

Test — MA3E1 Groups and Reps

Name: _____

Version 1

Student Number: _____

Tuesday 16 November 2010, 1–2pm

Department: _____

This test covers the material lectured up to and including chapter 5. You can bring any notes you like, but no electronic gadgets or library books.

Every correct answer is worth 1 mark; every incorrect answer or no answer worth 0 marks. Workings or proofs are neither required nor can give marks. Be clever and answer all multiple choice questions even if you have no clue!

Give your answers on this sheet and hand it in. You are free to make scratchy notes, but if you make many, please make them on another sheet. No name means no marks. Good luck!

- Let ρ be the representation of D_6 defined by $\rho(r) = \frac{1}{2} \begin{pmatrix} -1 & 3 \\ -1 & -1 \end{pmatrix}$, $\rho(s) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Suppose that ρ is afforded by (V, A) where V is some D_6 -module and $A = (v_1, v_2)$ a basis for V . Find an explicit D_6 -invariant inner product $\langle \cdot, \cdot \rangle$ on V , for example by giving the 2×2 matrix $(\langle v_i, v_j \rangle)_{ij}$.

- Give an example of a non-cyclic group G and an element $g \in G$ such that $\langle\langle g \rangle\rangle = G$.

- Consider the group, defined by a presentation, $G = \langle a, b \mid a^9, b^9, a^4b^5 \rangle$. Find all homomorphisms $G \rightarrow S_3$ where S_3 is the symmetric group on $\{1, 2, 3\}$.

- Consider the statements:

- Let $f: V \rightarrow W$ be a homomorphism of $\mathbb{C}G$ -modules. Then f is 0 or an isomorphism.
- Let ρ_1, \dots, ρ_k be irreducible representations of a finite group G and $\sigma = n_1 \rho_1 \oplus \dots \oplus n_k \rho_k$ where $n_i \in \mathbb{Z}_{\geq 0}$. Then $(\chi_\sigma, \chi_\sigma)_G = n_1^2 + \dots + n_k^2$.

Which of the following is correct?

- Consider the representation ρ of $C_2 = \langle c \mid c^2 \rangle$ defined by: $\rho(c) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find all intertwiners from ρ to itself in explicit form.

- (1) is false, (2) is true.
- (1) is true, (2) is true.
- (1) is false, (2) is false.
- (1) is true, (2) is false.

6. Let p be a prime number. Find all cyclic subgroups of the dihedral group D_{2p} . How many cyclic subgroups are there?

7. Consider the statements:

(1) Let $\rho: G \rightarrow \text{GL}(n, \mathbb{C})$ be a representation of a finite group G . Then $|\det(\rho(x))| = 1$ for all $x \in G$.

(2) There exists a representation $\rho: D_6 \rightarrow \text{GL}(n, \mathbb{C})$ such that $\det \rho(r) = \exp(2\pi i/3)$.

Which of the following is correct?

- a. (1) is true, (2) is false.
- b. (1) is true, (2) is true.
- c. (1) is false, (2) is false.
- d. (1) is false, (2) is true.

8. Let G be a set and $G \times G \rightarrow G: (x, y) \mapsto xy$ an associative binary operation. Let $1 \in G$ be such that $1x = x1 = x$ for all $x \in G$. Consider the following axioms which may or may not be satisfied:

(1) For all $x \in G$ there exists $y \in G$ such that $xy = 1$.

(2) For all $x \in G$ there exists $y \in G$ such that $yx = 1$.

(3) For all $x \in G$ there exists $y \in G$ such that $yx = xy = 1$.

By 'defines a group' we mean 'is equivalent to the definition of groups that you learned'.

Which is correct?

- a. (1) defines a group.
- b. (1), (2) don't define a group;
(3) defines a group.
- c. (1) doesn't define a group;
(1), (2) define a group.
- d. (3) doesn't define a group.