Beyond Normal Form Invariance: First Mover Advantage in Two-Stage Games with or without Predictable Cheap Talk*

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Abstract

Von Neumann (1928) not only introduced a fairly general version of the extensive form game concept. He also hypothesized that only the normal form was relevant to rational play. Yet even in Battle of the Sexes, this hypothesis seems contradicted by players' actual behaviour in experiments. Here a refined Nash equilibrium is proposed for games where one player moves first, and the only other player moves second without knowing the first move. The refinement relies on a tacit understanding of the only credible and straightforward perfect Bayesian equilibrium in a corresponding game allowing a predictable direct form of cheap talk.

*A key idea used here appeared in Hammond (1982), which was an extensive revision of notes originally prepared for a seminar at the Mathematical Economics Summer Workshop of the Institute for Mathematical Studies in the Social Sciences, Stanford University, in July 1981. Eric van Damme, Elon Kohlberg, David Kreps, and Robert Wilson aroused and then revived my interest in this topic, while Hervé Moulin, Richard Pitbladdo, Kevin Roberts, Stephen Turnbull, and other seminar participants made helpful comments, though their views may not be at all well represented here. Research support from the National Science Foundation at that time is gratefully acknowledged. The earlier paper has a serious flaw, however, because its "sophisticated" equilibria could fail to be Nash equilibria. Much later, Luis Rayo and other members of my graduate game theory course at Stanford made me aware that there was some experimental corroboration of the ideas presented here, thus re-awakening my interest once again.
My gratitude to all those named above, while recognizing that the usual disclaimer absolving them of responsibility applies even more than usual. Some ideas in the first part of this later version were included in my presentation to the conference honouring Kotaro Suzumura at Hitotsubashi University in March 2006. The remainder of this presentation appears elsewhere as the basis of Hammond (2006).

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1 Motivation and Introduction

1.1 Von Neumann's Standard Paradigm

Following von Neumann's (1928) pioneering idea, in modern economic analysis and other social science multi-person decision problems are nearly always modelled as non-cooperative games in strategic form. This *standard paradigm* relies on two key assumptions, of which the first can be stated as follows:

Assumption 1. *The multi-person decision problem is fully described by a game in extensive form, whose structure is commonly known to all players in the game.*

Von Neumann (1928) himself devised one framework for such a description, later incorporated in *The Theory of Games and Economic Behavior*. Kuhn (1953) pointed out the implicit assumption that the order of different players' information sets was commonly known to all players at all stages of the game, and extended the von Neumann description to relax this assumption. Much more generally, we can now envisage an extensive form game as a stochastic process subject to the control of different players, with each player's information at each time described by a filtration. One key assumption, however, is that this stochastic process fits within Kolmogorov's (1933) framework, where everything random can be fitted within one overall probability space. As I have argued in Hammond (2006), this fails to allow for the possibility of having events that no player can foresee, and which may indeed even be impossible for any ideal observer to foresee.

1.2 Normal Form Invariance

The second assumption, which is also due to von Neumann (1928), can be stated as follows:

Assumption 2. *It loses no generality to reduce the game in extensive form to the corresponding game in strategic or normal form, where each player makes a single strategic plan that covers all eventualities in the extensive form.*

It is perhaps worth going back all the way to von Neumann's original article, as adapted in von Neumann and Morgenstern (1953), in order to see how he justified normalizing the extensive form. First, normal form strategies are described on p. 79:

> "Imagine now that each player . . . , instead of making each decision as the necessity for it arises, makes up his mind in advance for all possible contingencies; i.e., that the player . . . begins to play with a complete plan: a plan which specifies what choices he will make in every possible situation, for every possible actual information which he may possess at that moment in conformity with the pattern of information which the rules of the game provide for him for that case. We call such a plan a *strategy*."

Then pages 79–84 proceed to simplify the description of an extensive form game, to arrive at the normal form of the game in which each player makes just one move, and all moves are chosen simultaneously. In fact (p. 84):

> "Each player must make his choice [of strategy] in absolute ignorance of the choices of the others. After all choices have been made, they are submitted to an umpire who determines . . . the outcome of the play for [each] player.
>
> Observe that in this scheme no space is left for any kind of further 'strategy.' Each player has one move, and one move only; and he must make it in absolute ignorance of everything else."

Normalizing an extensive form game in this way is an extremely powerful device. And if the players of a game really do simultaneously submit their choices of a strategy to an umpire, who then sees that the players never deviate from their announced choices, then von Neumann and Morgenstern's claim on p. 85 seems entirely justified:

> " . . . we obtained an all-inclusive formal characterization of the general game of n persons . . . . We followed up by developing an exact concept of strategy which permitted us to replace the rather complicated general scheme of a game by a much more simple special one, which was nevertheless shown to be fully equivalent to the former . . . . In the discussion which follows it will sometimes be more convenient to use one form, sometimes the other. It is therefore desirable to give them specific technical names. We will accordingly call them the *extensive* and the *normalized* form of the game, respectively.
>
> Since these two forms are strictly equivalent, it is entirely within our province to use in each particular case whichever is

> technically more convenient at that moment. We propose, indeed, to make full use of this possibility, and must therefore re-emphasize that this does not in the least affect the absolute general validity of all our considerations."

It is this simplification that gives such power to familiar "normal form" concepts like Nash equilibrium, as well as to less familiar ones like trembling-hand perfect equilibrium (Selten, 1975), proper equilibrium (Myerson, 1978), correlated equilibrium (Aumann, 1987), rationalizable strategies (Bernheim, 1984; Pearce, 1984). Also, Mailath, Samuelson, and Swinkels (1993) show how even such ostensibly extensive form concepts as Selten's (1965) concept of subgame perfection, or Kreps and Wilson's (1982) concept of sequential equilibrium, have their (reduced) normal form counterparts.

Game theorists do relax normal form invariance somewhat by using extensive form solution concepts which, for example, require players to respond credibly when other players deviate from expected behaviour. This was the motivation for Selten's (1965) concept of subgame perfect equilibrium. See also Amershi, Sadanand and Sadanand (1985, 1989a, b, 1992), Hammond (1993), Sadanand and Sadanand (1995), Battigalli (1997), Battigalli and Siniscalchi (1999, 2002), and Asheim and Dufwenberg (2003), among other works that cast doubt on the normal form invariance hypothesis.

1.3 Outline of Paper

The purpose of this paper is to present a theoretical argument supporting the view that normal form invariance may be unduly restrictive. To do so, Section 2 considers a simple "battle of the sexes" game, where experimental evidence suggests that the first move does confer an advantage. My claim is that this may be due to what would happen in the unique credible equilibrium of an associated game where cheap talk is possible after the first move, but before the second.

Section 3 begins by setting out a general class of two-stage games, where one player moves first, and the only other player moves second, but without knowing the first player's move. It goes on to consider the effect on such games of allowing cheap talk. Because the second player lacks any information that the first does not possess, only cheap talk by the first player is relevant. Also, we are looking for an equilibrium that the players can infer even without cheap talk. So we require cheap talk to be "predictable" in the sense that it results from a pure strategy which is independent of the first player's (hidden) action.

Not surprisingly, any perfect Bayesian equilibrium in the game with predictable cheap talk must induce a Nash equilibrium in the corresponding game without cheap talk. On the other hand, any Nash equilibrium without cheap talk can be extended into a perfect Bayesian equilibrium with a suitable "inflexible" belief structure under which the second player ignores all cheap talk. Thus, cheap talk alone fails to refine the set of Nash equilibria. The set of perfect Bayesian equilibria with cheap talk itself needs refining.

In order to facilitate such a refinement, Section 4 discusses how a particular version of the revelation principle allows general predictable cheap talk to be replaced by "direct" cheap talk, where the first player suggests what beliefs the second player should hold about the first move, as well as what (mixed) strategy the second player should follow. A perfect Bayesian equilibrium with general predictable cheap talk is then converted into a "straightforward" equilibrium with direct cheap talk, where the second player accepts the first player's suggestions.

Restricting attention to such straightforward perfect Bayesian equilibria, Section 5 finally introduces a credibility refinement. This requires that the first player cannot improve the equilibrium with direct cheap talk by offering a "cogent" suggestion for playing a different Nash equilibrium. The resulting "credible" equilibrium with cheap talk leads to an optimal Nash equilibrium for the first player. When this equilibrium is unique, actual cheap talk can be replaced by tacit cheap talk. While these results may be hardly surprising, they do show how tacit communication can explain first mover advantage in games like Battle of the Sexes.

Section 6 considers the idea of "virtual observability", which is the idea that, as in Battle of the Sexes, tacit communication may convert the game into one of perfect information, with the second player knowing the first move. Three examples show that virtual observability is rather special.

The concluding Section 7 discusses possible extensions and other future work that relaxes normal form invariance.

2 Battle of the Sexes

2.1 Two Different Extensive Forms

The two games in Figs. 1 and 2 are different extensive form versions of the well known "battle of the sexes" game, whose normal form is given in Fig. 3. There are two pure strategy Nash equilibria, namely (B, b) and (S, s). There is also one mixed strategy Nash equilibrium in which player 1 plays B with probability $\frac{2}{3}$, whereas player 2 plays b with probability $\frac{1}{3}$.

Nevertheless, experiments strongly suggest that the player who moves first enjoys an advantage, in so far as (B, b) is played more often than (S, s) in Fig. 1, but less often in Fig. 2. Note too that these results are usually ascribed to "positional order" or "presentation" effects that are seen as psychological or behavioral rather than fully rational responses to a change in the extensive form of the game.[1]

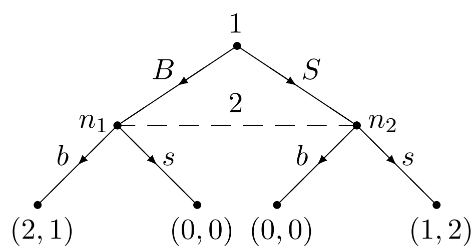

Figure 1: Battle of the Sexes where Player 1 Moves First

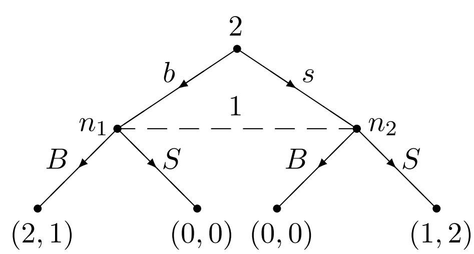

Figure 2: Battle of the Sexes where Player 2 Moves First

[1]A "preliminary" experiment along these lines is described by Amershi, Sadanand, and Sadanand (1989b). Kreps (1990, p. 100) writes of "casual experiences playing this game with students". Later formal experiments yielding similar results were reported in Cooper *et al.* (1989, 1993). See also Schotter, Weigelt and Wilson (1994), Güth, Huck and Rapoport (1998), Muller and Sadanand (2003), Weber, Camerer and Knez (2004). The paper by Güth *et al.* even includes an experiment in which a form of cheap talk is explicitly allowed. The experimental design, however, includes the wording "B learns about A's decision" in the instructions. This appears to bias the results by offering the subjects too little encouragement to recognize the possibility of sending or receiving a deceptive message.

	b	s
B	$2,1$	$0,0$
S	$0,0$	$1,2$

Figure 3: Battle of the Sexes in Normal Form

2.2 Direct Cheap Talk in Battle of the Sexes

Consider the extensive form of Fig. 1, where player 1 moves first, and this is common knowledge. Suppose that, during the time interval that succeeds player 1's move but precedes player 2's, the two players are allowed to communicate and indulge in unrestricted and mutually comprehensible "cheap talk".

Let $\Delta(\{B, S\})$ denote the set of probability distributions on the pair set $\{B, S\}$. Following the ideas of the revelation principle in the form due to Myerson (1982), as amended by Kumar (1985), where player 1's message m to suggest conditional probabilistic beliefs $\pi(\cdot|m) \in \Delta(\{B, S\})$ that player 2, at his only information set, should attach to player 1's earlier move. Player 1 may also suggest player 2's response. Moreover, we can limit attention to "straightforward" equilibria, where player 1's report is truthful, and player 2 accepts 1's suggested strategy.

Then there are essentially just three possible conversations that might ensue, corresponding to the three different Nash equilibria of the normal form. In two of these conversations, player 1 reports the pure strategy (B or S) that she has just chosen, and may or may not add an appropriate recommendation as to what strategy player 2 should choose next (b or s). The third possibility allows player 1's message m to suggest conditional probabilistic beliefs $\pi(\cdot|m) \in \Delta(\{B, S\})$ that player 2, at his only information set, should attach to player 1's earlier move.

Since player 2 has less information than player 1, there is really nothing that player 2 can contribute to the conversation. So we will consider only what player 1 says, and reduce the set of possible messages m she might send to $M = \Delta(\{B, S\})$. When player 1 reports her true choice of B or S, this is a special case where either $m(B) = 1$ or $m(S) = 1$.

2.3 Equilibrium with Cheap Talk

The perfect Bayesian equilibria (PBEs) of this extensive form game can be found by using backward induction. Any such equilibrium involves specifying the conditional probabilistic beliefs $\pi(\cdot|m) \in \Delta(\{B, S\})$ that player 2 attaches to player 1's earlier move at his information set. Player 2's best response is b for sure if $\pi(B|m) > \frac{2}{3}$ and s for sure if $\pi(S|m) > \frac{1}{3}$. But any mixed strategy by player 2 is best if $\pi(B|m) = \pi(S|m) = \frac{2}{3}$.

Depending upon the message space M, there are many PBEs in principle. But we now restrict attention to those satisfying the restriction that $\pi(B|m) = m(B)$ and $\pi(S|m) = m(S)$ — i.e., player 2 believes exactly what player 1 suggests he should believe. Obviously, player 2 must also choose a best response given these conditional beliefs, and for player 2 to accept player 1's suggestion, the message m should be a possible equilibrium strategy for player 1.

There are exactly three candidates for such equilibria with predictable cheap talk, corresponding to the three different Nash equilibria of the normal form:

1. corresponding to the pure strategy equilibrium (B, b), a message with

$$\pi(B|m) = m(B) = 1 \text{ and } \pi(S|m) = m(S) = 0,$$

   which yields the two players' expected payoffs of $(2, 1)$;

2. corresponding to the pure strategy equilibrium (S, s), a message with

$$\pi(B|m) = m(B) = 0 \text{ and } \pi(S|m) = m(S) = 1,$$

   which yields the two players' expected payoffs of $(1, 2)$;

3. corresponding to the mixed strategy equilibrium, a message with

$$\pi(B|m) = m(B) = \tfrac{2}{3},\ \pi(S|m) = m(S) = \tfrac{1}{3},$$

   which yields the two players' expected payoffs of $(\frac{2}{3}, \frac{2}{3})$ after player 2 plays her equilibrium mixed strategy in which b occurs with probability $\frac{1}{3}$.

In this game with cheap talk, however, only the first message seems credible. The reason is that, if all three candidate equilibria were credible, player 1 would expect player 2 to respond appropriately to her message, and so she would definitely choose the first message. But then, if player 2 hears any message except "I have played B and recommend that you play b", he should wonder whether player 1 has really not played B, or whether player 1 has somehow misspoken after playing B. Thus, player 2's best response to any other message actually becomes unclear, and we are therefore left with just one *credible* PBE.

Finally, we argue that if direct cheap talk would produce a unique direct message, both players are sufficiently "sophisticated" to work out in advance what it will be. But then there is really no need for cheap talk at all. Player 2 can work out the unique equilibrium message that he would receive in the game with predictable cheap talk, and player 1 should know this also. They reach the unique PBE with predictable cheap talk by tacitly inferring what would happen if cheap talk were actually permitted.

3 General Two-Stage Games

3.1 The Basic Extensive Game

Consider next a general two-stage game, with two players 1 and 2. Suppose all the following facts are common knowledge. Player 1 begins the game by choosing an action a_1 from the finite set A_1. Then player 2 at his only information set, without seeing a_1, finishes the game by choosing an action a_2 from the finite set A_2. The two players' respective payoffs are denoted by $u_i(a_1, a_2)$ (for $i = 1, 2$). For $i = 1, 2$, let $\Delta(A_i)$ denote the set of probability distributions over the finite set A_i. Allowing for mixed strategies, this gives us the basic game which we denote by

$$G_0 = \langle \{1, 2\}, \Delta(A_1), \Delta(A_2), u_1, u_2 \rangle \tag{1}$$

3.2 Introducing Relevant and Predictable Cheap Talk

Cheap talk is introduced by allowing the two players to choose simultaneous normal form message strategies $m_i \in M_i$ (for $i = 1, 2$) after player 1 has chosen a_1, but before player 2 chooses a_2. Of course, the main claim of this paper is precisely that considering normal form strategies really *is* restrictive. Nevertheless, restricting message strategies like this is permissible in establishing the negative result.[2]

Not all cheap talk will be relevant, however. In particular, because player 2 has no information to convey, nothing he says can affect the game

[2]This may appear to contradict Aumann and Hart (2003). Their model, however, involves messages that are sent by choosing one among only a finite set of "keystrokes". Also, the only example of an equilibrium involving "long cheap talk" that they provide is presented in their Section 2.8. In a particular signalling game, it amounts to finding a mixed message strategy with infinite support.

in the equilibria we consider. So we will model only player 1's cheap talk, and denote her message space simply by M.

Also, we are looking for a unique predictable equilibrium of the game with cheap talk. Note, however, no mixed message strategy for player 1 could work this way; player 2 could not predict what message results from such randomization. Indeed, player 1 cannot even make her message depend on the action that results from a mixed action strategy. That is why we restrict attention to "predictable" cheap talk resulting in one fixed message, independent of player 1's action.

3.3 Strategies with Predictable Cheap Talk

A general behavioral strategy profile in this extensive game of perfect recall proceeds in two successive stages as follows:

1. Player 1 chooses a mixed action strategy $\alpha_1 \in \Delta(A_1)$, along with a predictable message $m \in M$.
2. Player 2 forms conditional beliefs $\pi(\cdot|m) \in \Delta(A_1)$ and then chooses a mixed strategy $\alpha_2(\cdot|m) \in \Delta(A_2)$ as a function of player 1's message m. Thus $\pi(\cdot|\cdot) \in [\Delta(A_1)]^M$ and $\alpha_2(\cdot|\cdot) \in [\Delta(A_2)]^M$, where $[\Delta(A_i)]^M$ denotes the set of all mappings from M to $\Delta(A_i)$ $(i = 1, 2)$.

To summarize, the game with predictable cheap talk is

$$G = \langle \{1, 2\}, \Delta(A_1) \times M, \Delta(A_2), u_1, u_2 \rangle. \tag{2}$$

A *strategy–belief profile* in G is a quadruple

$$(\alpha_1(\cdot), m, \pi(\cdot|\cdot), \alpha_2(\cdot|\cdot)) \in \Delta(A_1) \times M \times [\Delta(A_1)]^M \times [\Delta(A_2)]^M. \tag{3}$$

Given any $m \in M$, the strategy–belief profile (3) in G induces the independent *action strategy distribution* $\alpha(\cdot|m) \in \Delta(A_1 \times A_2)$ given by

$$\alpha(a_1, a_2|m) = \alpha_1(a_1)\alpha_2(a_2|m) \text{ for all } (a_1, a_2) \in A_1 \times A_2. \tag{4}$$

3.4 Perfect Bayesian Equilibrium with Cheap Talk

The set of perfect Bayesian equilibria (PBEs) of the game in Section 3.3 can be found by backward induction, as usual. At the last stage 2, player 2 has observed m and uses Bayes' rule to derive the beliefs $\pi(\cdot|m) \in \Delta(A_1)$. In equilibrium, these must satisfy

$$\pi(a_1|m) = \alpha_1(a_1) \text{ for all } m \in M. \tag{5}$$

This is independent of m because, in order to be predictable, the message m must be independent of a_1, and so cannot convey any relevant information.

Given these beliefs, player 2 chooses a mixed strategy $\alpha_2^*(\cdot|m) \in \Delta(A_2)$ with the property that any $a_2 \in A_2$ for which $\alpha_2^*(a_2|m) > 0$ must satisfy

$$a_2 \in B_2(\pi(\cdot|m))) := \arg\max_{a_2 \in A_2} \sum_{\tilde{a}_1 \in A_1} \pi(\tilde{a}_1|m) u_2(\tilde{a}_1, a_2). \tag{6}$$

At stage 1, anticipating this choice, player 1 chooses a mixed action strategy $\alpha_1^* \in \Delta(A_1)$, together with a fixed message $m^* \in M$, such that for each a_1 with $\alpha_1^*(a_1) > 0$ one has

$$(a_1, m^*) \in B_1 := \arg\max_{(a_1,m) \in A_1 \times M} \sum_{\tilde{a}_2 \in A_2} \alpha_2^*(\tilde{a}_2|m) u_1(a_1, \tilde{a}_2). \tag{7}$$

The following result shows that any PBE of such a game must induce Nash equilibrium action strategies.

Lemma 1. *Suppose $(\alpha_1^*(\cdot), m^*, \pi^*(\cdot|\cdot), \alpha_2^*(\cdot|\cdot))$ is a PBE strategy–belief profile in the game G with predictable cheap talk. Then the induced mixed strategy profile $\alpha^*(\cdot|m^*) = \alpha_1^*(\cdot) \times \alpha_2^*(\cdot|m^*)$ in $\Delta(A_1 \times A_2)$ must be a Nash equilibrium of the game G_0 without cheap talk.*

Proof. Equations (5) and (6) together imply that $\alpha_2^*(\cdot|m^*)$ is a mixed strategy best response given $\pi^*(\cdot|m^*) = \alpha_1^*(\cdot)$. Equation (7), with m fixed at m^*, implies that $\alpha_1^*(\cdot)$ is a mixed strategy best response to $\alpha_2^*(\cdot|m^*)$. ■

Condition (7) would appear to refine the Nash equilibrium set by requiring player 1 to choose m^* optimally from M, in order to affect player 2's beliefs $\pi^*(\cdot|m^*)$ and his conditional mixed strategy best response $\alpha_2^*(\cdot|m^*)$. The following result, however, shows that PBE does not refine Nash equilibrium in the games we are considering. In particular, none of the three Nash equilibria in the Battle of the Sexes example of Section 2 has yet been excluded.

Lemma 2. *Let $\bar{\alpha}(\cdot) = \bar{\alpha}_1(\cdot) \times \bar{\alpha}_2(\cdot) \in \Delta(A_1 \times A_2)$ be any Nash equilibrium action strategy profile in the game G_0 without cheap talk. Then the corresponding game G with predictable cheap talk has a PBE strategy–belief profile which induces $\bar{\alpha}(\cdot)$.*

Proof. Consider the "unresponsive" strategy–belief profile

$$(\alpha_1^*(\cdot), m^*, \pi^*(\cdot|\cdot), \alpha_2^*(\cdot|\cdot)) \in \Delta(A_1) \times M \times [\Delta(A_1)]^M \times [\Delta(A_2)]^M$$

in G defined as follows: (i) $\alpha_1^*(\cdot) := \bar{\alpha}_1(\cdot)$; (ii) m^* is an arbitrary message in M; (iii) $\pi^*(\cdot|m) := \bar{\alpha}_1(\cdot)$ for all $m \in M$; (iv) $\alpha_2^*(\cdot|m) := \bar{\alpha}_2(\cdot)$ for all $m \in M$. It is easy to see that this must be a PBE. ■

We will refine the equilibrium set of G by focusing on strategy–belief profiles that are more responsive than those used to prove Lemma 2. In order to do so, we need to specify player 1's message space M more precisely.

4 Revelation Principle

4.1 Equivalent Direct Cheap Talk

The revelation principle will allow the use of a function $f : M \to \hat{M}$ to map each cheap talk message m in the arbitrary space M into an equivalent *direct* cheap talk message $\hat{m} := f(m) \in \hat{M} := f(M)$, where

$$f(m) = (\beta(m), \gamma(m)) \text{ with } \beta(m) \in \Delta(A_1) \text{ and } \gamma(m) \in \Delta(A_2). \tag{8}$$

Here, following Kumar's (1985) extension of the revelation principle, the first component $\beta(m)$ of each direct message can be interpreted as the beliefs about player 1's strategy that 1 suggests to 2. Following Myerson (1982), the second component $\gamma(m)$ can be interpreted as the ensuing (mixed) strategy that 1 suggests to 2.[3]

More specifically, given the arbitrary cheap talk message space M, consider any strategy–belief profile $(\alpha_1(\cdot), m, \pi(\cdot|\cdot), \alpha_2(\cdot|\cdot))$ as in (3). We map this to a unique strategy–belief profile

$$(\hat{\alpha}_1(\cdot), \hat{m}, \hat{\pi}(\cdot|\cdot), \hat{\alpha}_2(\cdot|\cdot)) \in \Delta(A_1) \times \hat{M} \times [\Delta(A_1)]^{\hat{M}} \times [\Delta(A_2)]^{\hat{M}} \tag{9}$$

with direct cheap talk that is specified as follows:

1. $\hat{\alpha}_1(\cdot) = \alpha_1(\cdot)$;
2. the cheap talk message $m \in M$ is replaced by $\hat{m}$ as in (8), where $\beta : M \to \Delta(A_1)$ and $\gamma : M \to \Delta(A_2)$ are defined by
$$\beta(\cdot|m) := \pi(\cdot|m) \quad \text{and} \quad \gamma(\cdot|m) := \alpha_2(\cdot|m); \tag{10}$$
3. the belief system $\pi(\cdot|\cdot) \in [\Delta(A_1)]^M$ for player 2 is replaced by $\hat{\pi}(\cdot|\cdot) \in [\Delta(A_1)]^{\hat{M}}$, where for all $m \in M$ one has
$$\hat{\pi}(\cdot|\hat{m}) = \hat{\pi}(\cdot|\beta(m), \gamma(m)) = \beta(\cdot|m) = \pi(\cdot|m). \tag{11}$$

[3] Following Forges (1986), many later writers describe direct messages as "canonical".

4. the conditional strategy $\alpha_2(\cdot|\cdot) \in [\Delta(A_2)]^M$ for player 2 is replaced by $\hat{\alpha}_2(\cdot|\cdot) \in [\Delta(A_2)]^{\hat{M}}$, where for all $m \in M$ one has

$$\hat{\alpha}_2(\cdot|\hat{m}) = \hat{\alpha}_2(\cdot|\beta(m), \gamma(m)) = \gamma(\cdot|m) = \alpha_2(\cdot|m). \tag{12}$$

Note that $\hat{\pi}(\cdot|\hat{m})$ and $\hat{\alpha}_2(\cdot|\hat{m})$ are well defined by (11) and (12) because $\hat{m} = f(m) = f(m')$ implies that $\beta(m) = \beta(m')$ and $\gamma(m) = \gamma(m')$, ensuring that $\hat{\pi}(\cdot|\hat{m}) = \beta(m) = \beta(m')$ and $\hat{\alpha}_2(\cdot|\hat{m}) = \gamma(m) = \gamma(m')$.

Obviously, the mapping specified above defines an equivalent game

$$\hat{G} = \langle \{1, 2\}, \Delta(A_1) \times \hat{M}_1, \Delta(A_2), u_1, u_2 \rangle \tag{13}$$

with direct cheap talk messages $\hat{m} \in \hat{M} = \Delta(A_1) \times \Delta(A_2)$. Furthermore:

Lemma 3. *For each $m \in M$ and corresponding $\hat{m} \in \hat{M}$ given by (10), the equivalent strategy profile $(\hat{\alpha}_1(\cdot), \hat{m}, \hat{\alpha}_2(\cdot|\cdot))$ in $\hat{G}$ induces the conditional action strategy profile $\hat{\alpha}(\cdot|\hat{m})$ in $\Delta(A_1 \times A_2)$ that is identical to $\alpha(\cdot|m)$.*

Proof. By (4) and (12),

$$\hat{\alpha}(a_1, a_2|\hat{m}) = \hat{\alpha}_1(a_1)\hat{\alpha}_2(a_2|\hat{m}) = \alpha_1(a_1)\alpha_2(a_2|m) = \alpha(a_1, a_2|m),$$

as required. ■

4.2 A Revelation Principle with Direct Cheap Talk

Definition 1. *In the game $\hat{G}$ with direct cheap talk, a PBE strategy–belief profile $(\hat{\alpha}_1^*(\cdot), \hat{m}^*, \hat{\pi}^*(\cdot|\cdot), \hat{\alpha}_2^*(\cdot|\cdot))$ is* **straightforward** *if it satisfies*

$$\hat{m}^* = (\hat{\alpha}_1^*(\cdot), \hat{\alpha}_2^*(\cdot|\hat{m}^*)) = (\hat{\pi}^*(\cdot|\hat{m}^*), \hat{\alpha}_2^*(\cdot|\hat{m}^*)).$$

Thus, a straightforward PBE with direct cheap talk is one where player 1 suggests what player 2 should believe and what (mixed) action he should choose. Moreover, player 2 accepts these suggestions. In this sense, player 1 is *believed* and player 2 is *obedient*.

The following result extends to our setting the versions of the revelation principle due to Myerson (1982) and Kumar (1985).

Theorem 1. *Suppose $(\alpha_1^*(\cdot), m^*, \pi^*(\cdot|\cdot), \alpha_2^*(\cdot|\cdot))$ is any PBE strategy–belief profile in the game G with predictable cheap talk. Then the equivalent strategy–belief profile $(\hat{\alpha}_1^*(\cdot), \hat{m}^*, \hat{\pi}^*(\cdot|\cdot), \hat{\alpha}_2^*(\cdot|\cdot))$ in the game $\hat{G}$ with direct cheap talk, constructed as in Section 4.1, is a straightforward PBE.*

Proof. At the last stage 2 of the game $\hat{G}$, given any $m \in M$ and corresponding $\hat{m} \in \hat{M}$, equations (5), (6) and (11) together imply that $\hat{\pi}^*(\cdot|\hat{m}) = \pi^*(\cdot|m) = \alpha_1^*(\cdot) = \hat{\alpha}_1^*(\cdot)$ and that

$$\hat{\alpha}_2^*(\cdot|\hat{m}) = \alpha_2^*(\cdot|m) \in \Delta(B_2(\pi^*(\cdot|m))) = \Delta(B_2(\hat{\pi}^*(\cdot|\hat{m}))).$$

At stage 1 of $\hat{G}$, if $a_1 \in A_1$ satisfies $\hat{\alpha}_1^*(a_1) = \alpha_1^*(a_1) > 0$, then (7) implies

$$(a_1, \hat{m}^*) \in \hat{B}_1 := \arg\max_{(a_1,\hat{m}) \in A_1 \times \hat{M}} \sum_{\tilde{a}_2 \in A_2} \hat{\alpha}_2^*(\tilde{a}_2|\hat{m}) u_1(a_1, \tilde{a}_2).$$

Thus, all the conditions for the specified strategy–belief profile to be a PBE of $\hat{G}$ have been verified. Also, applying equations (8), (10), (11) and (12) in turn gives

$$\begin{aligned} \hat{m}^* = f(m^*) &= (\beta(m^*), \gamma(m^*)) = (\pi^*(\cdot|m^*), \alpha_2^*(\cdot|m^*)) \\ &= (\hat{\pi}^*(\cdot|\hat{m}^*), \hat{\alpha}_2^*(\cdot|\hat{m}^*)) = (\hat{\alpha}_1^*(\cdot), \hat{\alpha}_2^*(\cdot|\hat{m}^*)) \end{aligned}$$

which verifies that the PBE is straightforward. ∎

5 Credible Equilibria with Direct Cheap Talk

5.1 Definitions

The revelation principle is especially useful in allowing a two-stage game with predictable cheap talk to be converted to a game with direct cheap talk. Nevertheless, Lemma 2 applies even when the cheap talk is direct. For this reason, an extra consideration is needed to refine the set of Nash equilibria. We shall insist that player 2 be rather more flexible in responding to some of the messages that player 1 might send. This is our next topic.

Definition 2. *A direct message* $\hat{m} = (\beta, \gamma) \in \hat{M} = \Delta(A_1) \times \Delta(A_2)$ *is* **cogent** *if, when viewed as mixed strategies, the pair* (β, γ) *constitutes a Nash equilibrium.*

Thus, plausibility requires player 1 to suggest a best response γ by player 2 given beliefs β, and to suggest beliefs that are compatible with player 1 choosing a best response β to the suggested strategy γ for player 2.

The next definition considers what happens when player 2 feels bound to heed cogent direct messages that improve player 1's expected outcome, as in our analysis of Battle of the Sexes.

Definition 3. *A straightforward PBE strategy–belief profile in the game* $\hat{G}$ *with direct cheap talk is* **credible** *if there is no cogent alternative direct message* $\hat{m} = (\beta, \gamma)$ *and corresponding strategy* $\alpha_1 = \beta$ *that allows player 1 to reach a better straightforward PBE strategy–belief profile.*

5.2 First Mover Advantage

We now characterize credible PBEs of $\hat{G}$ as the Nash equilibria of G that confer maximum first mover advantage.

Definition 4. *In the game* G_0 *without cheap talk, the Nash equilibrium mixed strategy pair* $(\alpha_1^*, \alpha_2^*) \in \Delta(A_1) \times \Delta(A_2)$ *is* **optimal for player 1** *if, given the set* $E \subseteq \Delta(A_1) \times \Delta(A_2)$ *of all Nash equilibria in* G_0*,*

$$(\alpha_1^*, \alpha_2^*) \in \arg\max_{(\alpha_1,\alpha_2)\in E} \sum_{a_1\in A_1} \sum_{a_2\in A_2} \alpha_1(a_1)\alpha_2(a_2)u_1(a_1, a_2). \tag{14}$$

Theorem 2. *In the game* $\hat{G}$ *with predictable direct cheap talk, a strategy–belief profile* $(\hat{\alpha}_1^*(\cdot), \hat{m}^*, \hat{\pi}^*(\cdot|\cdot), \hat{\alpha}_2^*(\cdot|\cdot))$ *is a credible PBE if and only if there exists a corresponding Nash equilibrium* $(\hat{\alpha}_1^*, \hat{\alpha}_2^*) \in \Delta(A_1) \times \Delta(A_2)$ *that is optimal for player 1 in the game* G_0 *without cheap talk.*

Proof. Let $(\hat{\alpha}_1^*(\cdot), \hat{m}^*, \hat{\pi}^*(\cdot|\cdot), \hat{\alpha}_2^*(\cdot|\cdot))$ be any straightforward PBE strategy–belief profile in $\hat{G}$, not necessarily credible. Applying Lemma 1 to $\hat{G}$ rather than G shows that it must induce a Nash equilibrium action profile $\hat{\alpha}(\cdot|\hat{m}^*) = (\hat{\alpha}_1^*(\cdot), \hat{\alpha}_2^*(\cdot|\hat{m}^*))$ in G_0. Suppose there is an alternative Nash equilibrium (α_1, α_2) in G_0 which is better for player 1 than $\hat{\alpha}(\cdot|\hat{m}^*)$. Then the message $\hat{m} \in \hat{M} = \Delta(A_1) \times \Delta(A_2)$ with $\hat{m} = (\alpha_1, \alpha_2)$ is cogent, so the straightforward PBE cannot be credible after all.

Conversely, let $(\alpha_1^*, \alpha_2^*) \in \Delta(A_1) \times \Delta(A_2)$ be any Nash equilibrium in the game G_0 without cheap talk. As in Lemma 2, there is an "unresponsive" PBE strategy–belief profile which induces (α_1^*, α_2^*). Provided that the Nash equilibrium is optimal for player 1, there is no cogent predictable cheap talk message $\hat{m} \in \hat{M} = \Delta(A_1) \times \Delta(A_2)$ that can lead to a better outcome for player 1. Hence the straightforward PBE strategy–belief profile must be credible. ∎

5.3 First Mover Advantage without Cheap Talk

Suppose the game $\hat{G}$ with predictable direct cheap talk has a unique credible PBE. Then the two players can reasonably expect each other to infer what this direct cheap talk would be, even in the game G_0 without cheap talk.

The following definition singles out the corresponding Nash equilibrium of this game.

Definition 5. *A Nash equilibrium of the game G_0 without cheap talk is a* **sophisticated equilibrium** *if it is induced by a credible PBE of the corresponding game $\hat{G}$ with predictable direct cheap talk, and moreover this credible PBE is unique.*

Theorem 3. *Suppose (α_1^*, α_2^*) is a uniquely optimal Nash equilibrium for player 1 in G_0. Then (α_1^*, α_2^*) is the unique sophisticated equilibrium.*

Proof. Theorem 2 implies that there is a unique credible PBE of $\hat{G}$, and that this equilibrium induces (α_1^*, α_2^*). ∎

The following is an example of a game with no sophisticated equilibrium. Not surprisingly, cheap talk plays a key role here in enabling coordination on one of the two Nash equilibria that are equally good for player 1. But if the two players' payoffs after (L, ℓ) were $(1 + \epsilon, \delta)$ instead, for any $\epsilon > 0$ and any $\delta > 0$, then (L, ℓ) would be the unique sophisticated equilibrium.

	ℓ	r
L	$1, 1$	$0, 0$
R	$0, 0$	$1, 1$

Figure 4: A Game with No Sophisticated Equilibrium

6 The Special Case of Virtual Observability

6.1 Definition

Corresponding to our basic game G_0 without cheap talk, there is a game

$$G_1 := \langle \{1, 2\}, \Delta(A_1), [\Delta(A_2)]^{A_1}, u_1, u_2 \rangle \tag{15}$$

of perfect information, where player 2 is informed of 1's move and so can make his mixed strategy $\alpha_2 \in \Delta(A_2)$ a function of player 1's action a_1. Now, in the Battle of Sexes example of Section 2, there is a unique credible equilibrium where both players effectively act as though player 1's move could be observed. It is a case where the same pure strategy profile $(a_1, a_2) \in A_1 \times A_2$ in the game G_0 happens to be induced by both the unique credible equilibrium of $\hat{G}$ and the unique subgame perfect equilibrium of G_1. Weber, Camerer and Knez (2004) call this "virtual observability". The next three examples remind us that virtual observability is really very special.

6.2 Mixed Strategies

Consider the simple and familiar example of matching pennies, whose normal form is shown in Figure 5. There is a unique Nash equilibrium, associated with a unique PBE strategy–belief profile in the corresponding game of predictable direct cheap talk. The only direct message $\hat{m} = (\beta, \gamma)$ that is sent in this unique equilibrium has $\beta(H) = \beta(T) = \gamma(h) = \gamma(t) = \frac{1}{2}$. Obviously, the need for mixed strategies in equilibrium implies that virtual observability cannot hold.

	h	t
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

Figure 5: Matching Pennies

6.3 Duopoly: Cournot versus Stackelberg

It is fairly obvious that if one duopolist is able to choose his quantity before the other, and if both know this and know the first duopolist's quantity, then the only sophisticated dynamic equilibrium has the first duopolist acting as a Stackelberg leader and the second as a follower. Where the first duopolist's quantity remains hidden, however, the normal form of the game corresponds to one in which the duopolists choose their quantities simultaneously. Then the Cournot–Nash equilibrium is the appropriate sophisticated equilibrium. Moreover, it will turn out that credibility imposes a binding constraint on the leader's strategy choice.

For example, suppose that each duopolist $i \in \{1, 2\}$ has a profit function

$$\Pi_i(q_i, q_j) = \alpha_i q_i - \beta q_i q_j - \tfrac{1}{2} q_i^2$$

which is quadratic in its own quantity q_i and also depends on the other's quantity q_j. Suppose too that each duopolist is risk neutral and so content to maximize expected profits. Finally, suppose that the three parameters α_1, α_2 and β are positive and satisfy the restrictions $\alpha_1 > \beta\alpha_2$, $\alpha_2 > \beta\alpha_1$, and $\beta < 1$. Even if the first duopolist pursues a mixed strategy, the second duopolist's optimal choice satisfies $q_2 = \alpha_2 - \beta\, \mathbb{E}\, q_1$, where $\mathbb{E}$ denotes the mathematical expectation. Thus, the first duopolist's expected profit is

$$\mathbb{E}\Pi_1 = (\alpha_1 - \beta q_2)\mathbb{E} q_1 + \beta^2 (\mathbb{E} q_1)^2 - \tfrac{1}{2}\mathbb{E} q_1^2.$$

This is maximized by choosing the Stackelberg leader's pure strategy with $q_1 = (\alpha_1 - \beta\alpha_2)/(1 - 2\beta^2)$. This exceeds the unique Cournot equilibrium quantity, which is $q_1 = (\alpha_1 - \beta\alpha_2)/(1 - \beta^2)$. It follows that virtual observability fails, even though there is a unique Nash equilibrium and it uses pure strategies.

6.4 Multiple Equilibria

	h	t	e
H	$4,0$	$0,4$	$0,0$
T	$0,4$	$4,0$	$0,0$
E	$0,0$	$0,0$	$1,1$

Figure 6: Extended Matching Pennies

The game in Fig. 6 is matching pennies played for a stake of \$4 supplied by a third party. The game is also extended by allowing each (steady handed) player to choose "edge" as well as heads or tails. If just one player chooses edge, the stake is withdrawn, and neither wins anything. But if both choose edge the third party pays each \$1 for being imaginative.

In the corresponding game G_1 with perfect information where player 1 moves first, player 2 would choose: t in response to H; h in response to T; and e in response to E. So G_1 has (E, e) as a unique subgame perfect equilibrium. This is not induced by a PBE strategy–belief profile of $\hat{G}$, however, because a better Nash equilibrium of G_0 for player 1 is the familiar mixed strategy equilibrium with $\alpha_1(H) = \alpha_1(T) = \alpha_2(h) = \alpha_2(t) = \frac{1}{2}$, for which player 1's expected payoff is 2 rather than 1. Once again, virtual observability fails, and in this case it does so even though the unique subgame perfect equilibrium is a Nash equilibrium in pure strategies.

7 Concluding Remarks

7.1 Beyond Experimental Anomalies

Experimental economists have recognized that there is a first mover advantage in Battle of the Sexes and similar games. They ascribe this advantage, however, to "positional" or "presentational" effects, suggesting the need to look beyond orthodox rationality concepts in order to explain their experimental results.

This paper, by contrast, introduces a "sophisticated" refinement of Nash equilibrium capable of explaining first mover advantage. This refinement, like the "manipulated Nash equilibrium" concept explored in Amershi, Sadanand and Sadanand (1985, 1989a, b, 1992) and in Sadanand and Sadanand (1995), depends on the extensive form of the game, and so it violates von Neumann's hypothesis of normal form invariance. Unlike manipulated Nash equilibrium, however, the tacit communication that underlies forward induction arguments is explicitly modelled through a corresponding game with cheap talk. This cheap talk is required to be predictable so that it can remain tacit.

Nevertheless, the precise relationship between sophisticated and manipulated Nash equilibrium deserves further exploration. The ideas presented here should also be explored in a much broader class of games, perhaps starting with the "recursive games" considered in Hammond (1982).

7.2 Future Research

The introduction pointed out the two assumptions of the von Neumann paradigm, which underlies much of modern game theory. This paper has criticized normal form invariance, the second of these. But the first, claiming that games can be modelled with a single extensive form, is also questionable, as discussed in Hammond (2006). So, of course, is a third key assumption, that players are fully rational, and so will always find the optimal action at each information set. The following passage is taken from a novel by an author who won the Nobel Prize for Literature in 1968. It applies to the perfect information two-person zero sum game of Go, which orthodox game theory predicts should be played perfectly, and so perfectly predictably.

> "'This is what war must be like,' said Iwamoto gravely.
>
> He meant of course that in actual battle the unforeseeable occurs and fates are sealed in an instant. Such were the implications of White 130. All the plans and studies of the players, all the predictions of us amateurs and of the professionals as well had been sent flying.
>
> As an amateur, I did not immediately see that White 130 assured the defeat of the 'invincible Master.'"
>
> Yasunari Kawabata (1954) *The Master of Go*, translated from the author's own shortened version by Edward G. Seidensticker (New York: Alfred A. Knopf, 1972); end of chapter 37.

Such considerations remind us how far the three standard assumptions take us from reality. To conclude, it seems that the systematic study of games and economic behaviour has barely progressed beyond a promising but possibly misleading beginning.

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