

# MA241 Combinatorics – Marking Sheet 4

Deadline: Monday, 27 November 2006, 3:00.

For this sheet, B1 (10 marks in total) and B4b (15 marks in total) will be assessed.

**(B1).** Compute the generating function of  $(a_0, a_1, \dots)$  defined by  $a_0 = 0$ ,  $a_1 = 1$  and

$$a_n = -2a_{n-1} + 8a_{n-2} + 4^n \quad (n \geq 2).$$

**SOLUTION.** [10 marks in total]. First we make the recurrence formula correct for all  $n \geq 0$ :

$$a_n = -2a_{n-1} + 8a_{n-2} + 4^n - [n = 0] - 3[n = 1]. \quad [5 \text{ marks}]$$

Multiply with  $z^n$  and sum over all  $n \geq 0$ :

$$\begin{aligned} A(z) &= -2zA(z) + 8z^2A(z) + \frac{1}{1-4z} - 1 - 3z, & [5 \text{ marks}] \\ A(z) &= \frac{(1-4z)^{-1} - 1 - 3z}{1+2z-8z^2} = \frac{1-1+4z-3z+12z^2}{(1-4z)(1+2z-8z^2)} \\ &= \frac{z(1+12z)}{(1-4z)(1+4z)(1-2z)}. \quad \square \end{aligned}$$