

# The groups of order $p^7$

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Eamonn O'Brien and I have classified the groups of order  $p^7$ , and have built a Magma database of the groups. The 2328 groups of order  $2^7$  were found by James, Newman and O'Brien in 1990. There are 9310 groups of order  $3^7$  and 34297 groups of order  $5^7$ . For  $p > 5$  the number of groups of order  $p^7$  is

$$\begin{aligned} &3p^5 + 12p^4 + 44p^3 + 170p^2 + 707p + 2455 \\ &+ (4p^2 + 44p + 291) \gcd(p-1, 3) + (p^2 + 19p + 135) \gcd(p-1, 4) \\ &+ (3p + 31) \gcd(p-1, 5) + 4 \gcd(p-1, 7) + 5 \gcd(p-1, 8) + \gcd(p-1, 9). \end{aligned}$$

For  $p > 5$ , we actually classify the nilpotent Lie rings of order  $p^7$ , and then use the Baker-Campbell-Hausdorff formula to translate Lie ring presentations into group presentations.

We make extensive use of Magma programs both in classifying the Lie rings of order  $p^7$ , and in checking the accuracy of our results.