Promoting versatile learning of higher order concepts in algebra using the computer

David Tall & Michael Thomas
Mathematics Education Research Centre
University of Warwick
Coventry

This paper[1] reports a comparison between a traditional skill-based approach to algebra and an alternative approach using a combination of computer programming, activities enacting the computer processes manipulating variables, and the use of computer software which evaluates algebraic expressions written in either traditional notation or BASIC. The results of the experiments indicate the value of this approach in improving early learners' understanding of higher level algebraic concepts. Our hypothesis is that the improved conceptualisation of algebra resulting from the computer paradigm, with its emphasis on mental imagery and a global/holistic viewpoint, will lead to more versatile learning.

Mental images need not be only pictures in the traditional sense. The ideas we are discussing apply equally well to such things as symbolic concepts, provided that they are capable of being visualised and mentally manipulated. In Tall & Thomas [1986], a "dynamic approach" to algebra is described which involves a combination of activities designed to give children the ability to visualise the concept of an algebraic variable. This involves programming activities (in BASIC), such as:

```
A = 3
PRINT A +1
```

to see what the effect will be. Again the computer acts in a reasonable and predictable fashion; if A=3, then PRINT A+1 will print 4. Coupled with these activities are games involving the physical storage of a number in a box drawn on card and marked with a letter, so that a variable may be visualised as a placeholder for a number, and software which enables mathematical formulae to be evaluated for given numerical values of the letters involved. Matched pairs of 11/12 year olds were selected; one set studied the dynamic algebra module whilst their matched pairs did other (non-algebraic) work. The results, as one might expect, showed the experimental students performing at a higher

---

[1]The contents of this paper (*Proceedings of B.S.R.L.M.*, Warwick May 1988) are taken from "Versatile Learning and the Computer", to appear in the journal "Focus". The results will be presented in more detail in the paper "Longer term Effects of the Use of the Computer in the Teaching of Algebra" at PME 12 in July 1988.

conceptual level where algebraic notation was concerned. For example the successful responses to the question:

When does M+P+N=N+M+R? Always, never, sometimes when ...

improved from 0% to 42.9% (as compared to 27% of 15 year olds in Hart, [1981]).

In order to test the long-term effects of the approach, a further algebra test was given over one year after the initial experiment. A summary of the results and a comparison with their previous results are given in table 1. This demonstrates that, more than one year after their work on basic concepts of algebra in a computer environment, they were still performing significantly better.

Test		Experim. Mean	Control Mean	p (1-tail)
Post Test	(N=21)	41.2 %	25.3 %	<0.0005
Delayed Post Test	(N=20)	43.9 %	32.6 %	<0.005
one year later	(N=10)	55.8 %	47.3 %	<0.025

Table 1

This lends strong support to the idea that the introduction of a module of work such as the dynamic algebra package, with its emphasis on conceptualization and use of mental images rather than skill acquisition, can provide significant long-term conceptual benefits.

Skills and Higher Order Concepts

A second teaching experiment was held in which a dynamic algebra approach using the computer was compared with more traditional teaching methods [Thomas 1988]. The subjects of this second experiment were children taken from six mixed ability forms in the first year of a 12-plus entry comprehensive school. On the basis of an algebra pre-test it was possible to organise 57 matched pairs covering the full ability range.

In the first stage of the comparison the experimental group used the dynamic algebra module during their normal mathematics periods, using computers in small groups over a three week period. At the same time the control group used a traditional skill-based module developed in the school over some years, covering basic simplification of expressions and elementary equation solving in one unknown. Immediately following the work they were given a post-test identical to the pre-test.

The results superficially showed that there was no significant difference in overall performance, but on skill-based questions related to the content of the traditional module, the control group performed significantly better, whilst on questions traditionally considered to be conceptually more demanding, related to the experimental material, the experimental group performed better. For instance the question "simplify 3b-b+2a" was answered correctly by 61% of the control group, but only 29% of the experimental group. Meanwhile the question "for what values of a is a+3>7" was answered correctly by 31% of the experimental group and only 12% of the control group. The experimental group also performed better on more general problems less related to the taught subject matter, for example 50% of them responded correctly to the question "What is the perimeter of a rectangle D by 4" compared to 27% of the control.

Some six months later the pupils were all given the same traditional revision course on their earlier algebra, without any use of the computer. Both groups were re–tested and a comparison of matched pairs now showed the experimental students performing significantly better than the control students. In traditional manipulative skills they made up their deficit, whilst retaining a measure of superiority in conceptually higher order tasks (table 2).

Manipulative & Higher order skills	Experim. Mean	Control Mean	p (1-tail)
Manipulative Skills : Post Test (N=48)	65.1 %	68.7%	n.s.
Delayed Post Test (N=47)	75.3%	72.4 %	n.s.
Higher order Skills : Post Test (N=48)	35.7%	29.9 %	<0.025
Delayed Post Test (N=47)	42.2%	36.3%	<0.025

Table 2

The differential effects of the two treatments could be considered as a manifestation of the skills versus conceptual understanding dichotomy, in terms of the levels of understanding defined by Küchemann [1981]. His level 1 involves purely numerical skills or simple structures using letters as objects, level 2 involves items of increased complexity but not letters as specific unknowns. Level 3 requires an understanding of letters as specific unknowns; level 4 requires an understanding of letters as generalized numbers or variables. It is only at levels 3 and 4 that children are really involved in algebraic thinking rather than arithmetic. In the experiment the control pupils outperform the experimental pupils at levels 1 and 2, whilst the experimental pupils outperform the control pupils at the higher levels.

This suggests that there are differential effects from the two approaches in respect of surface algebraic skills (in which the control students have greater facility) and deeper conceptual understanding (in which the experimental students perform better). An alternative and, we suggest, more viable explanation is that the traditional levels of difficulty depend on the approach to the curriculum and may be altered by the use of the computer to encourage versatile learning.

References

Hart K.M. (ed.) 1981: *Children's Understanding of Mathematics: 11-16*, John Murray, London.

Küchemann D.E. 1981: 'Algebra', in Hart K.M. (ed.),1981, pages 102-119.

Tall D.O. & Thomas M.O.J. 1986: 'The Value of the Computer in Learning Algebra Concepts', *Proceedings of the 10th Conference of P.M.E.*, London.

Thomas M.O.J. 1988: *A Conceptual Approach to the Early Learning of Algebra Using a Computer*, unpublished Ph.D. Thesis, The University of Warwick, (in preparation).