Test — MA3E1 Groups and Reps

Version **1** Wednesday 24 February 2010

Name: ____________________

Student Number: ____________________

This test covers the material lectured up to and including chapter 5. You can bring any notes you like, but no electronic gadgets or library books. Marks are given as follows.

	correct answer	incorrect answer	no answer
true/false questions	1	-1	0
all other questions	1	0	0

Workings or proofs are neither required nor can give marks.

Give your answers on this sheet and hand it in. You are free to make scratchy notes, but please not on this sheet. No name means no marks. *Good luck!*

1. Let A be an alphabet of k elements. How many words u over A of length 4 are there such that $\mathrm{R}(u) = 1$ (that is, whose unique reduced lower bound is empty).

2. True or false (T/F)? Let G be a group (not necessarily finite). Let V be a finite-dimensional $\mathbb{C}G$-module and $\langle \cdot, \cdot \rangle$ a G-invariant inner product on V. Then V is a direct sum of simple $\mathbb{C}G$-submodules.

3. True or false (T/F)? Let ρ be a representation of A_4 (the alternating group) of degree n. Let $T \in M_n(\mathbb{C})$ be such that $T\,\rho(g^2) = \rho(g^2)\,T$ for all $g \in A_4$. Then T is an intertwiner $\rho \to \rho$.

4. True or false (T/F)? Let V be a complex vector space. If $\langle \cdot, \cdot \rangle$ is an inner product on V then so is $(v, w) \mapsto \langle iv, -iw \rangle$.

5. Let G be a finite group. Let A be the set of characters of G and B the set of irreducible characters of G. Which is right?
   a. Neither A nor B are necessarily finite.
   b. A and B are finite.
   c. B is finite but A not necessarily so.
   d. A is finite but B not necessarily so.

6. Find the order of the group $\langle x, y, z \mid x^2 = y^3,\ y^2 = z^3,\ z^2 = x^3 \rangle$.

7. True or false (T/F)? Let G be a group (not necessarily finite). Let V be a finite-dimensional $\mathbb{C}G$-module. Then V admits a G-invariant inner product.

8. Find the orders of all elements of the dihedral group of order 28 defined by $D_{28} = \langle r, s \mid r^{14}, s^2, (rs)^2 \rangle$.

9. True or false (T/F)? There exists a unique representation ρ of the group $\langle a, b \mid a^2 = b^2 \rangle$ such that

$$\rho(a) = \left(\begin{smallmatrix} 1 & -1 \\ -1 & 1 \end{smallmatrix}\right), \quad \rho(b) = \left(\begin{smallmatrix} -1 & 1 \\ 1 & -1 \end{smallmatrix}\right).$$

10. Let $(v_1, \ldots, v_4)$ be a basis of a vector space V on which S_4 (the symmetric group) acts by

$$s\Big(\sum_{i=1}^{4} a_i\, v_i\Big) = \sum_{i=1}^{4} a_i\, v_{si}$$

($a_i \in \mathbb{C}$, $s \in S_4$). Let χ be the character of V (that is, of any representation afforded by V). Compute a, b, c where

$$a = \chi(12), \quad b = \chi(123), \quad c = \chi((12)(34)).$$