An example of a computation of a Dehornoy graph

We will compute the Dehornoy graph Γ for the complemented presentation

$$\big(a, b, c \;\big|\; abbc = cbba,\ aba = bcb,\ cbc = bab\big). \tag{0.1}$$

At the same time this may help understand the definition of Dehornoy graphs in 15.2 which admittedly is a bit hard the way it is written.

Let Γ denote any generalised Dehornoy graph for the complemented presentation (0.1).

As the vertex set of Γ is supposed to contain $A = \{a, b, c\}$ (see part (a) of 15.2) we start with three edges, one for each of a, b, c. See figure 2(1). We gradually make the graph bigger as follows.

Whenever Γ contains two edges labelled x, y emanating from the same vertex (for example, $x = a$ and $y = c$ in figure 2(1)) it should be part of a cycle as prescribed by the relation the form $x \cdots = y \cdots$ (in our example $abbc = cbba$). This yields figure 2(2). Adding edges this way is called *operation A*.

By repeatedly applying operation A we arrive at figure 2(3) to which operation A cannot be applied any further.

Whenever Γ contains a path of edges whose labels read one side of a relation, say bab which is one side of the relation $bab = cbc$ (see the left of figure 1) the other side of the relation should also be there, in this case cbc. It connects start and finish another way (see the right of figure 1). Making the picture larger this way will be called *operation B*.

Operation B can be applied twice to part (3) of figure 2 to obtain part (4). Here, neither operation A nor B can be applied and we have arrived at the Dehornoy graph.

When you compute a Dehornoy graph this way, it may happen that two vertices turn out to be equal even if they didn't seem to be at first.

So an alternative definition of *generalised Dehornoy graph* is any graph, closed under operations A and B, containing at least the generators as in figure 2(1). The *Dehornoy graph* is the smallest among them.

Figure 1: Operation B

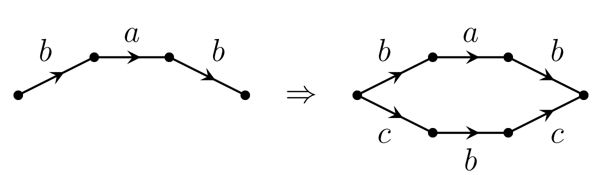

Figure 2: Computing a Dehornoy graph