# MA3E1 Groups and Representations
## Assignment 4
## Deadline: Friday 30 November 2012, 3pm

Write your name and student number on your solution sheet. Mention your department if it is not mathematics.

**(4.1)** Let $\rho$ be an irreducible representation of a finite group $G$. Prove $\sum_{g \in G} \rho(g) = 0$ unless $\rho$ is the trivial representation of degree 1.

**(4.2)** Let $H \leq K \leq G$ be finite groups and $p$ a class function on $H$. Prove $(p^K)^G = p^G$.

**(4.3)** Let $H \leq G$ be finite groups and $p$ a nonzero character of $H$. Prove that

$$\ker(p^G) = \bigcap_{x \in G} x^{-1}(\ker\, p)x.$$

Where does your proof break down if $p = 0$?

**(4.4)** In this exercise you will compute the character table for $G := S_5$ by induction from $A_5$ and $S_4$. Recall the **permutation character** $\chi^p$ which is the character of the representation $\rho^p \colon S_n \to \mathrm{GL}(n, \mathbb{C})$ defined by $(\rho^p s)e_i = e_{si}$ for all $s \in S_n$.

(a) For each conjugacy class in $S_5$, find its cardinality and give one element.

(b) Find two linear characters $\chi_1$, $\chi_2$ of $S_5$ where $\chi_1$ is trivial.

(c) Compute $\chi_3 := \chi^p - \chi_1$ and $\chi_4 := \chi_3\chi_2$ and prove that they are irreducible characters.

(d) Choose an irreducible character $\phi$ of $A_5$ of degree 3. Compute $\chi_5 := \phi^G$ and prove that it is irreducible.

(e) Let $\mu$ be the following irreducible character of $S_4$:

| | 1 | (12) | (12)(34) | (123) | (1234) |
|---|---|---|---|---|---|
| $\mu$ | 3 | $-1$ | $-1$ | 0 | 1 |

Compute $\chi_6 := \mu^G - \chi_5 - \chi_4$ and prove that it is an irreducible character.

(f) Finish the character table.