MA4F2 Braid Groups 2004–2005

Errata and hints and more examples

May 6, 2005

- Sheet 2, (B3). The presentation should read $\langle x, y \mid x^3,\ y^2,\ (xy)^3 \rangle$.
- Sheet 2, exercise (C1) is complete rubbish. Sorry!
- Page 46 of the printed notes, 12.13, last formula should read $2 \vee 13 = 132312$.
- In the printed notes, page 54, the last displayed formula should read

$$\begin{aligned} abbacaaca &= (abb)(a)(\underline{caa})(ca) \\ &= (abb)(\underline{a})(\underline{aab})(ca) = (abb)(aa)(ab)(ca). \end{aligned}$$

  Another more elaborate example is as follows:

$$\begin{aligned} abccbbbcc &= (ab)(cc)(bb)(\underline{bcc}) = (ab)(cc)(\underline{bb})(\underline{cca}) \\ &= (ab)(cc)(\underline{bbc})(ca) = (ab)(\underline{cc})(\underline{abb})(ca) \\ &= (ab)(\underline{cca})(\underline{bb})(\underline{ca}) = (ab)(\underline{cca})(\underline{bb})(\underline{ca}) \\ &= (\underline{ab})(\underline{bcc})(bbc)(a) = (abbc)(c)(\underline{bbc})(a) \\ &= (abbc)(\underline{c})(\underline{abb})(a) = (abbc)(ca)(bb)(a) \\ &= \Delta(ca)(bb)(a). \end{aligned}$$

- In the printed notes, page 54 the latter equation in 15.3(3) should read

$$\overline{R} = R \cup \left\{ \begin{array}{l} \Delta\Delta^{-1} \to \varnothing,\ \Delta^{-1}\Delta \to \varnothing, \\ a\Delta^{-1} \to \Delta^{-1}\phi(a) \end{array} \ \middle|\ a \in \Omega \right\}.$$

- Hint for (B2), sheet 3. First prove $b < bs_i \Leftrightarrow b(i) < b(i+1)$ for all $b \in \Sigma_n$.
- Hint for (B3), sheet 3. You may use without proving it that $(a, \Delta) \xrightarrow[R]{*} (\Delta, \phi(a))$ for all $a \in \Omega$ where ϕ is the automorphism of B_n given by $\phi(\sigma_i) = \sigma_{n-i}$.

- There was a serious misprint in (16.5). So proposition 16.3 should read:

  Proposition 16.3. The braid group B_n is presented by the generators $a_{ij} = a_{ji}$ $(1 \leq i < j \leq n)$ and relations

$$a_{ij}\, a_{k\ell} = a_{k\ell}\, a_{ij} \qquad (i <_i j <_i k <_i \ell) \qquad (16.4)$$
$$a_{ij}\, a_{jk} = a_{jk}\, a_{ki} \qquad (i <_i j <_i k). \qquad (16.5)$$

  Also note that by cycling (16.5) around one also obtains

$$a_{ij}\, a_{jk} = a_{jk}\, a_{ki} = a_{ki}\, a_{ij}.$$

- Hint for (B8), sheet 3. The hint in part (e) also applies to part (a).

- There were three misprints in the proof of 18.30, namely, the starred inverses were missing: We prove (2) by

$$p_{ij}\, p_{jm}\, p_{ij}^{-1*} = a_{ij}\,(a_{ij}\, p_{jm}\, a_{ij}^{-1})\, a_{ij}^{-1} = a_{ij}\, p_{im}\, a_{ij}^{-1} = p_{im}^{-1}\, p_{jm}\, p_{im}$$

  and (3) by

$$\begin{aligned} p_{ij}\, p_{im}\, p_{ij}^{-1*} &= a_{ij}\,(a_{ij}\, p_{im}\, a_{ij}^{-1})\, a_{ij}^{-1} = a_{ij}\,(p_{im}^{-1}\, p_{jm}\, p_{im})\, a_{ij}^{-1} \\ &= (a_{ij}\, p_{im}\, a_{ij}^{-1})^{-1}(a_{ij}\, p_{jm}\, a_{ij}^{-1})(a_{ij}\, p_{im}\, a_{ij}^{-1}) \\ &= (p_{im}^{-1}\, p_{jm}^{-1}\, p_{im})(p_{im})(p_{im}^{-1}\, p_{jm}\, p_{im}) \\ &= p_{im}^{-1}\, p_{jm}^{-1*}\, p_{im}\, p_{jm}\, p_{im}. \end{aligned}$$

- In the third line of 18.17 it should read

$$(x, y) \in S_H \times S_G.$$