UCLA **Winter 2017**
MATH 214A, Introduction to Algebraic Geometry **Martin Gallauer**

Homework 5

Due: February 17, 2017

1 There are 9 topological spaces of cardinality 3 up to homeomorphism. For each of these give an example of an affine scheme with this topology, or explain why no such example exists.

2 Let A be a boolean ring, i.e. $a^2 = a$ for each $a \in A$. Prove the following properties about $X = \mathrm{Spec}(A)$:

(a) Every point of X is closed.

(b) For each $x \in X$, $\kappa(x) \cong \mathbb{F}_2$.

(c) X is compact and totally disconnected.

Optionally, prove that $A \mapsto \mathrm{Spec}(A)$ determines an equivalence of categories between boolean rings (as a full subcategory of the category of rings) and compact totally disconnected spaces. (What is "the" quasi-inverse?)

3 (a) Let X be a scheme and A a local ring. Identify $X(A) := \hom_{\mathcal{SCH}}(\mathrm{Spec}(A), X)$ with the set of pairs (x, φ) where $x \in X$ and $\varphi : \mathcal{O}_{X,x} \to A$ is a local homomorphism.

(b) Notice that we get, for a given point $x \in X$, a canonical morphism $\mathrm{Spec}(\mathcal{O}_{X,x}) \to X$. What is the image of the underlying map on topological spaces?