Exercises on aglebraic groups - I

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All groups are over a filed k of characteristic 0.

1. Check that D_n (diagonal matrices), T_n (upper triangular), U_n (upper triangular unipotent) and SL_n are closed subgroups of GL_n.

2. Let G be and affine algebraic group. Show that any finite subgroup of G is an affine algebraic group.

3. Show that the only algebraic automorphisms of $\mathbb{G}_m$ are $x \mapsto x$ and $x \mapsto x^{-1}$.

4. Show that $\mathbb{G}_a$ and $\mathbb{G}_m$ are not isomorphic.

5. Show that $\mathbb{G}_a$, $\mathbb{G}_m$, D_n, T_n, U_n and SL_n are connected but O_n is not connected. Determine the connected component of the identity of O_n.

6. Let G be a connected algebraic group and N a *finite* normal subgroup. Show that N is contained in the centre of G.

7. Determine all algebraic group homomorphisms $\mathbb{G}_m \longrightarrow \mathbb{G}_m$.

8. Determoine the group of automorphisms of $\mathbb{G}_m^r$.

9.