MA3E1 Groups and Representations
Assignment 4
Deadline: Friday 2 December 2011, 3pm

Write your name and student number on your solution sheet. Mention your department if it is not mathematics.

(4.1) Let ρ be an irreducible representation of a finite group G. Prove $\sum_{g \in G} \rho(g) = 0$ unless ρ is the trivial representation of degree 1.

(4.2) Let $H \leq K \leq G$ be finite groups and p a class function on H. Prove $(p^K)^G = p^G$.

(4.3) Let $H \leq G$ be finite groups and p a nonzero character of H. Prove that

$$\ker(p^G) = \bigcap_{x \in G} x^{-1}(\ker\, p)x.$$

Where does your proof break down if $p = 0$?

(4.4) In this exercise you will compute the character table for $G := S_5$ by induction from A_5 and S_4. Recall the **permutation character** χ^p which is the character of the representation $\rho^p\colon S_n \to \mathrm{GL}(n, \mathbb{C})$ defined by $(\rho^p s)e_i = e_{si}$ for all $s \in S_n$.

(a) For each conjugacy class in S_5, find its cardinality and give one element.

(b) Find two linear characters χ_1, χ_2 of S_5 where χ_1 is trivial.

(c) Compute $\chi_3 := \chi^p - \chi_1$ and $\chi_4 := \chi_3\chi_2$ and prove that they are irreducible characters.

(d) Choose an irreducible character ϕ of A_5 of degree 3. Compute $\chi_5 := \phi^G$ and prove that it is irreducible.

(e) Let μ be the following irreducible character of S_4:

	1	(12)	(12)(34)	(123)	(1234)
μ	3	-1	-1	0	1

Compute $\chi_6 := \mu^G - \chi_5 - \chi_4$ and prove that it is an irreducible character.

(f) Finish the character table.