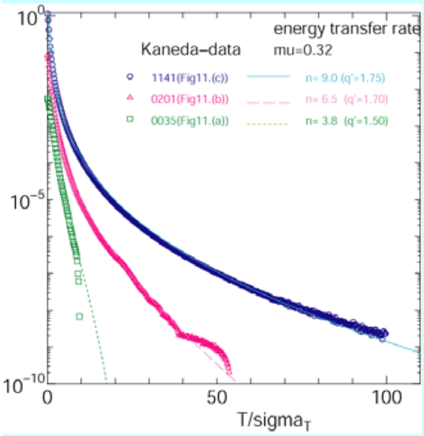
We will analyze the PDFs of energy transfer rates and of energy dissipation rates measured in the DNS on 4096^3 mesh size by Kaneda's group.
(Aoyama et al., J.Phys.Soc.Japan, 2005)

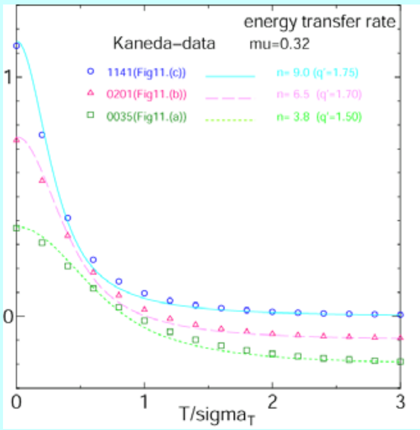
The Reynolds number for this DNS is $Re_\lambda = 1132$.

The observed PDFs of energy transfer rates are made symmetric by averaging the data on the left and the right hand sides.

DNS 4096^3 2/3

PDFs of energy transfer rates (symmetrized)





$$n = -1.04 \log_2 r/\eta + 13.0,$$

Closed circles

Experimental PDFs by Kaneda

r/η from top to bottom:

13.7, 78.1, 449

with η = 5.12 x 10^{-4}

Inertial range: 62.8< r/η <224

Lines

Theoretical PDF with q = 0.534

(μ = 0.320) by A&A model

(n, q') from top to bottom:

(9.00, 1.75), (6.50, 1.70), (3.80, 1.50)

Connection pts. ξ_n^* from top to bottom:

0.477, 0.637, 0.882 (α^*= 0.928)

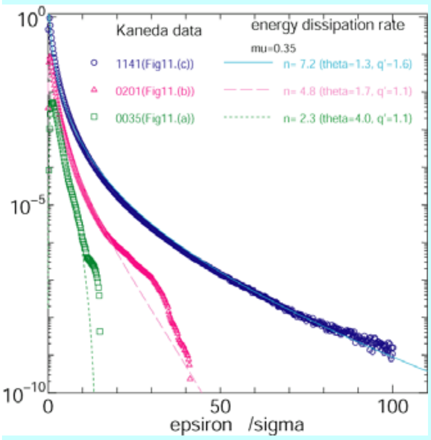
ξ_n^{max} from top to bottom:

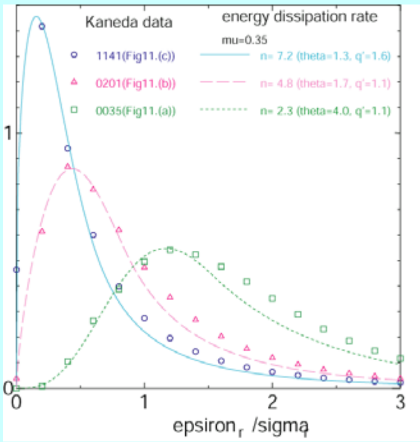
1400, 203, 25.7

For better visibility, the left and right PDFs are respectively shifted by −1 and by - 0.1 unit along the vertical axis.

DNS 4096^3 3/3

PDFs of energy dissipation rates





$$n = -0.995 \log_2 r/\eta + 11.1,$$

Closed circles

Experimental PDFs by Kaneda

r/η from top to bottom:

13.7, 78.1, 449

with η = 5.12 x 10^{-4}

Inertial range: 62.8< r/η <224

Lines

Theoretical PDF with q = 0.568

(μ = 0.345) by A&A model

(n, q', θ) from top to bottom:

(7.35, 1.59, 1.30), (4.90, 1.10, 1.70), (2.35, 1.10, 4.10)

Connection pts. ξ_n^* from top to bottom:

0.597, 0.839, 1.53 (α^*= 0.922)

ξ_n^{max} from top to bottom:

676, 95.7, 14.5

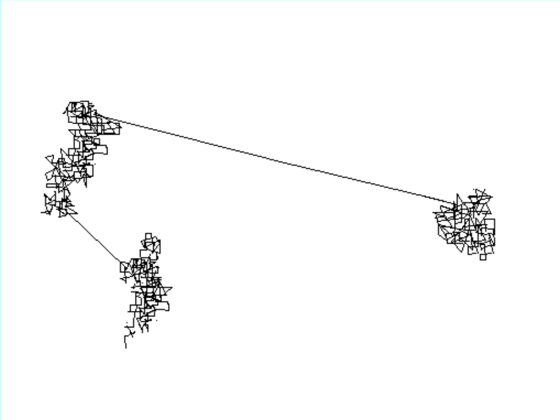
For better visibility, the left PDF is shifted by −1 unit along the vertical axis.

Summary and prospects 1/2

It has been revealed that there exist two main contributions to PDFs of those variables representing intermittent large deviations.

The tail part of the PDFs is determined mainly by the global structure of turbulence representing its intermittent character, which is the outcome of the multifractal distribution of singularities in real space.

The shape of the central part is a reflection of local structure of flow fields representing a wave and oscillation of vortex due to the interaction between vortices and so on.



Summary and prospects 2/2

We expect that MPDFA can be a clue to search for the fundamental process of intermittency, i.e., the origin of singularities and the reason why the singularities distribute themselves multifractal way, etc., which may provide us with a fruitful insight to produce something for the dynamical approach.

It is one of the attractive future problems to find out two different dynamics, i.e., the one determines the tail part of PDF, and the other the central part of PDF.

When the underlying dynamics of MPDFA is revealed by starting the consideration with N-S equation, it may provide us with new route to extract intermittency from the dynamical point of view, e.g., an appropriate RG pathway to intermittency.

Thank you for your attention.