Modeling Turbulent Pipe Flow

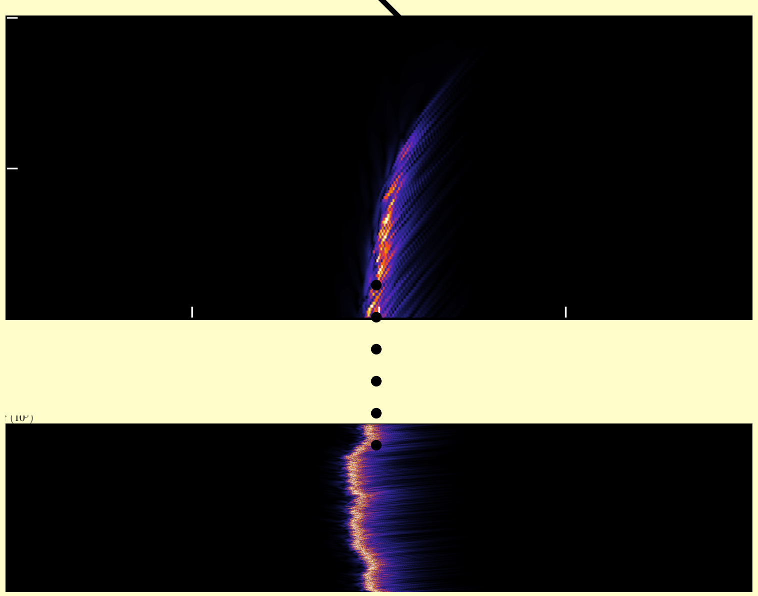
Slides from talk given July 19th, 2011
at BIFD 2011, Barcelona

Regimes of Transitional Pipe Flow

(From the work of many)

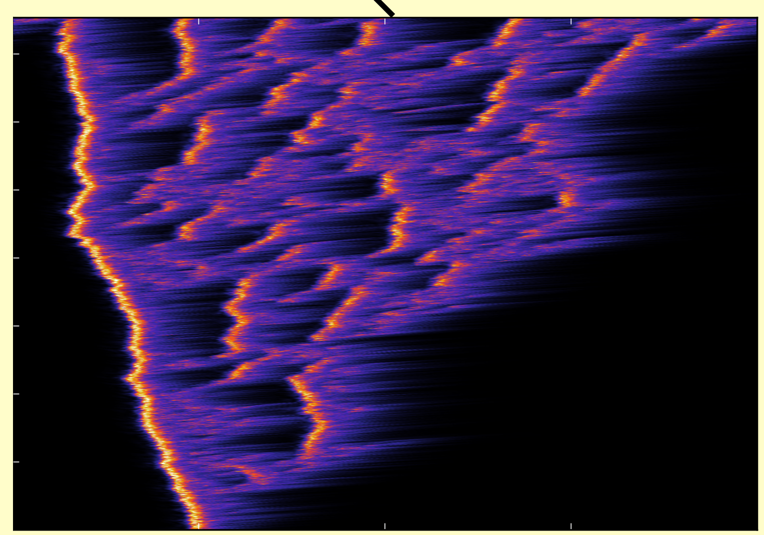
metastable puffs

laminar



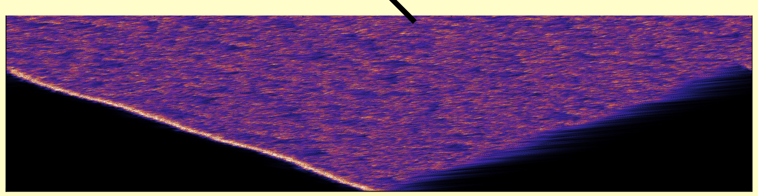
puff splitting

intermittent



slugs

uniform



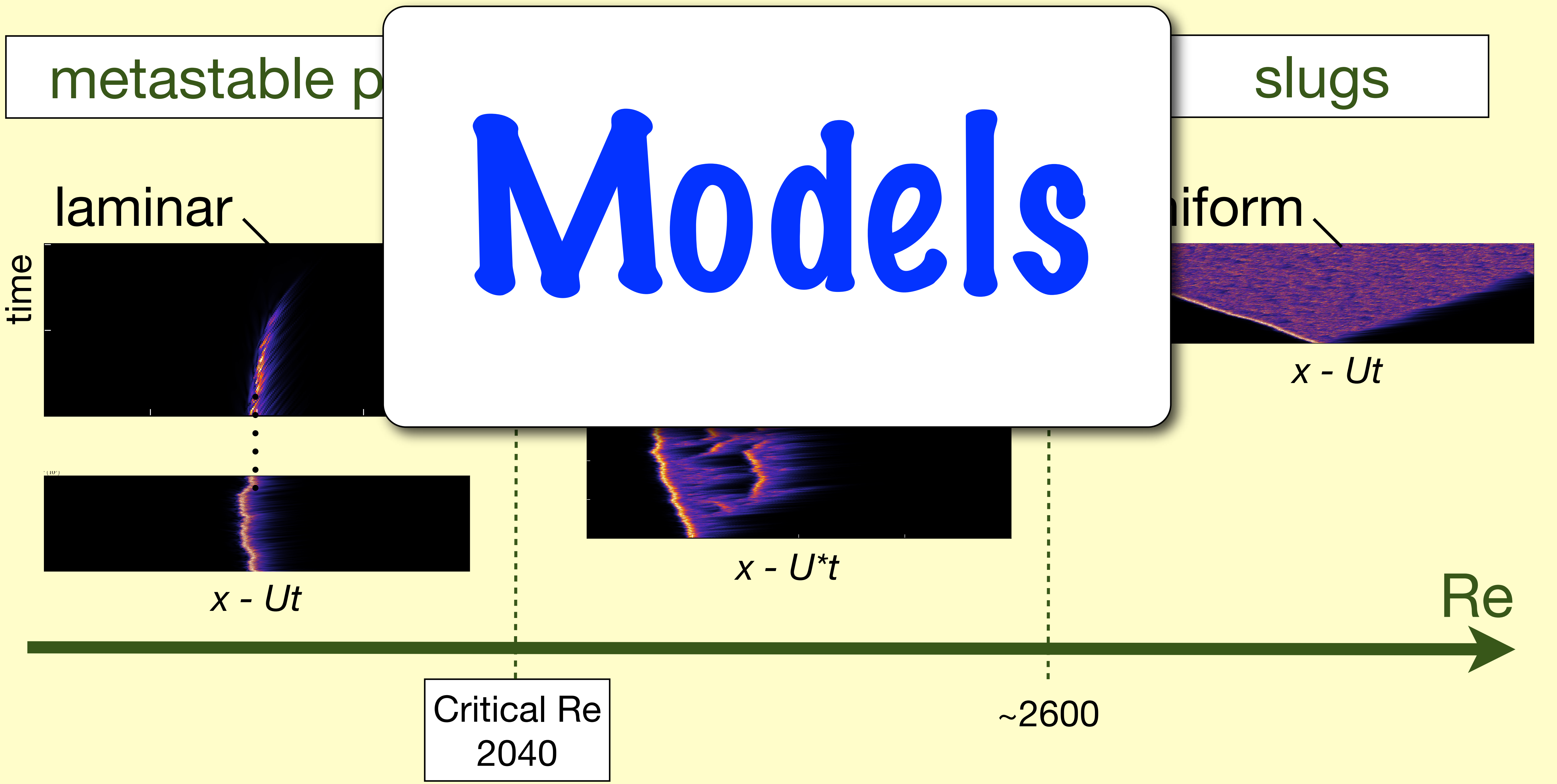
Re

Critical Re
2040

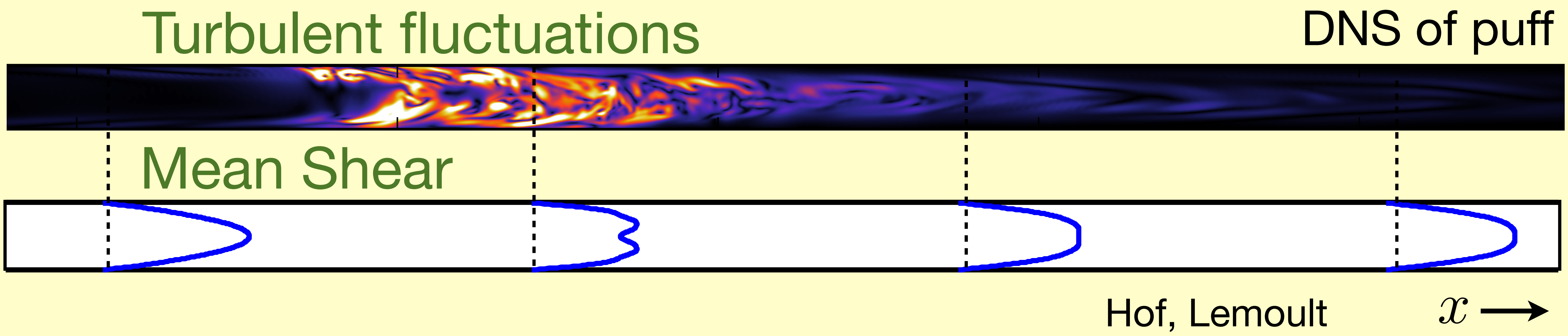
~2600

Regimes of Transitional Pipe Flow

(From the work of many)



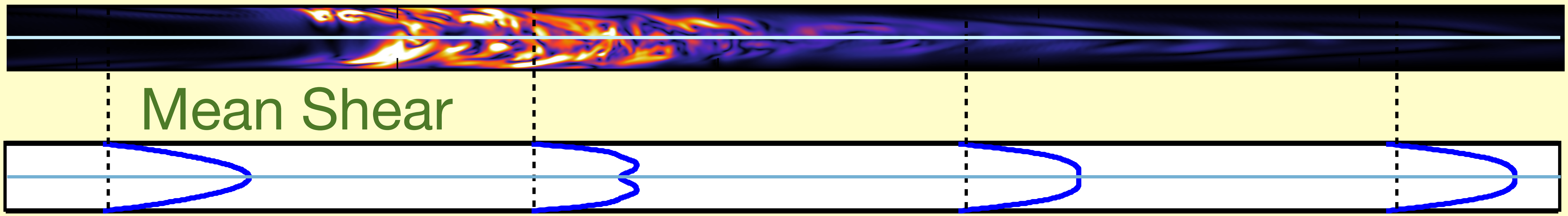
Two Fields:


Turbulent fluctuations
DNS of puff
Mean Shear
Hof, Lemoult
$x \longrightarrow$

Two Fields:

Turbulent fluctuations

DNS of puff

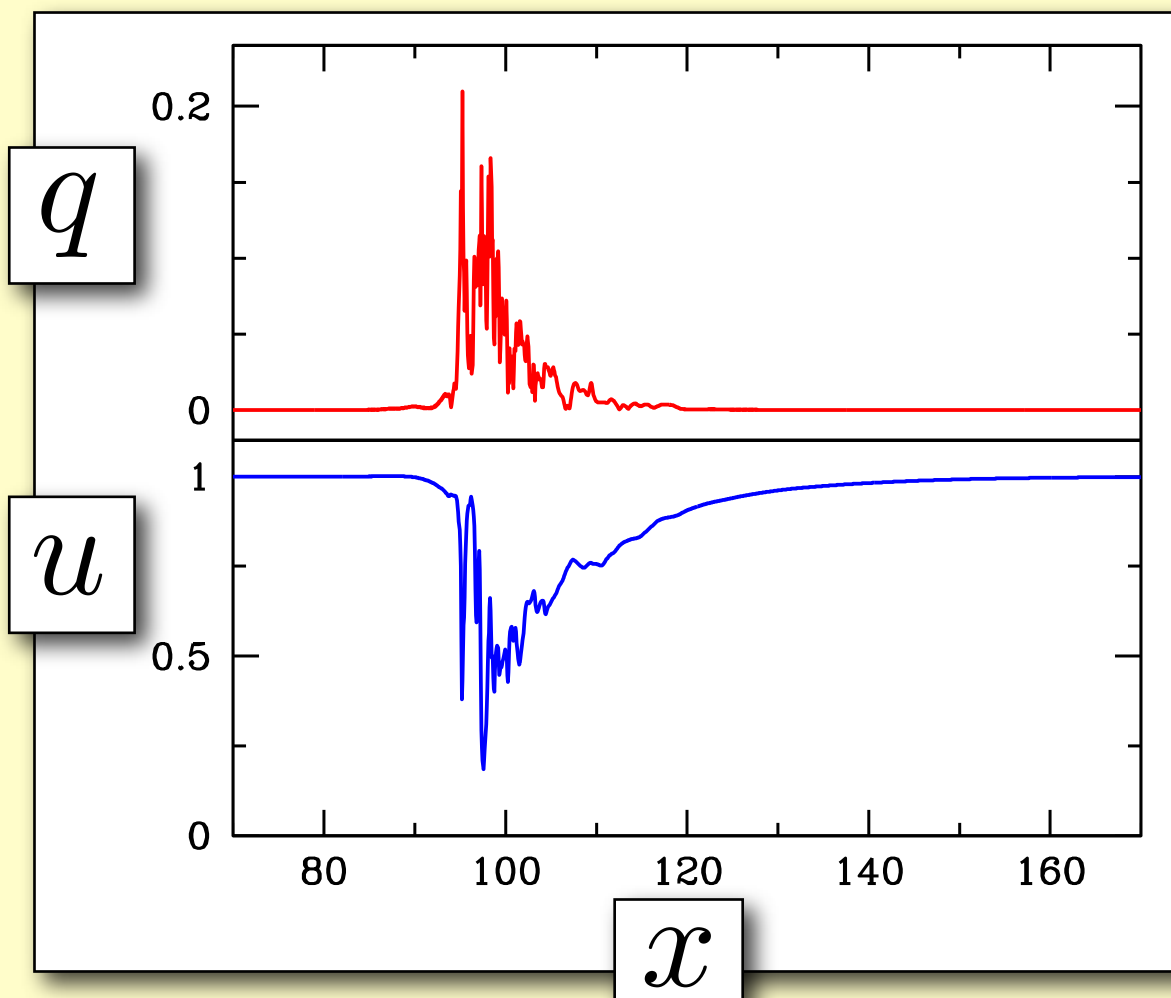


Hof, Lemoult

$x \longrightarrow$

Turbulence Intensity (centerline)

Mean Shear (centerline velocity)



From DNS

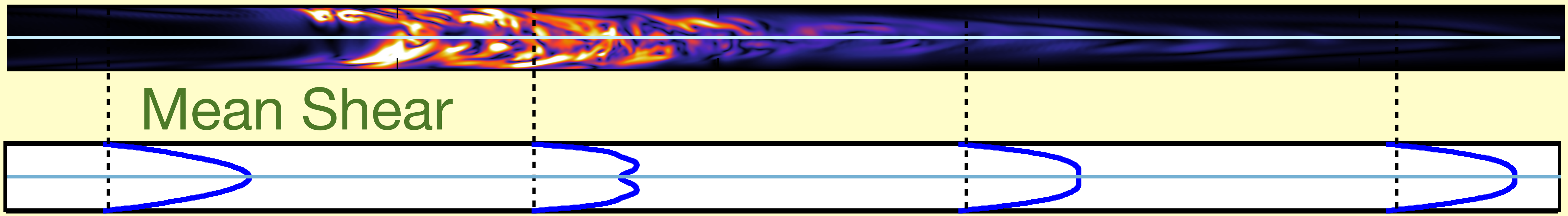
Obtained on pipe centerline

See *Phys. Rev. E* **84,** 016309 (2011) for full definition

Two Fields:

Turbulent fluctuations

DNS of puff



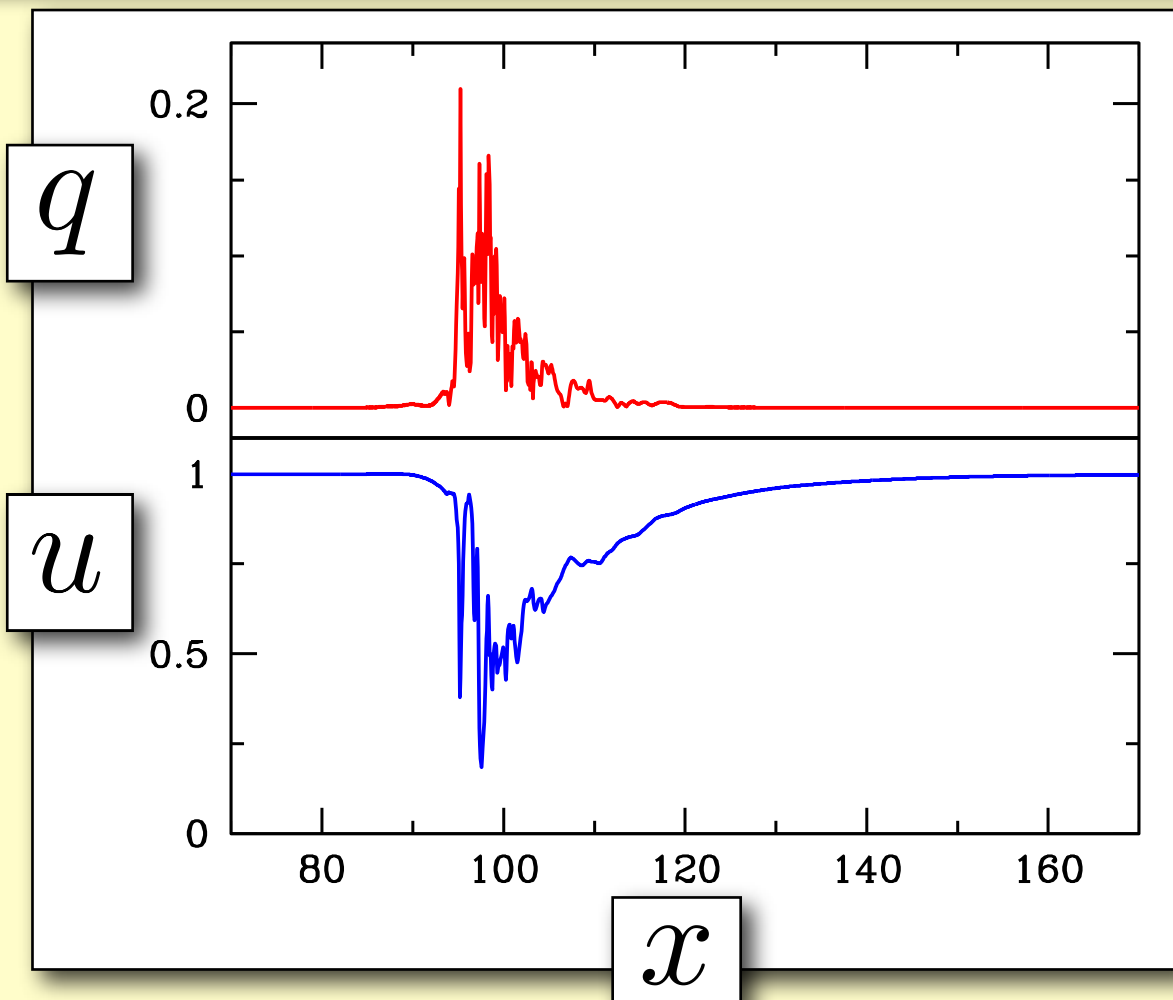
Mean Shear

$x \longrightarrow$

2 variables: q(x) and u(x)

Turbulence Intensity (centerline)

Mean Shear (centerline velocity)



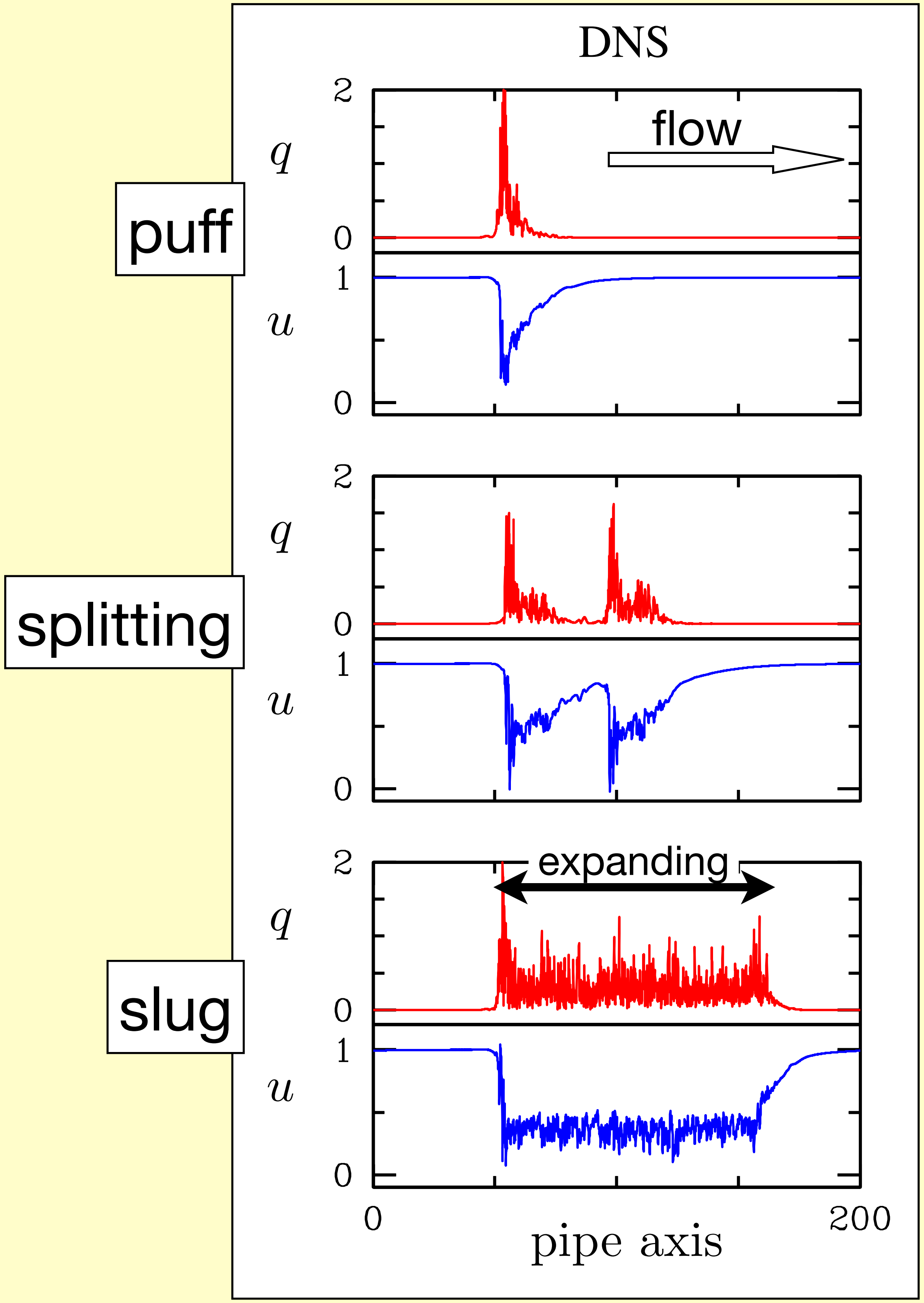
From DNS

Obtained on pipe centerline

See *Phys. Rev. E* **84,** 016309 (2011) for full definition

Physical Ideas

(Laufer (60's), Wygnanski *et al.* (70's), Sreenivasan *et al*. (70's -80's), Hof *et al.,* Eckhardt *et al* (00's))



- Sharp upstream front
  (turbulent energy extracted from laminar shear)
- Reverse transition on downstream side of puff
  (modified shear cannot sustain turbulence)
- No reverse transition on downstream side of slug
- Slow recovery following excitation
  (mean shear recovers slowly)
- State of recovery controls susceptibility to excitation
- Turbulence is locally transient (chaotic saddle)

PDE Model

$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$
$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 uq - \partial_x u$$

Reaction-Advection-Diffusion Equation

PDE Model

$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$
$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 u q - \partial_x u$$

Reaction-Advection-Diffusion Equation

Step-by-step Explanation

PDE Model

$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$
$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 uq - \partial_x u$$

First consider model without spatial derivatives.

ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

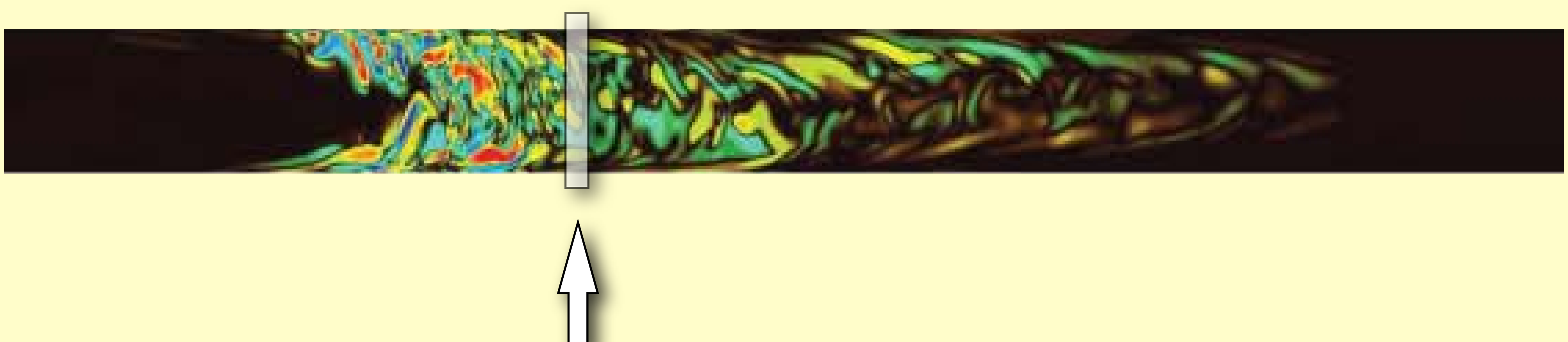
The model reduces to ODEs for the local dynamics

ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 u q$$

The model reduces to ODEs for the local dynamics

This is the core of the model.
It describe how
turbulence and mean shear behave locally in space.

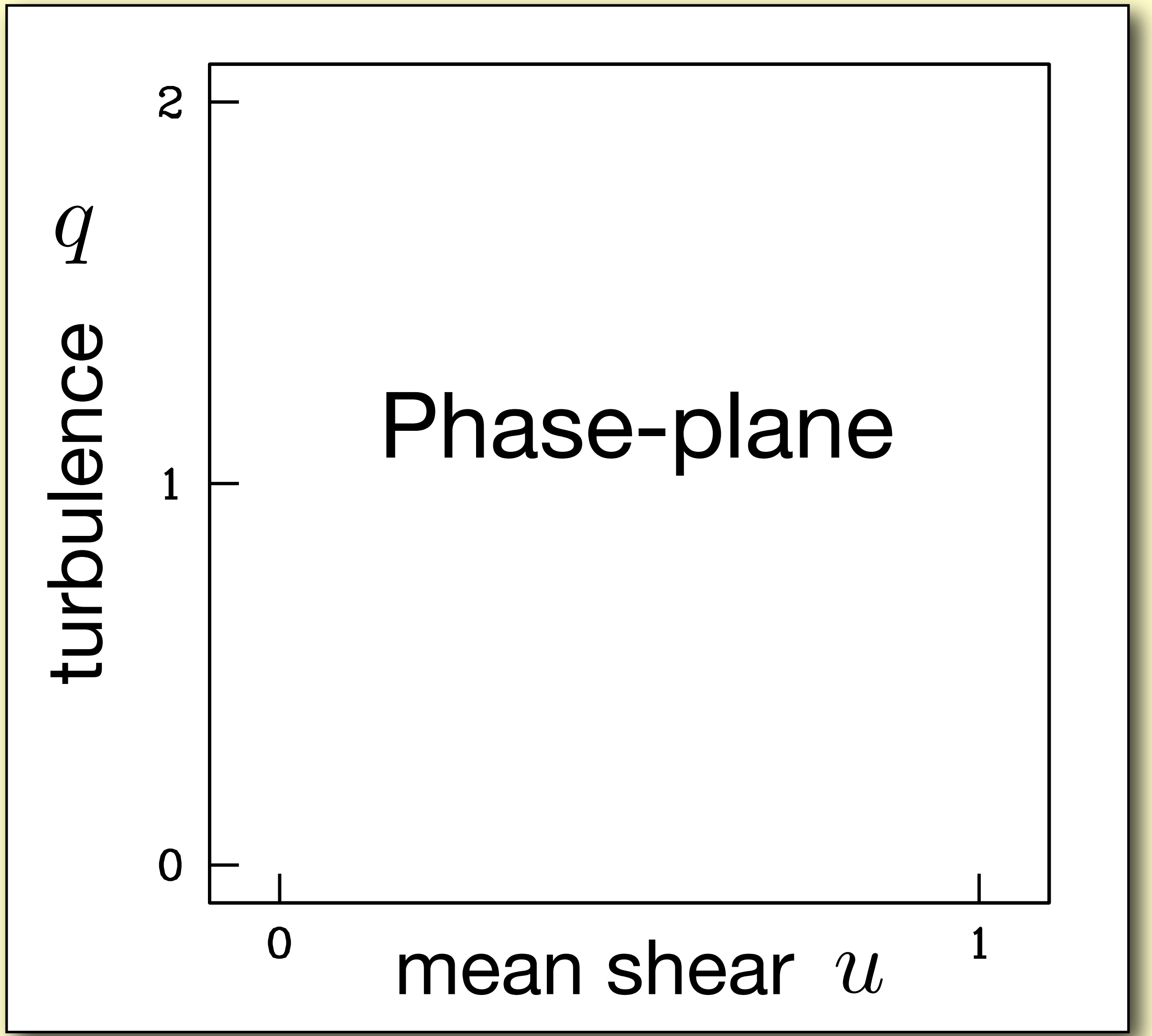


local region

ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 u q$$

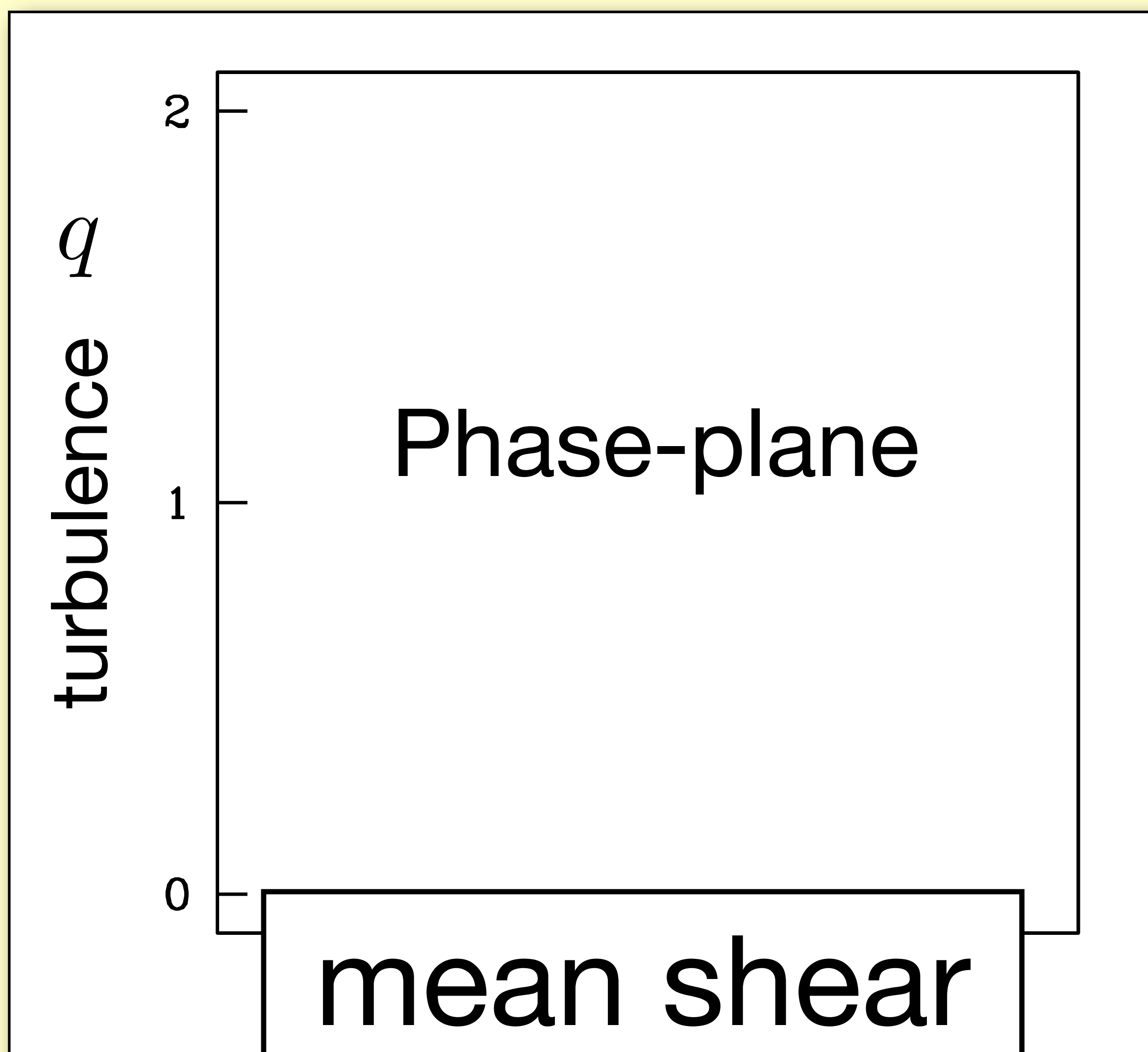
Perform a phase-plane analysis



ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

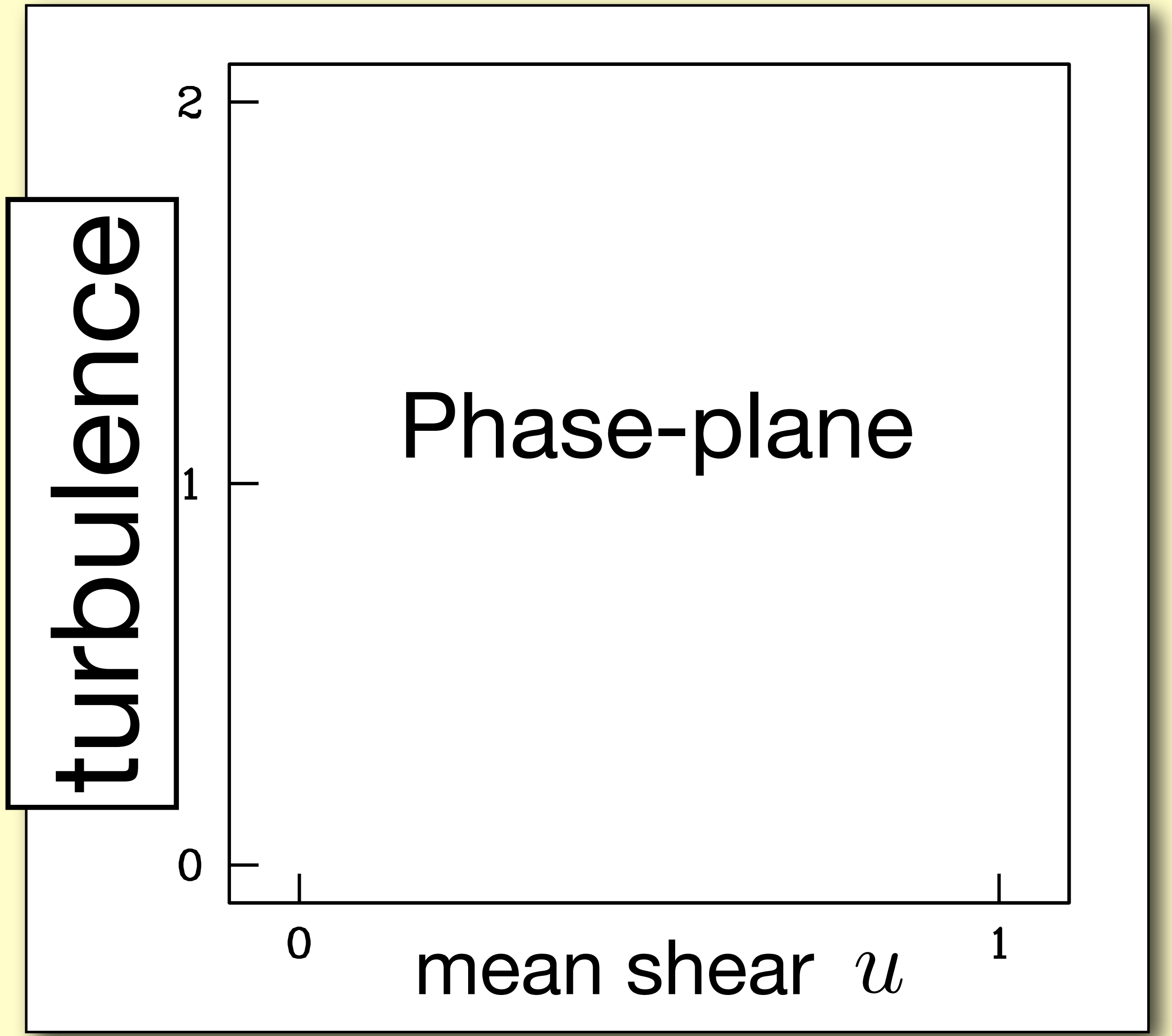
Perform a phase-plane analysis



ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

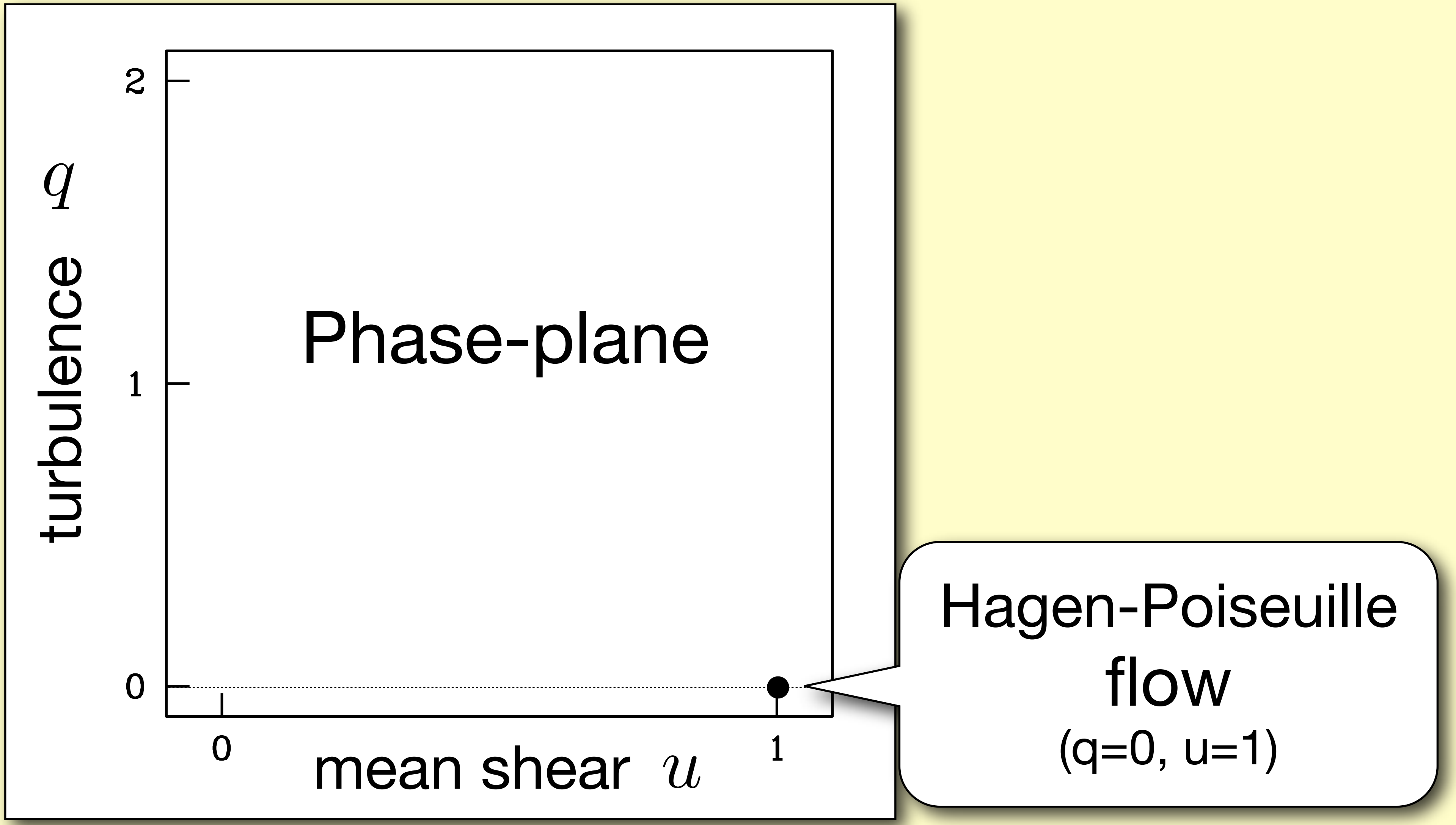
Perform a phase-plane analysis



ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Perform a phase-plane analysis

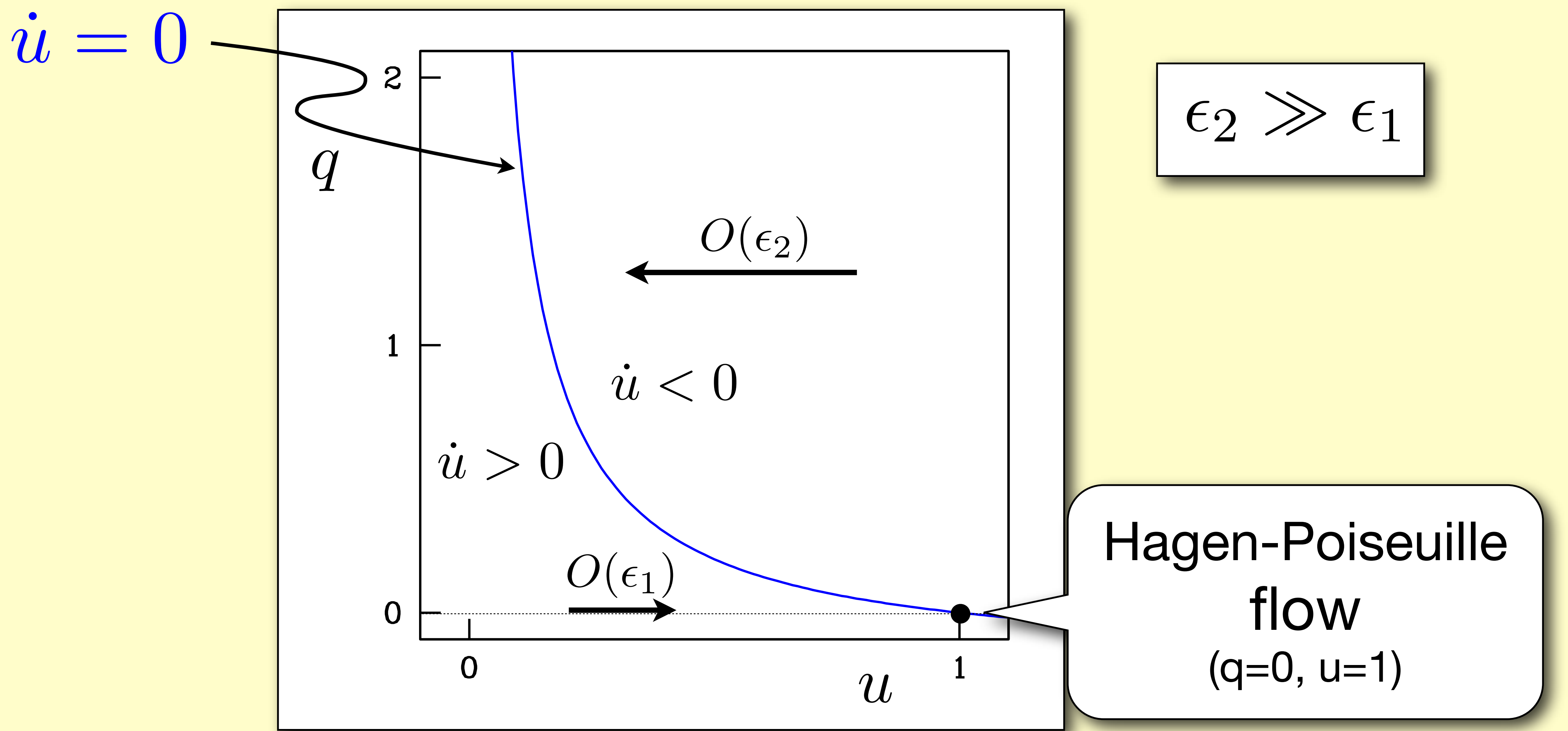


ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Consider first the *u*-dynamics (mean shear)

$$\epsilon_2 \gg \epsilon_1$$



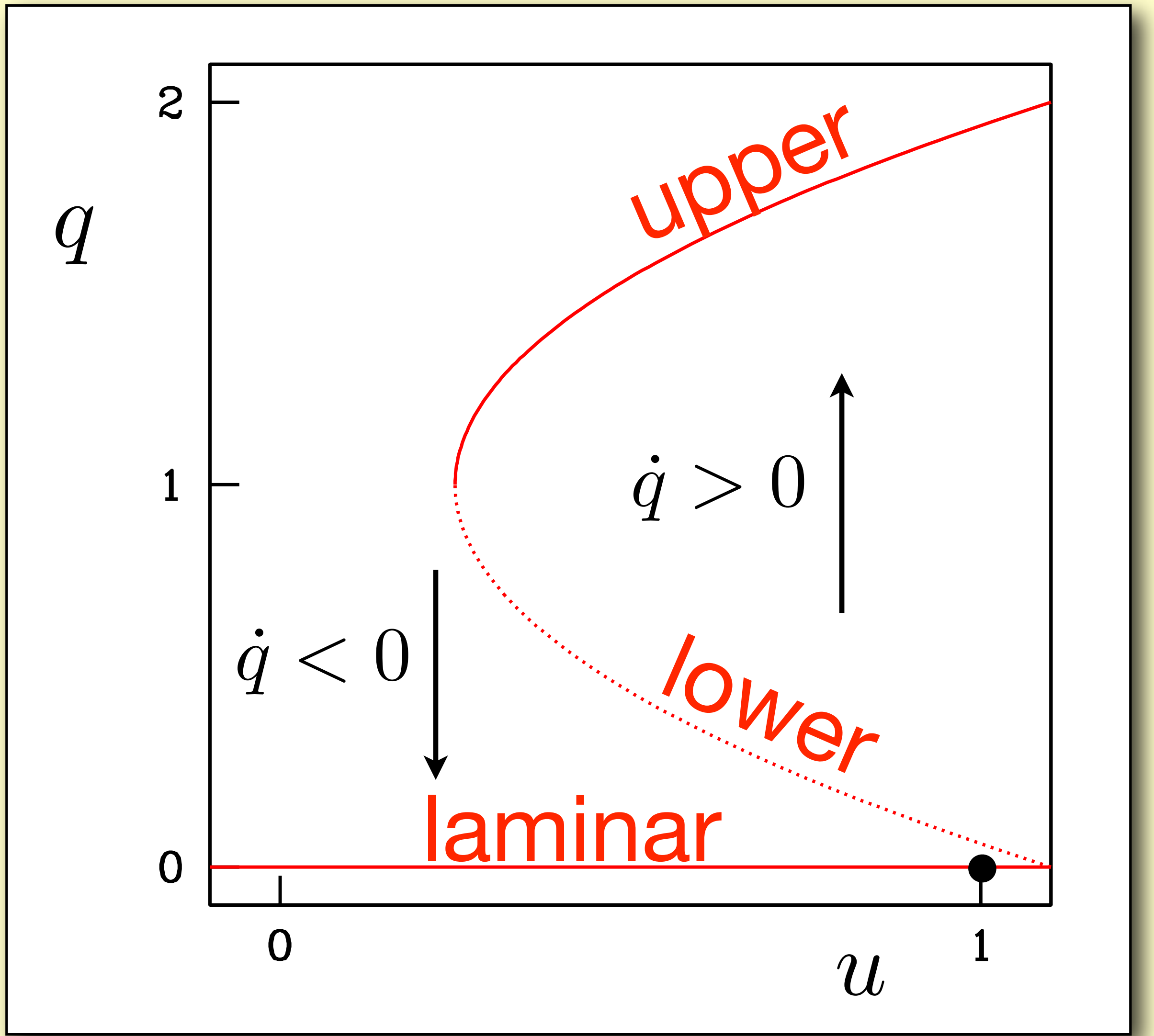
ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Then the q-dynamics (turbulence)

Cubic q equation, so 3 branches:

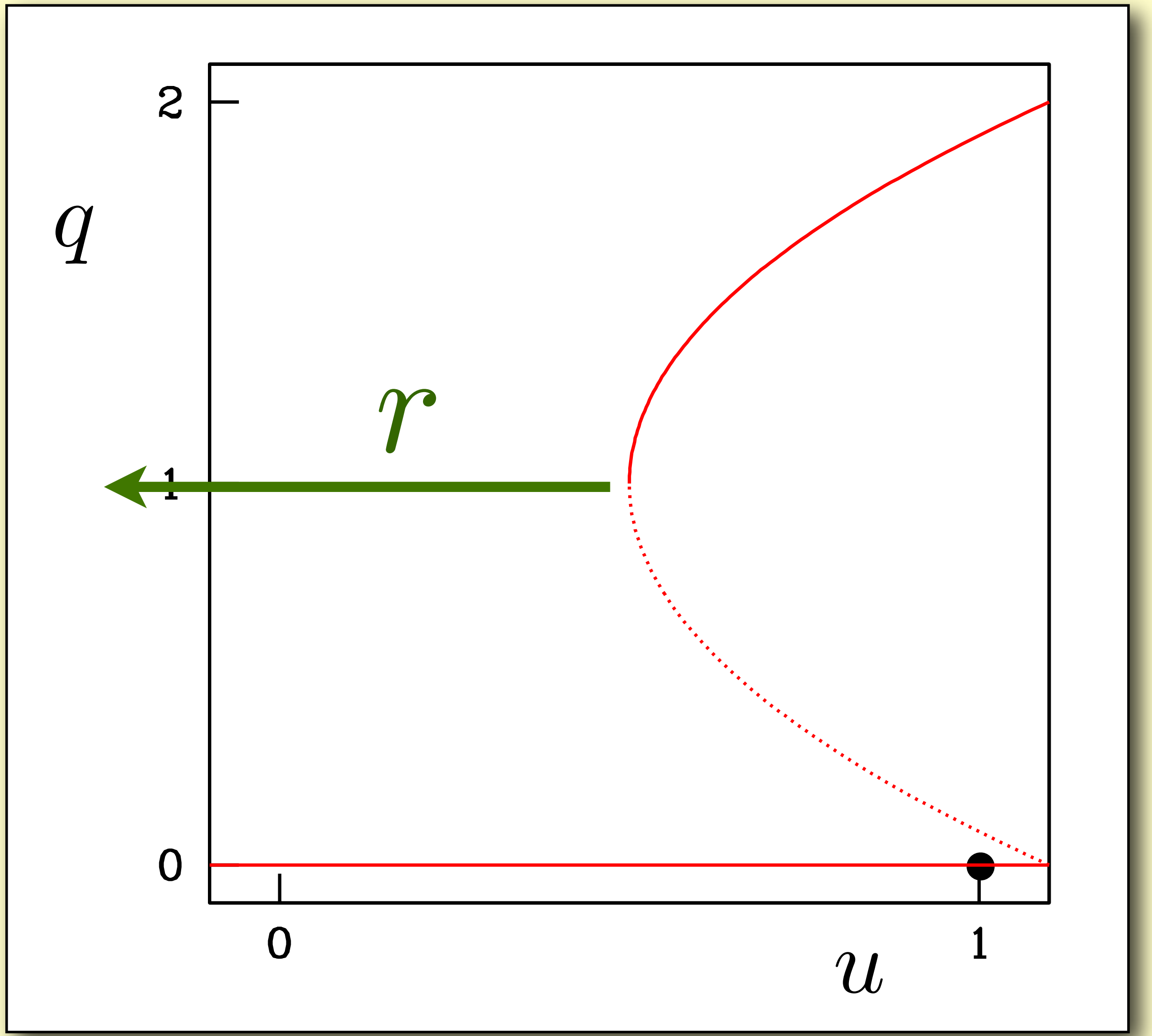
- upper (stable)
- lower (unstable)
- laminar (stable)



ODEs

$$\dot{q} = q\left(u + r - 1 - (r + \delta)(q - 1)^2\right)$$
$$\dot{u} = \epsilon_1(1 - u) - \epsilon_2 uq$$

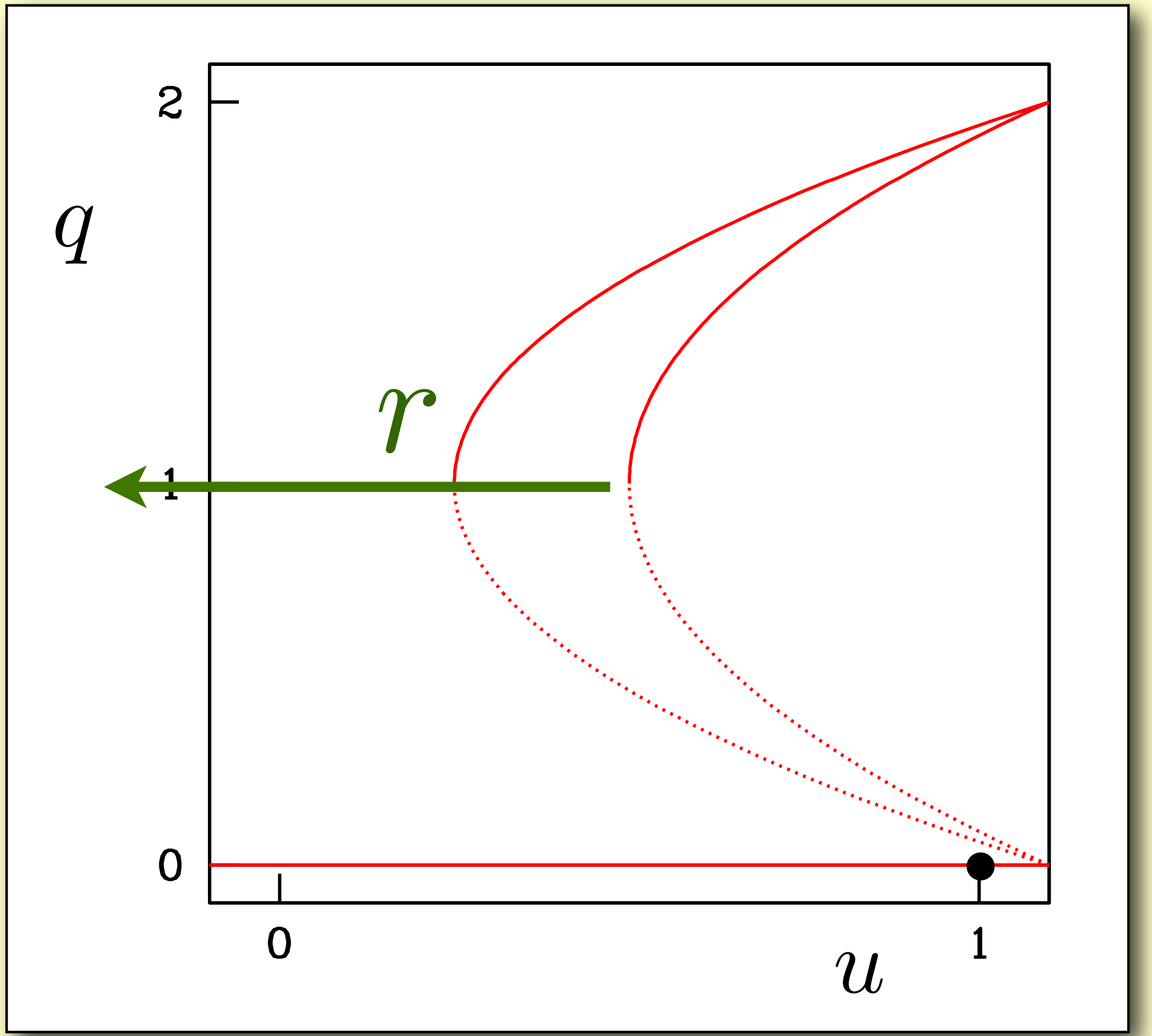
Parameter r "Reynolds number"



ODEs

$$\dot{q} = q\left(u + r - 1 - (r + \delta)(q - 1)^2\right)$$
$$\dot{u} = \epsilon_1(1 - u) - \epsilon_2 uq$$

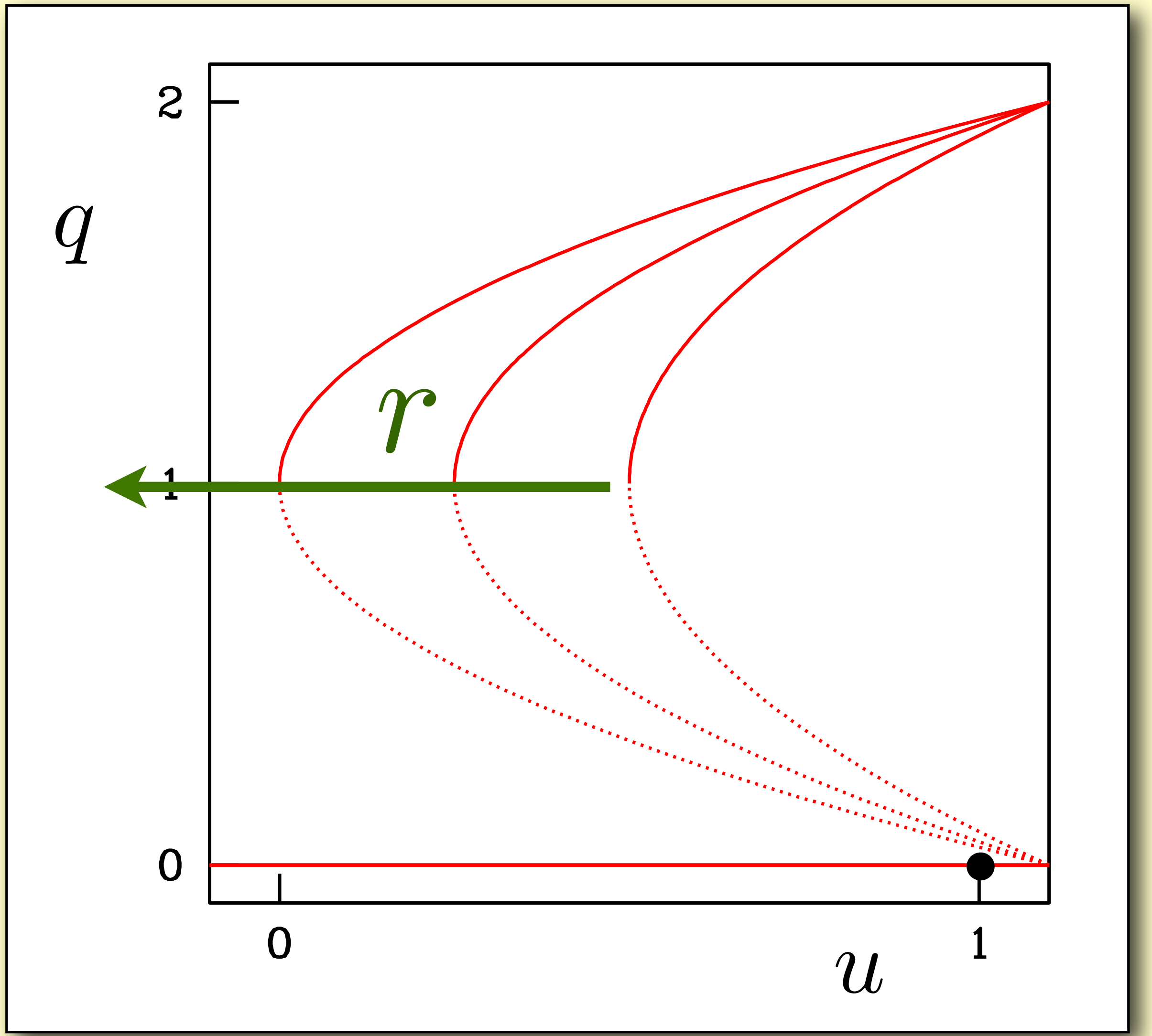
Parameter r “Reynolds number”



ODEs

$$\dot{q} = q\left(u + r - 1 - (r + \delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Parameter r "Reynolds number"

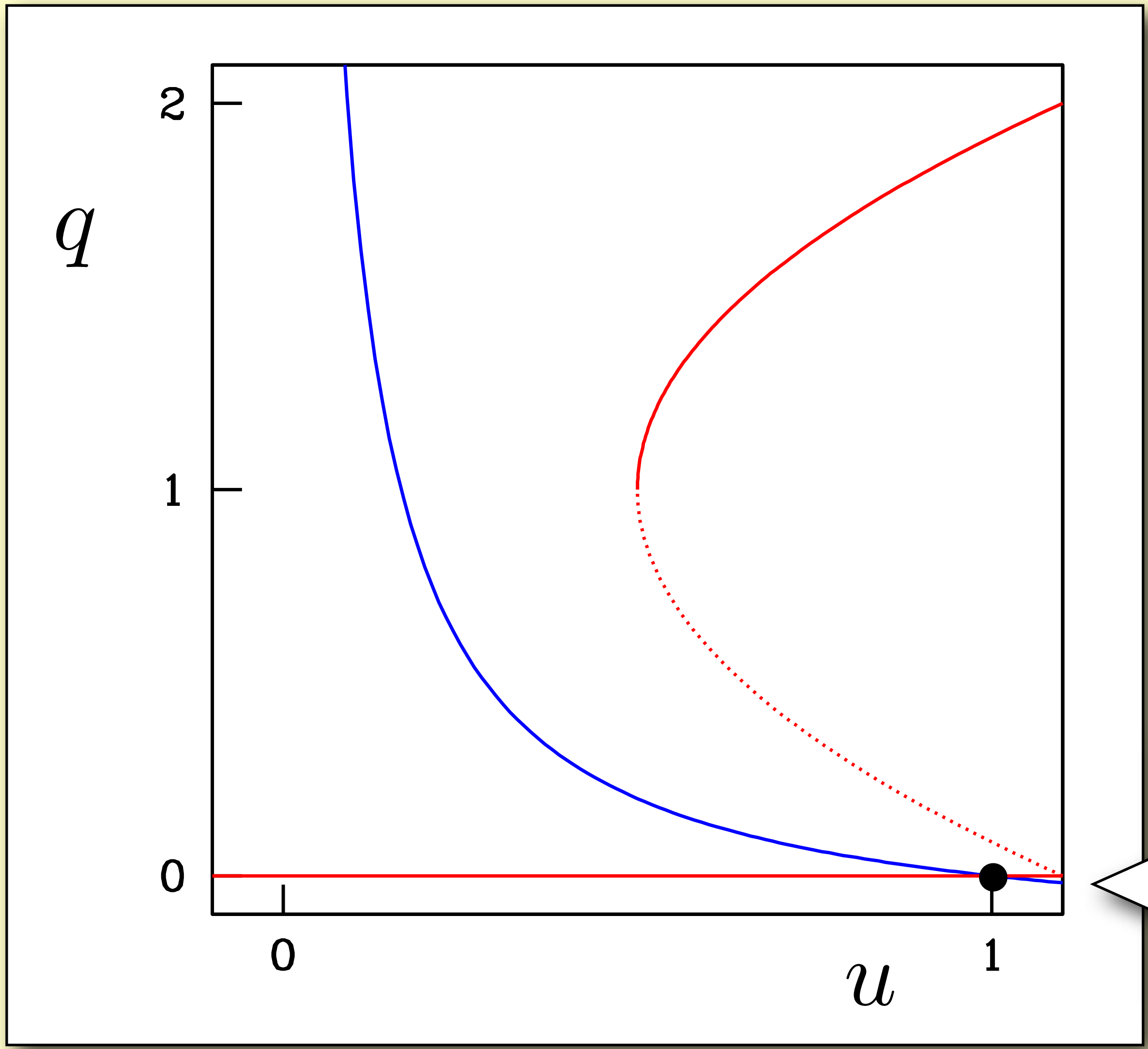


ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$

$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Blue and red curves intersect at fixed points: $\dot{u} = \dot{q} = 0$



Always a fixed point corresponding to

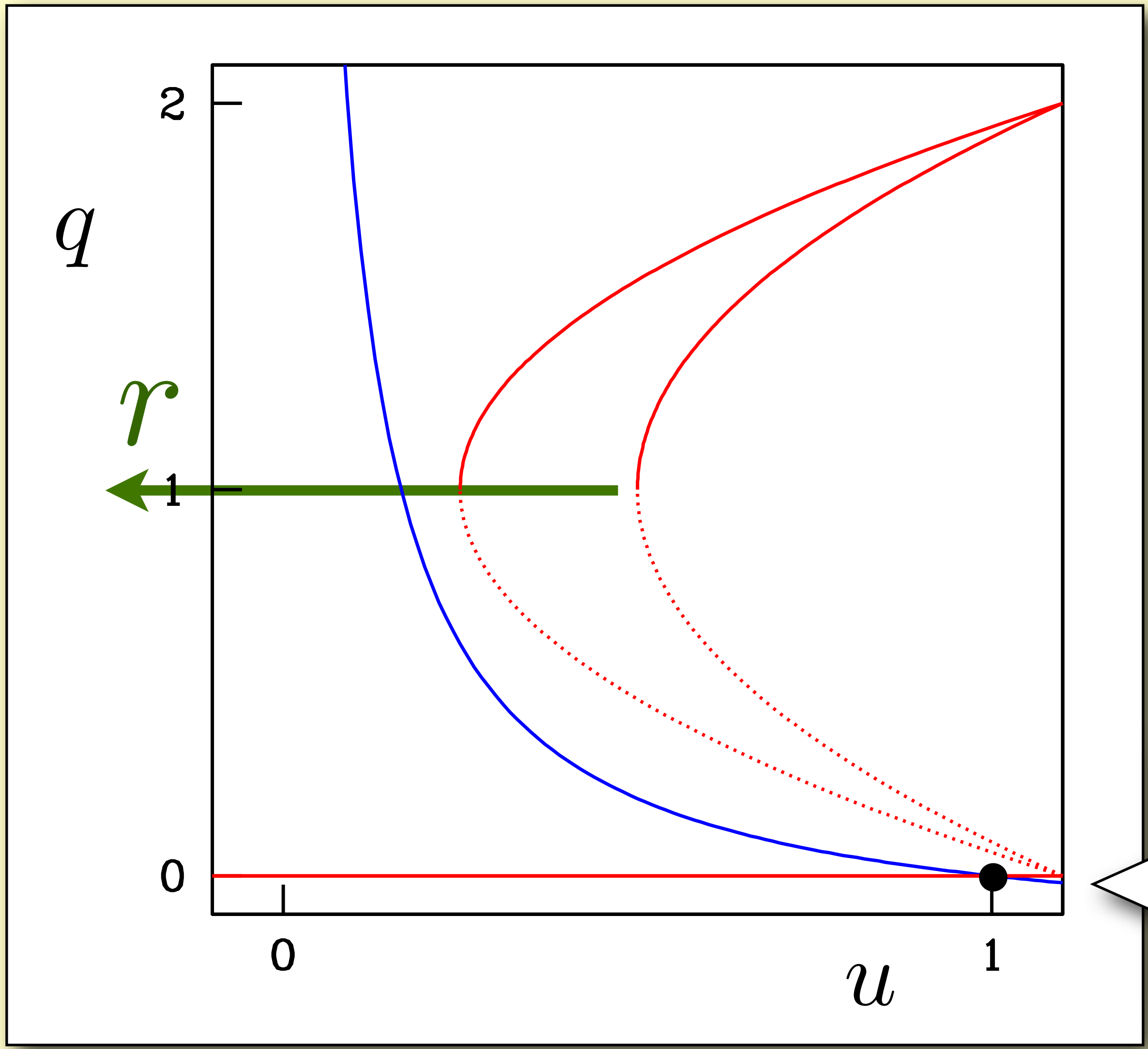
Hagen-Poiseuille flow
(q=0, u=1)

ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Blue and red curves intersect at fixed points: $\dot{u} = \dot{q} = 0$

Increasing r



Always a fixed point corresponding to

Hagen-Poiseuille flow
(q=0, u=1)

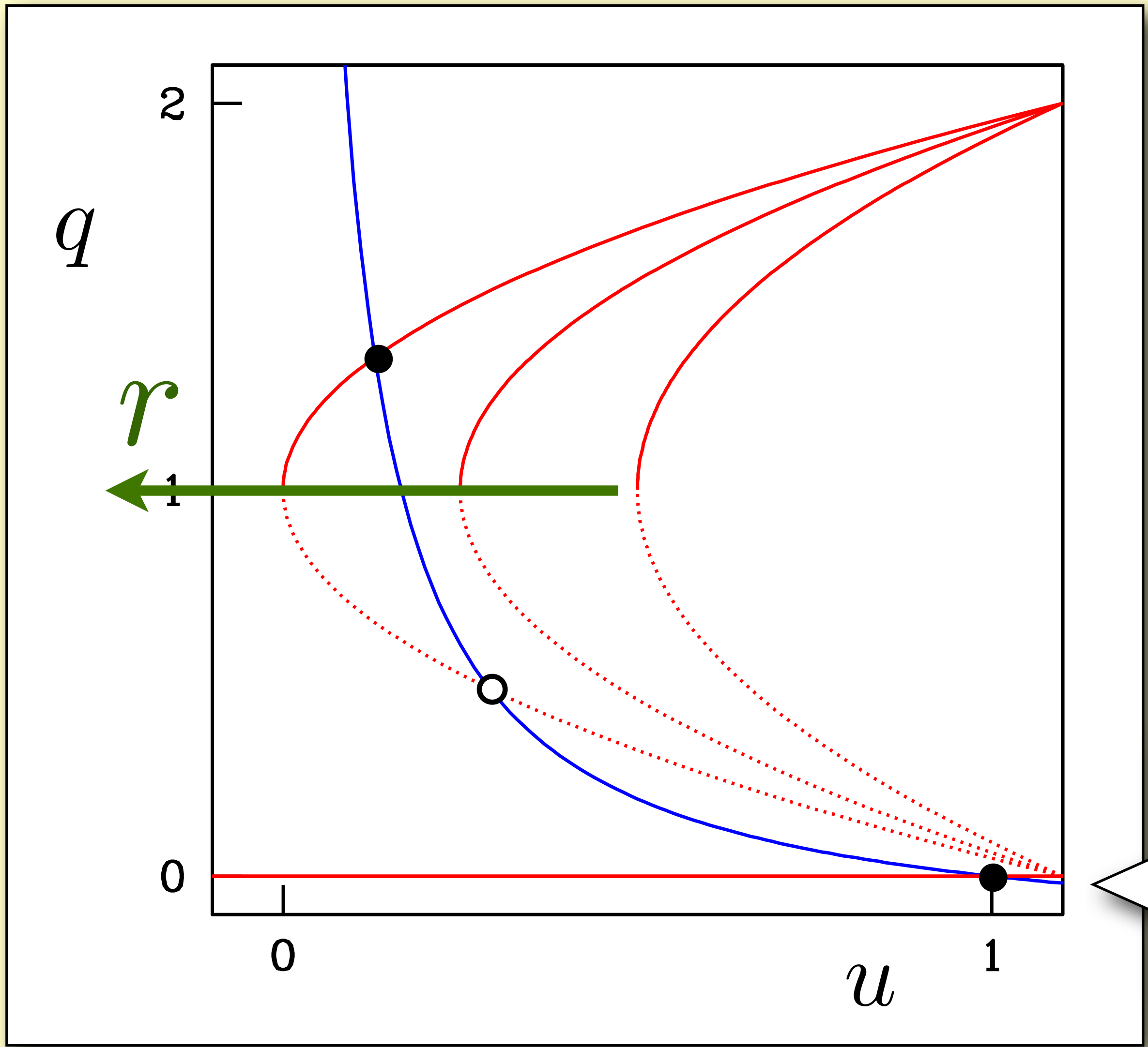
ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$

$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Blue and red curves intersect at fixed points: $\dot{u} = \dot{q} = 0$

Beyond critical value r_c two more fixed points appear.



Always a fixed point corresponding to

Hagen-Poiseuille flow
(q=0, u=1)

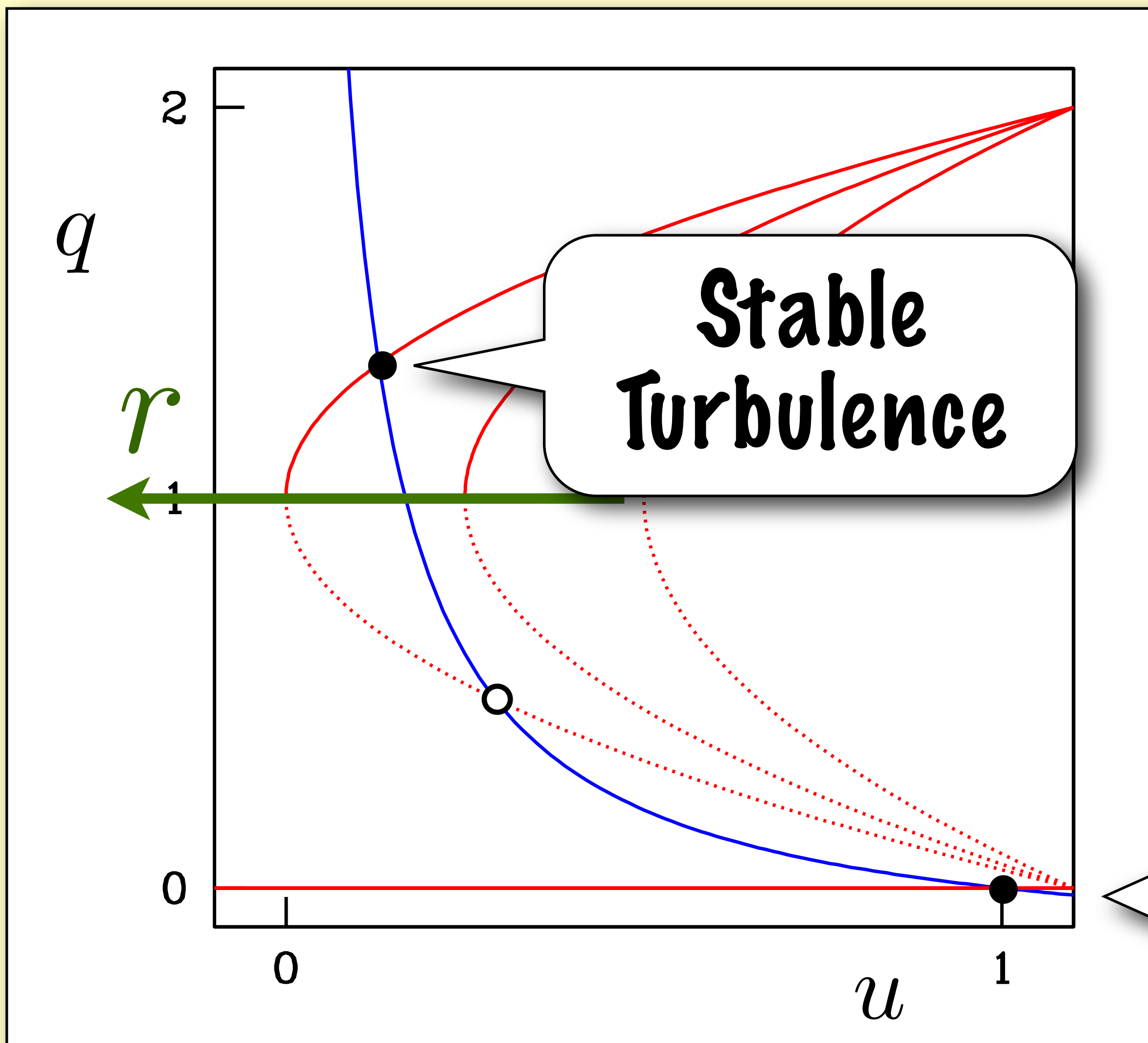
ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$

$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Blue and red curves intersect at fixed points: $\dot{u} = \dot{q} = 0$

Beyond critical value two more fixed points appear.



Always a fixed point corresponding to

Hagen-Poiseuille flow

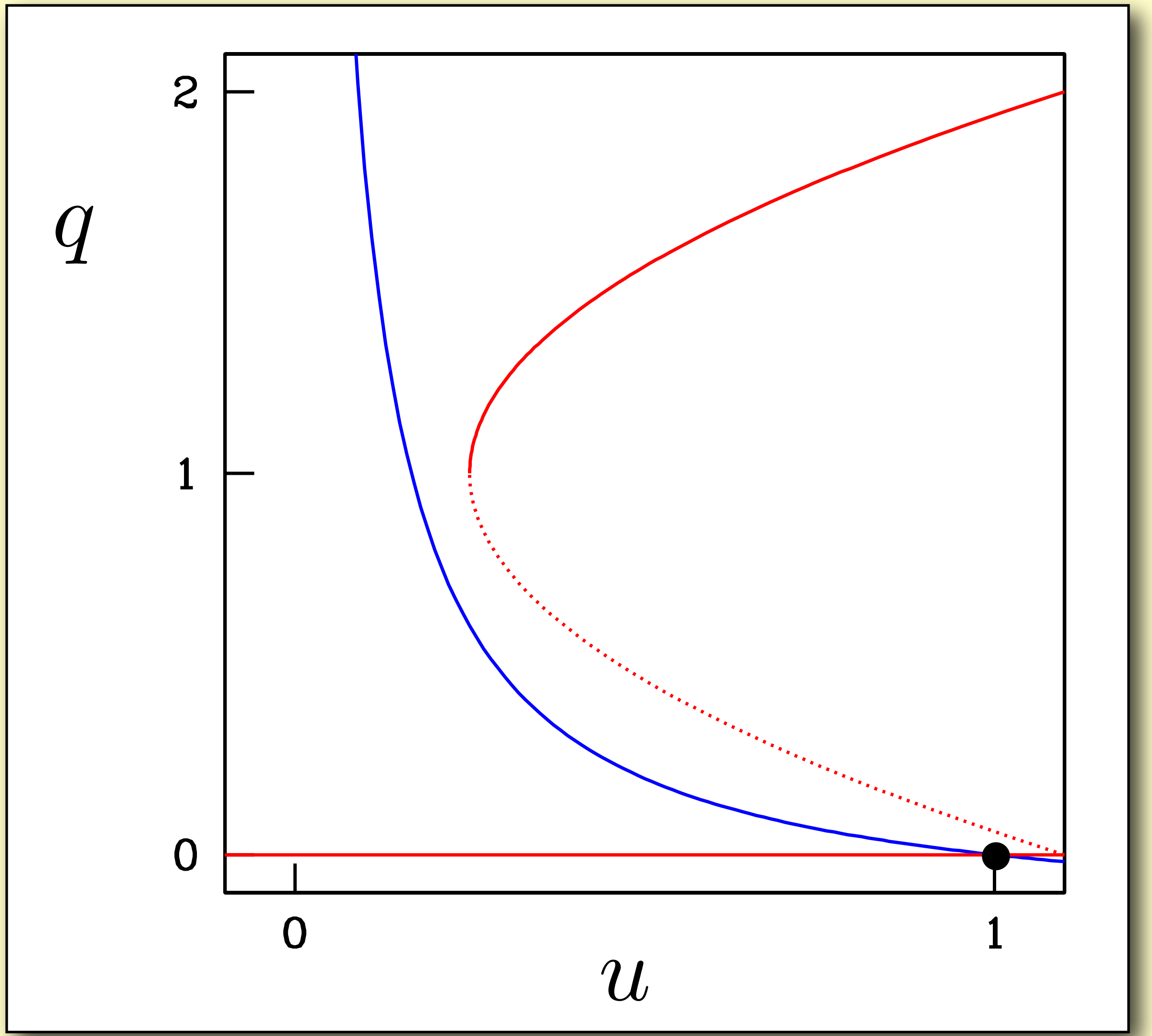
(q=0, u=1)

ODEs

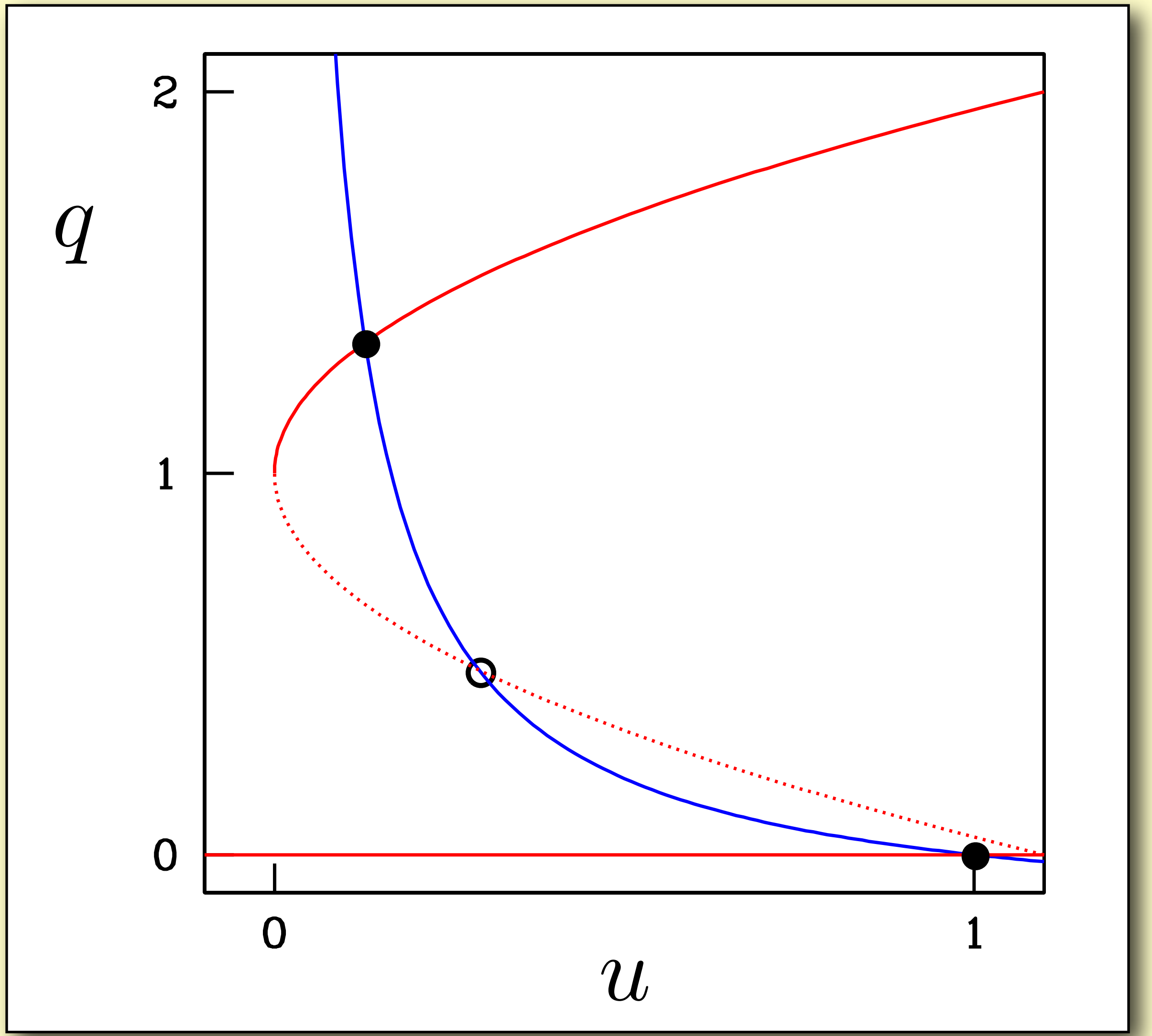
$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Two cases:

Excitable $r < r_c$



Bistable $r > r_c$

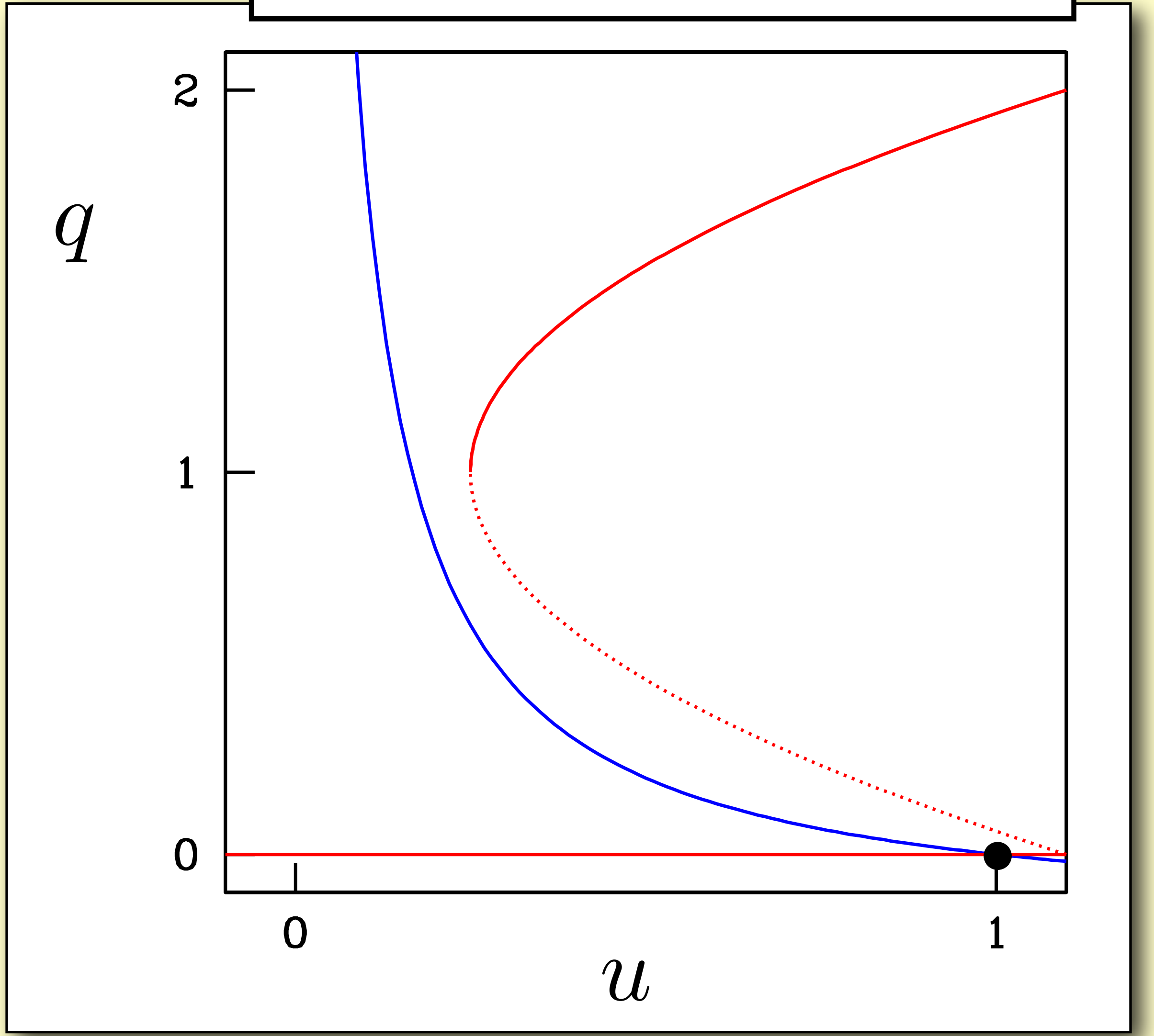


ODEs

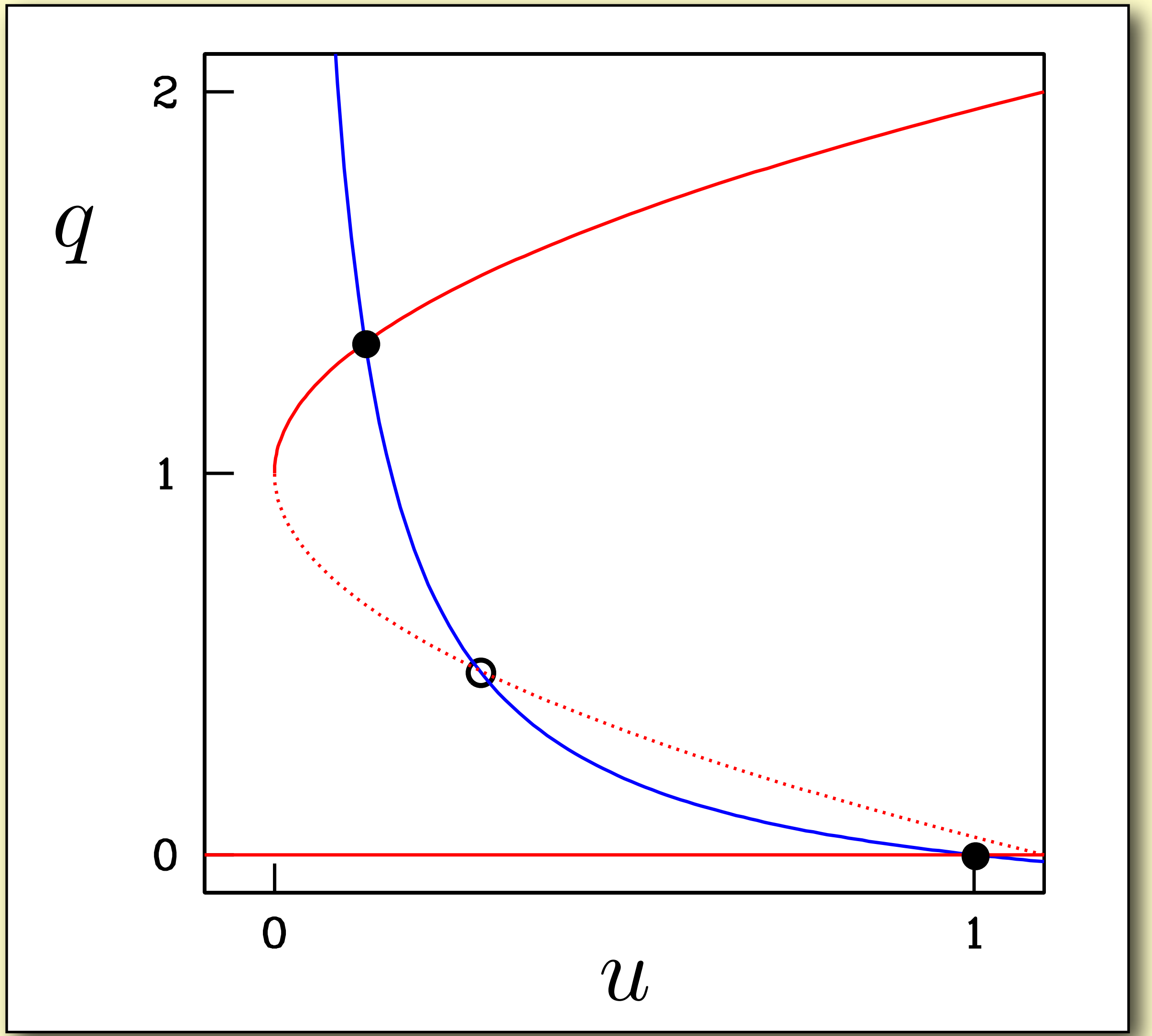
$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Two cases:

Excitable $r < r_c$



Bistable $r > r_c$

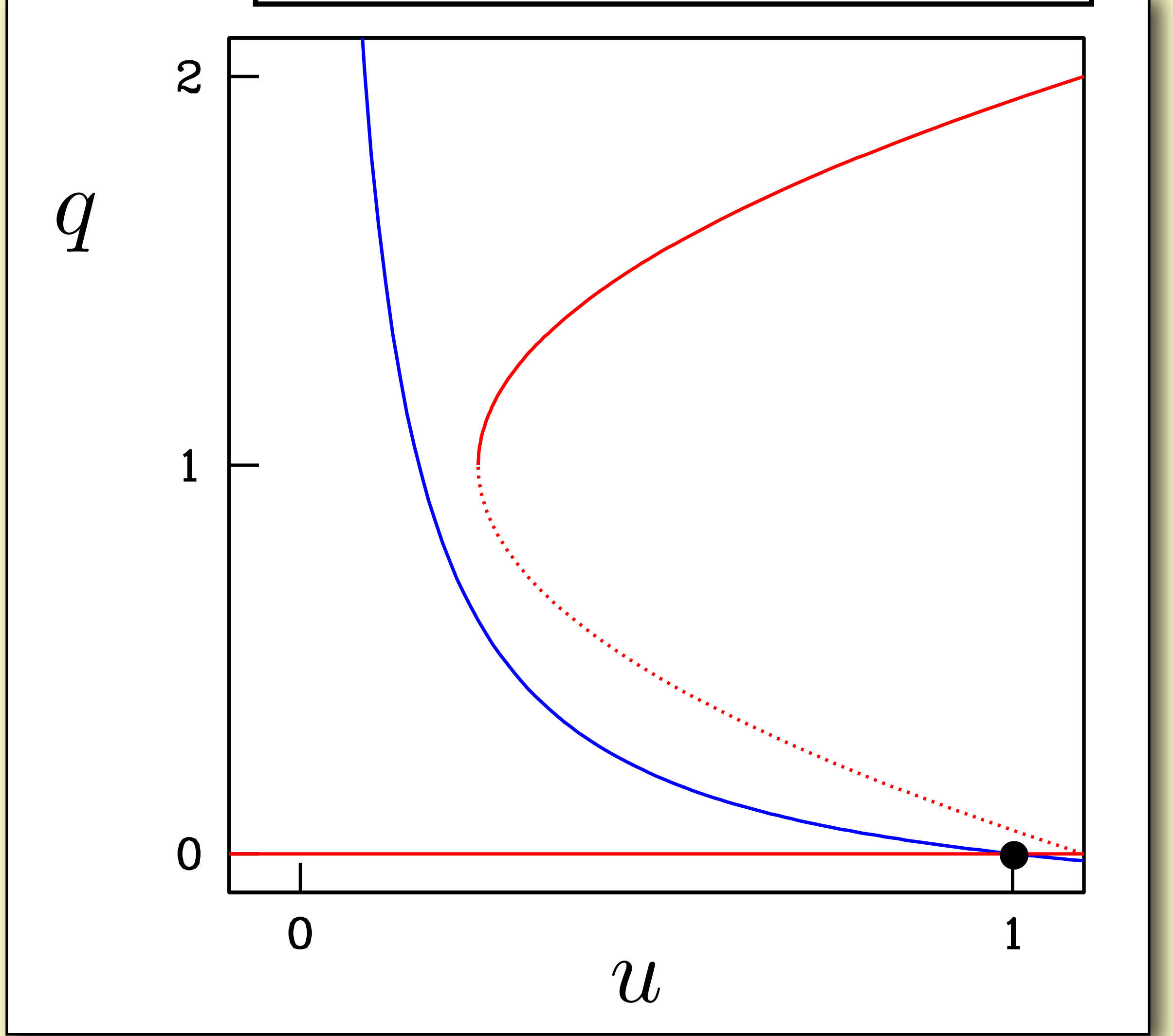


ODEs

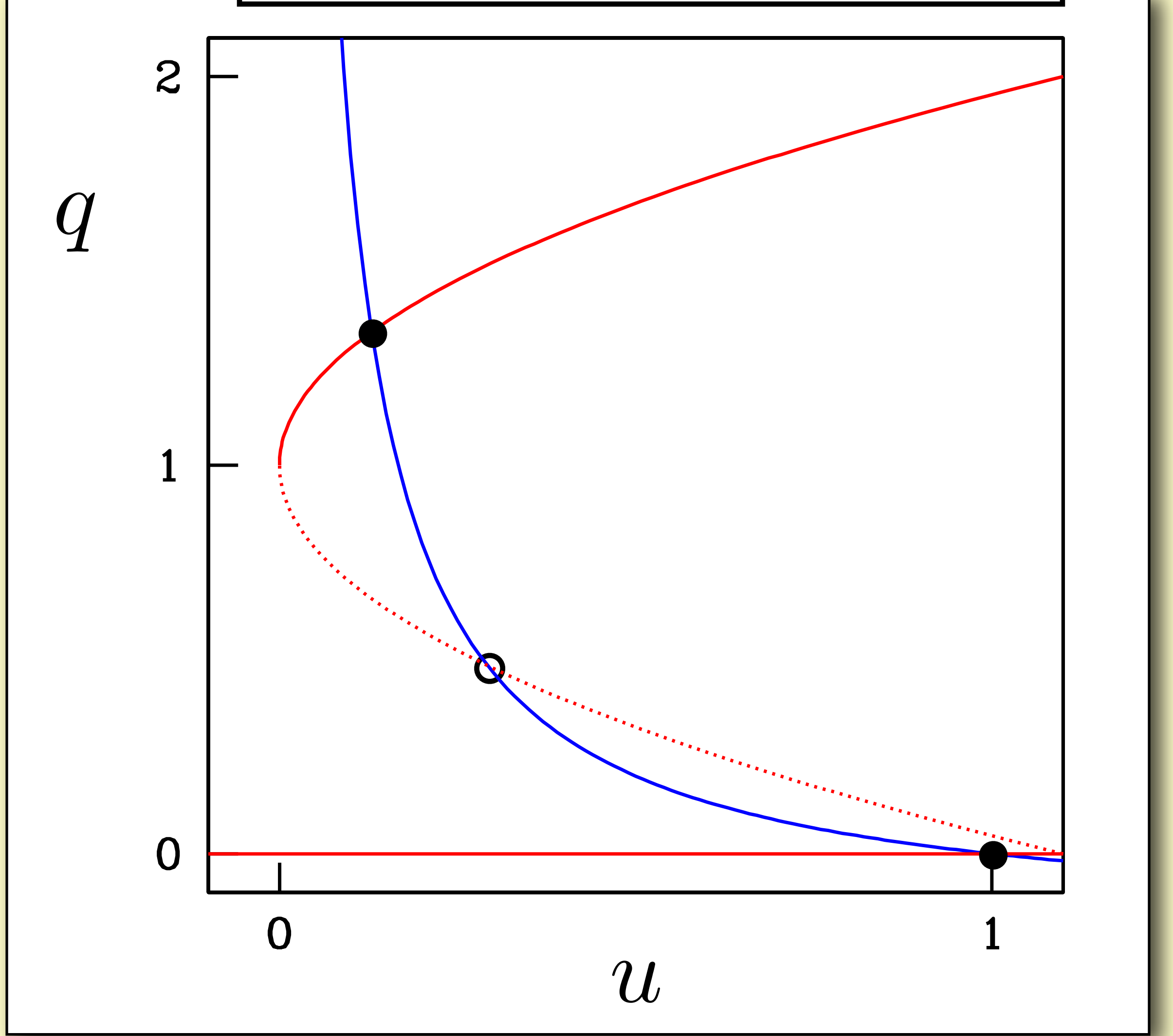
$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Two cases:

Excitable $r < r_c$



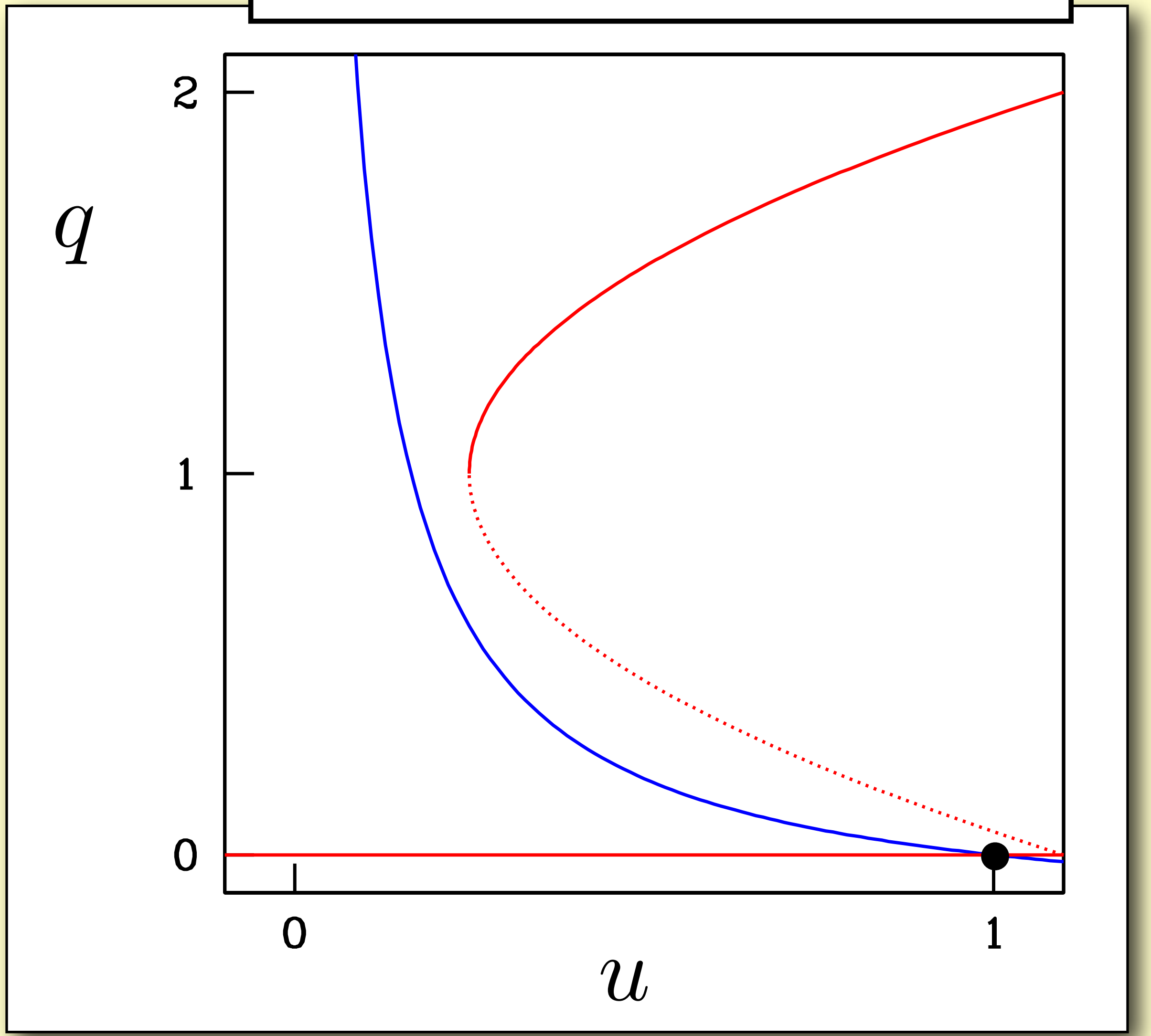
Bistable $r > r_c$

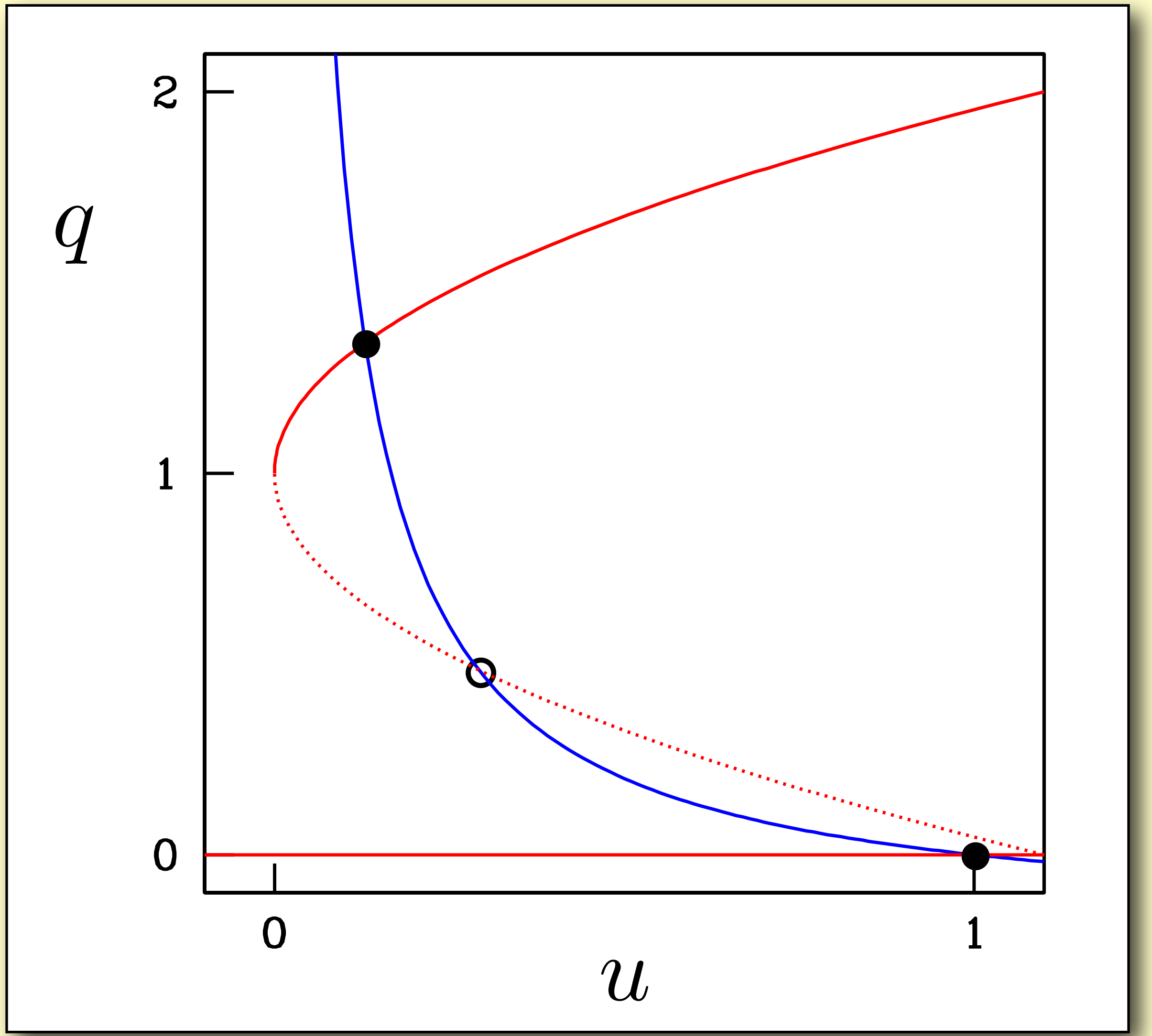


ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Excitable $r < r_c$

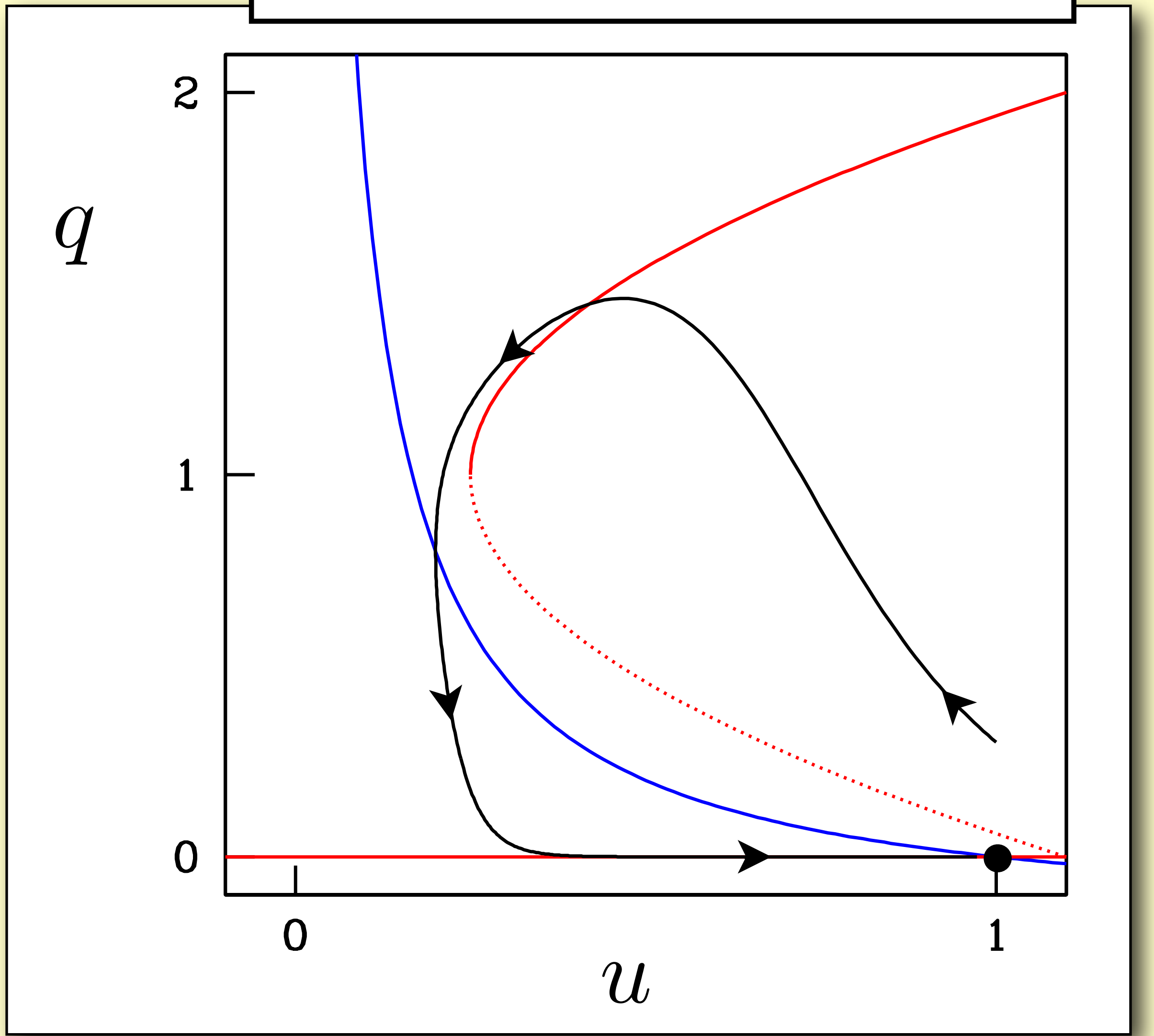


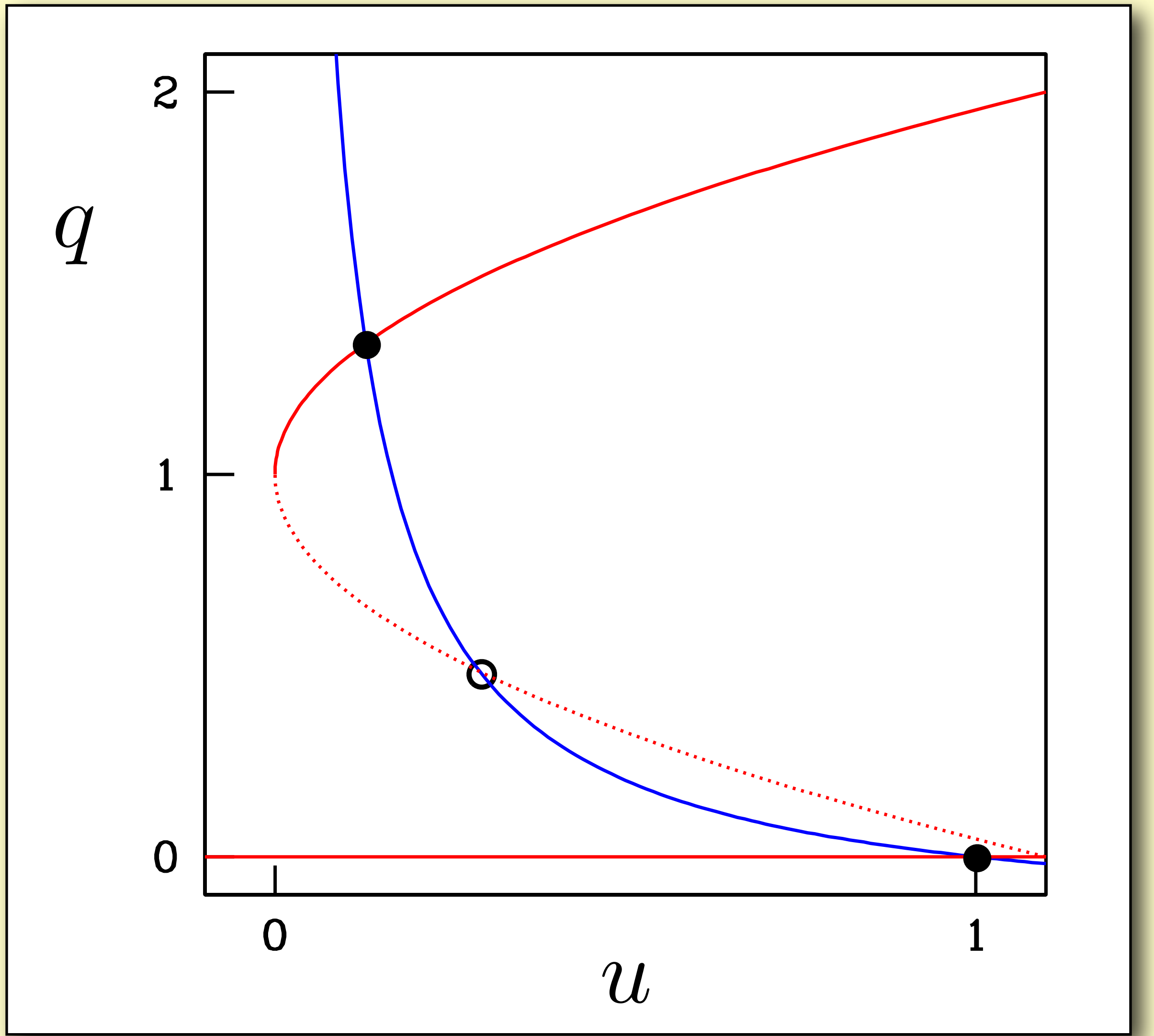


ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Excitable $r < r_c$

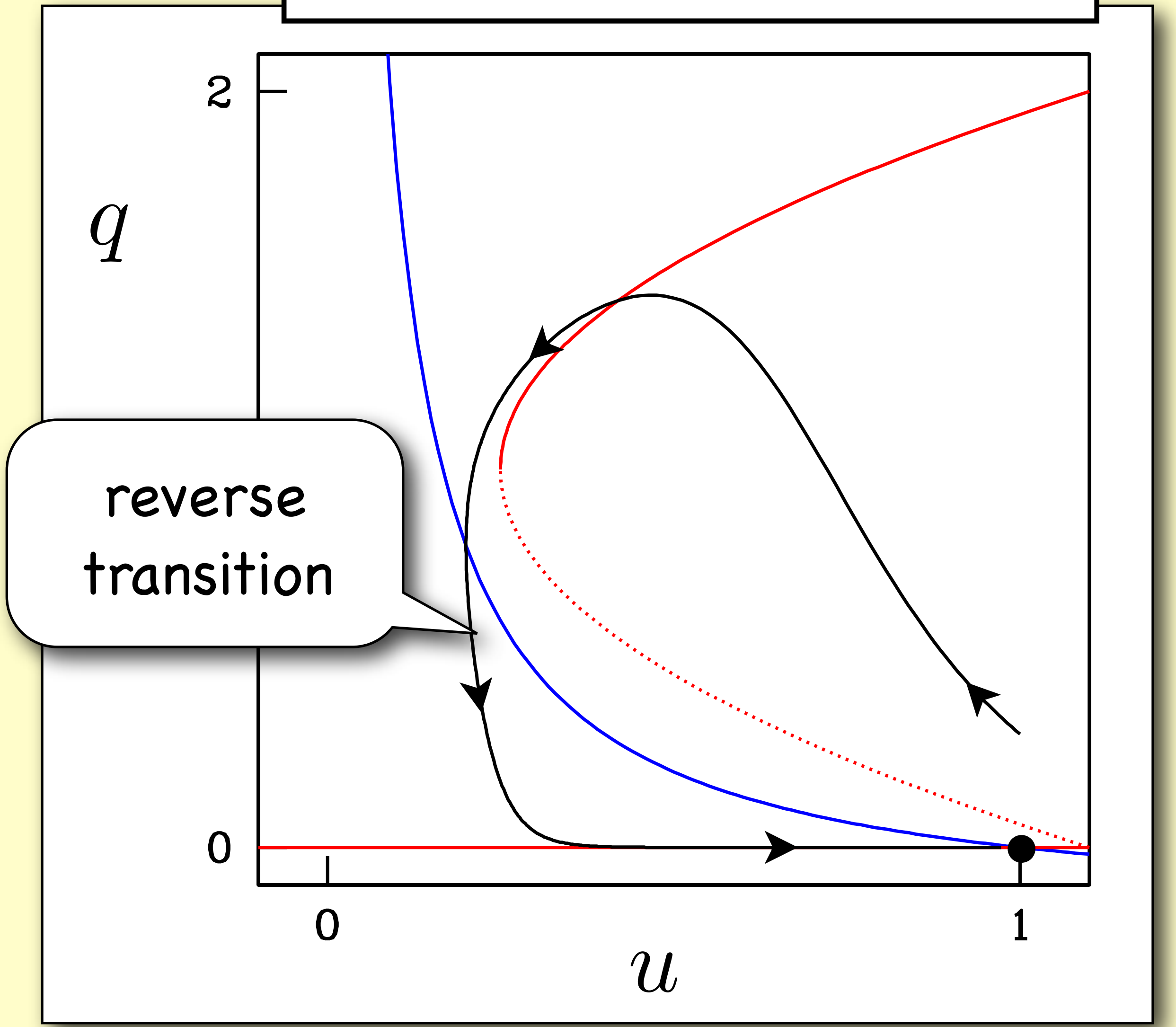


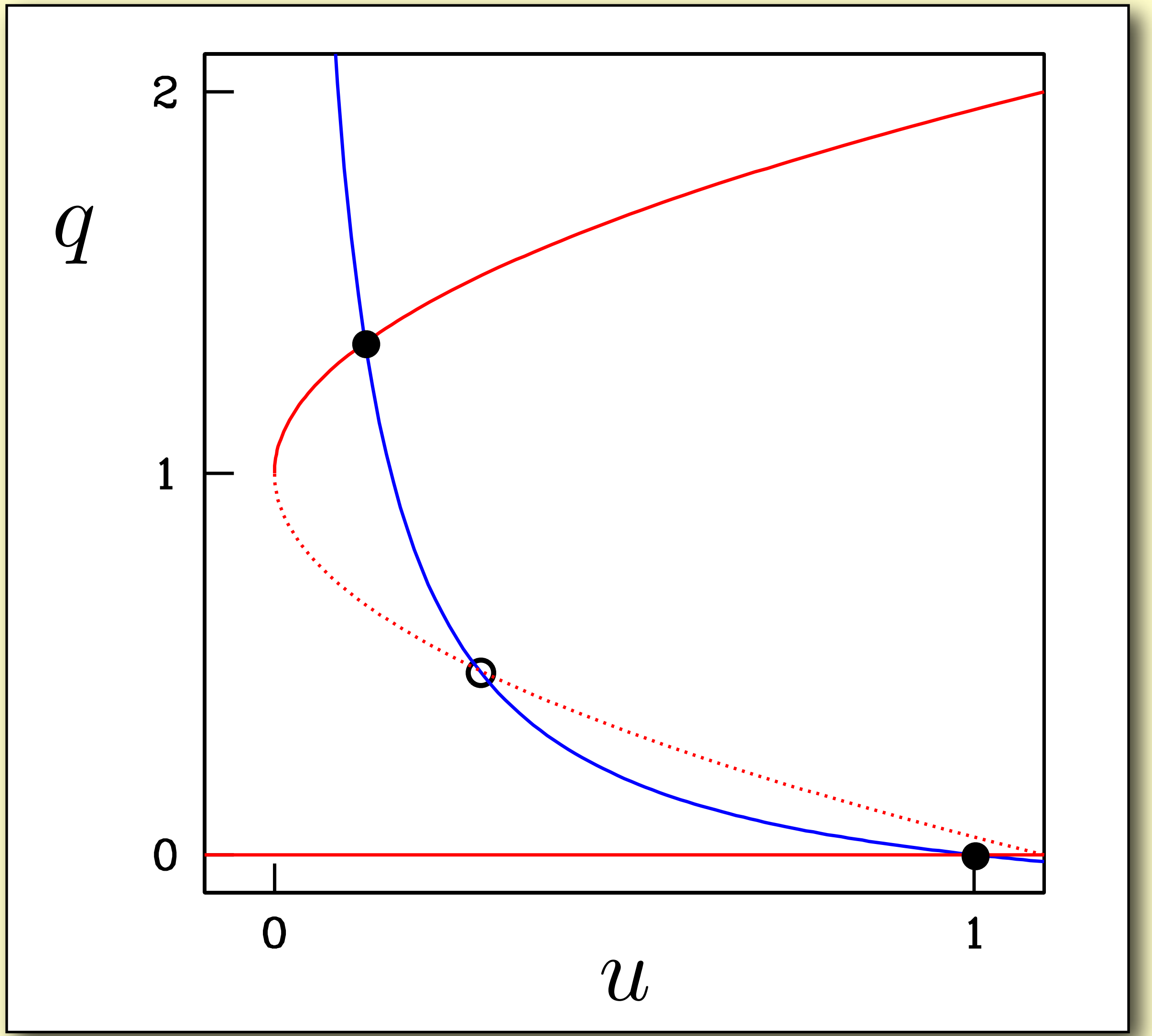


ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Excitable $r < r_c$

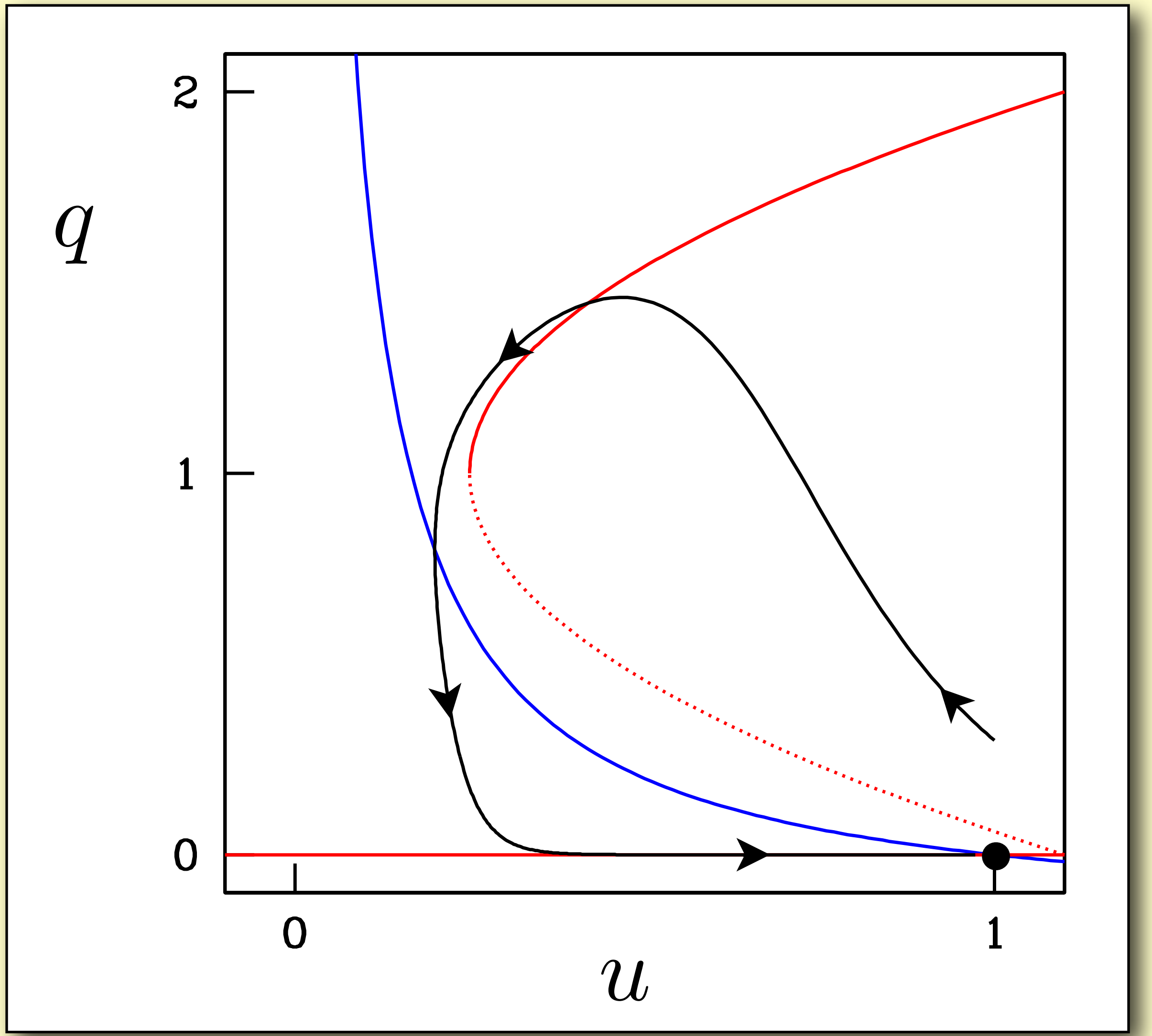




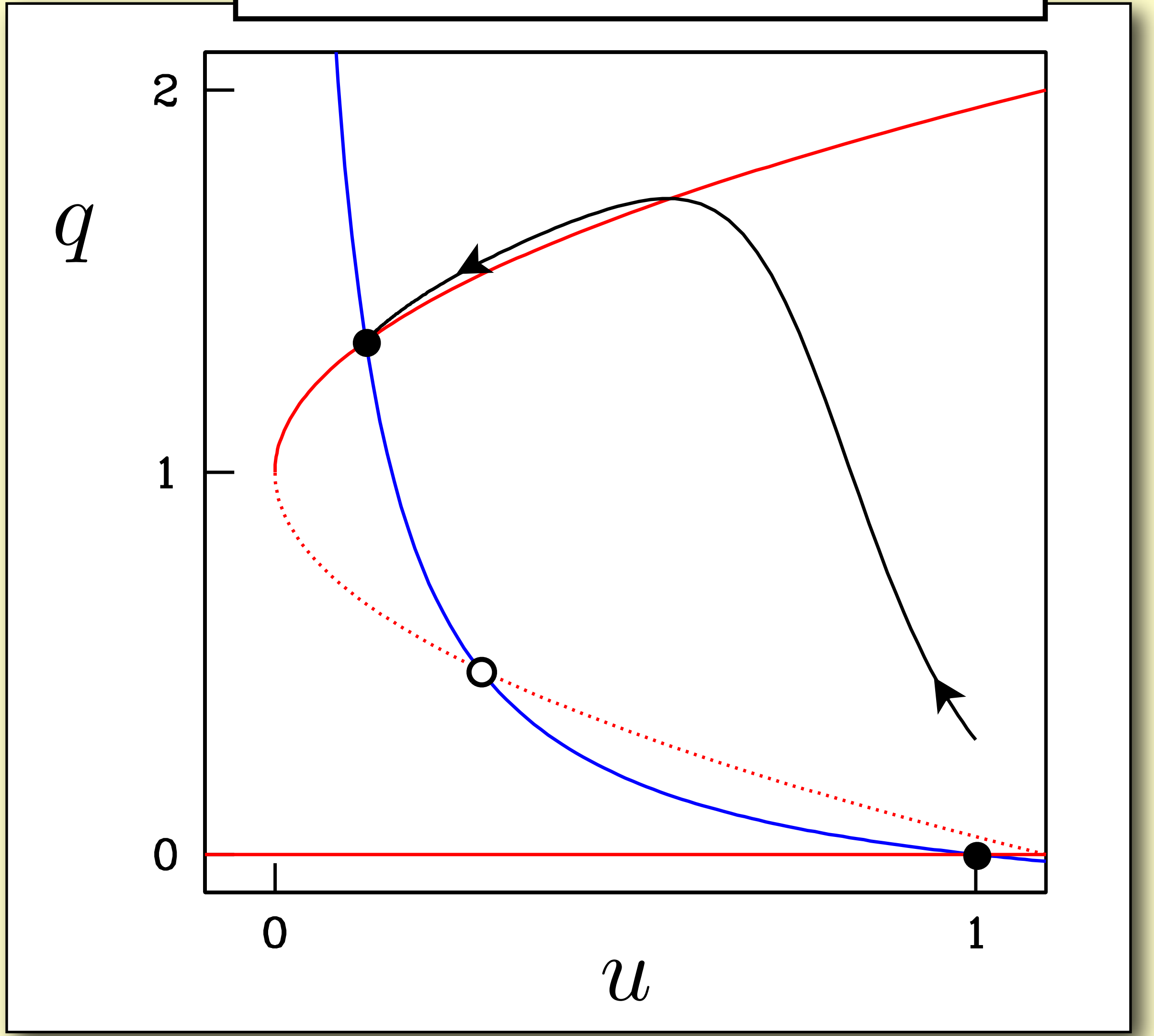
ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Excitable $r < r_c$



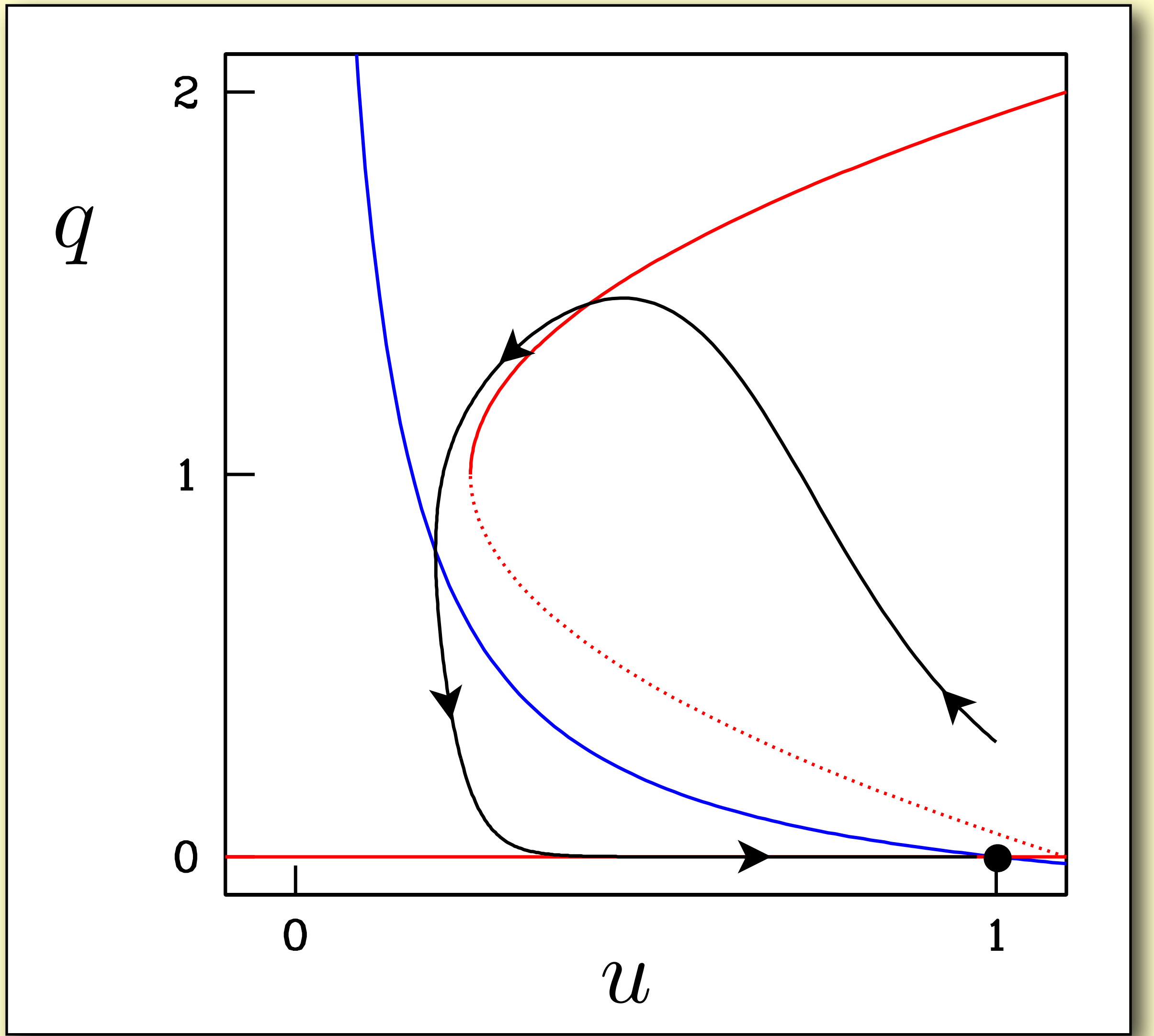
Bistable $r > r_c$



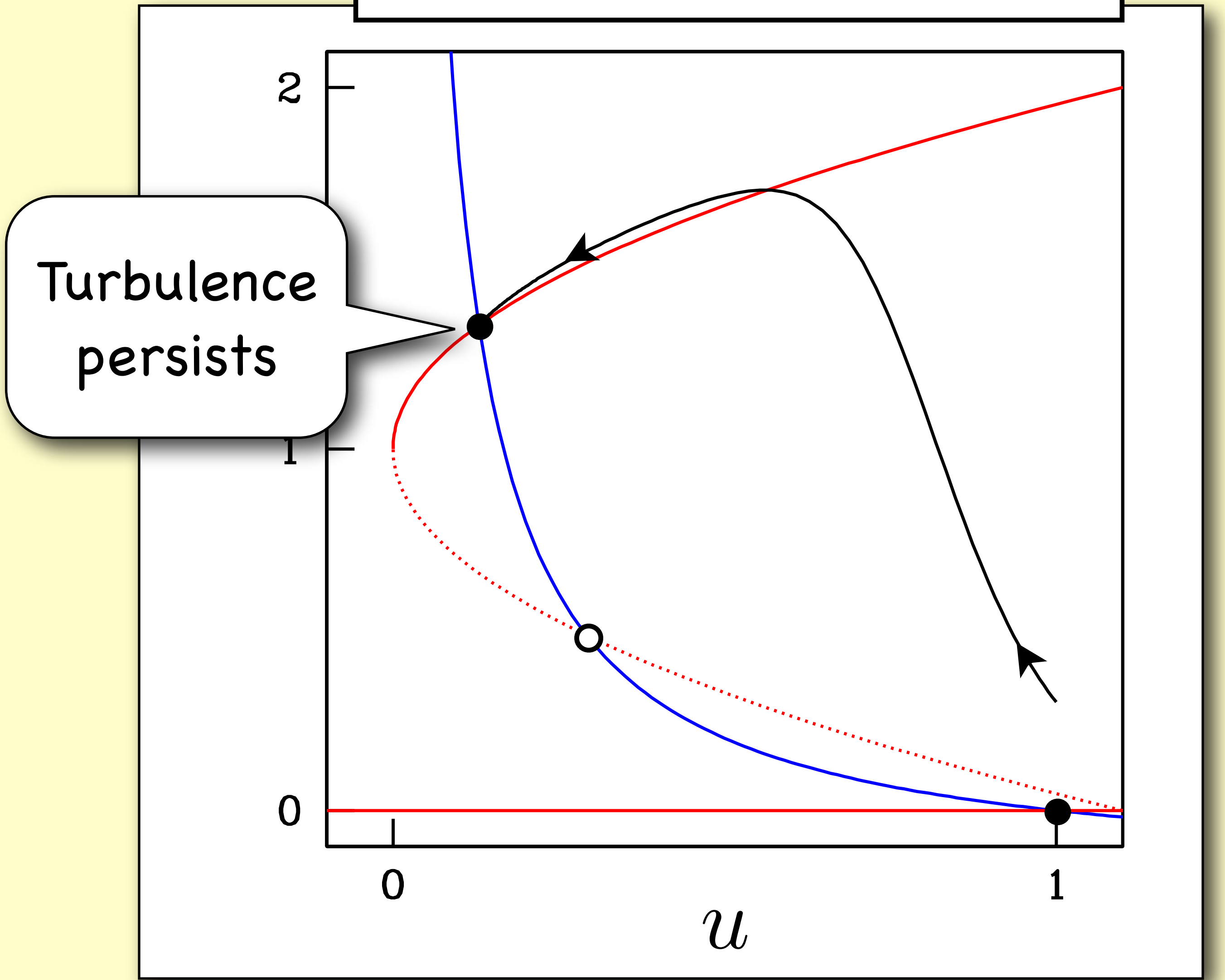
ODEs

$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Excitable $r < r_c$



Bistable $r > r_c$

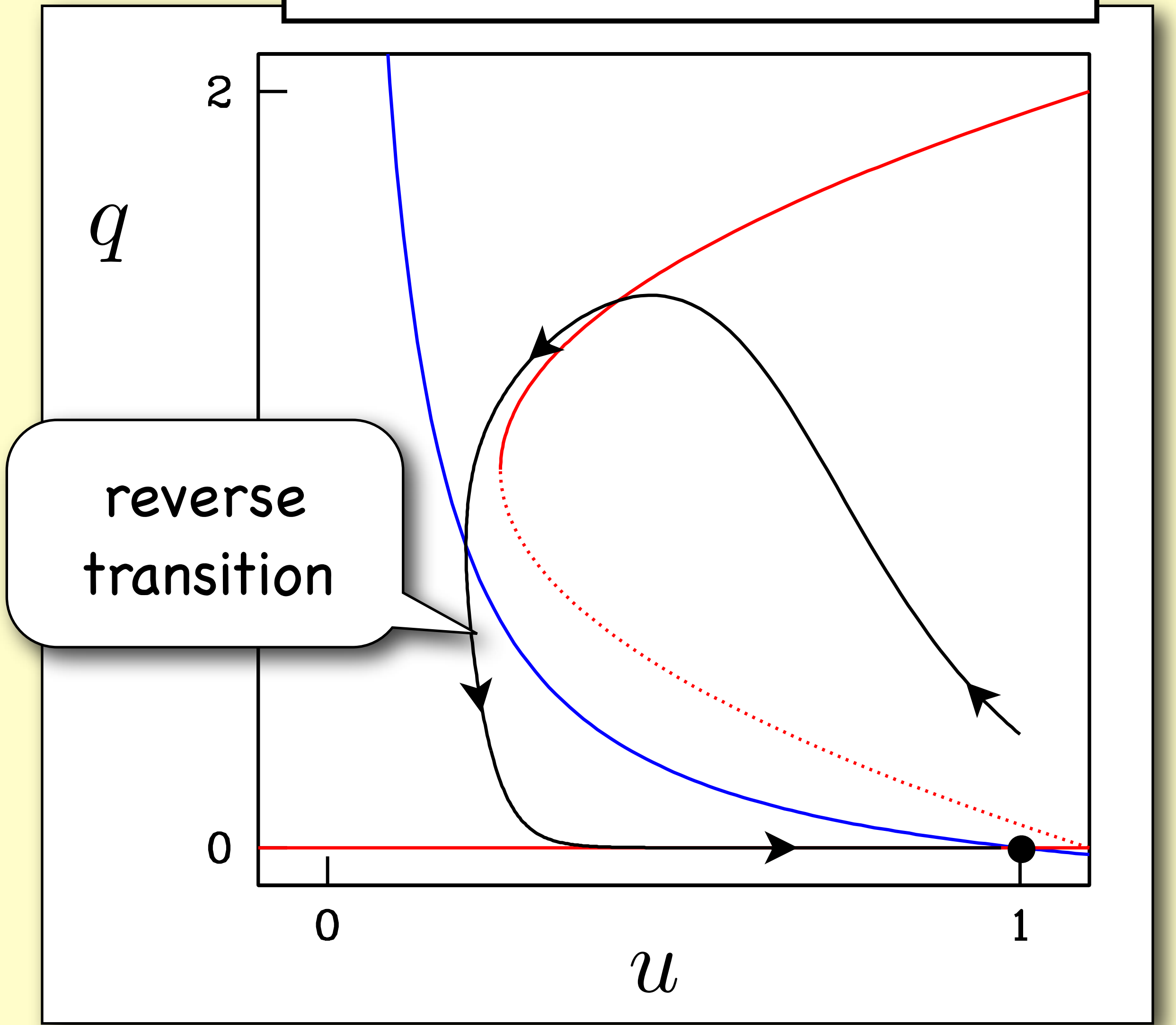


ODEs

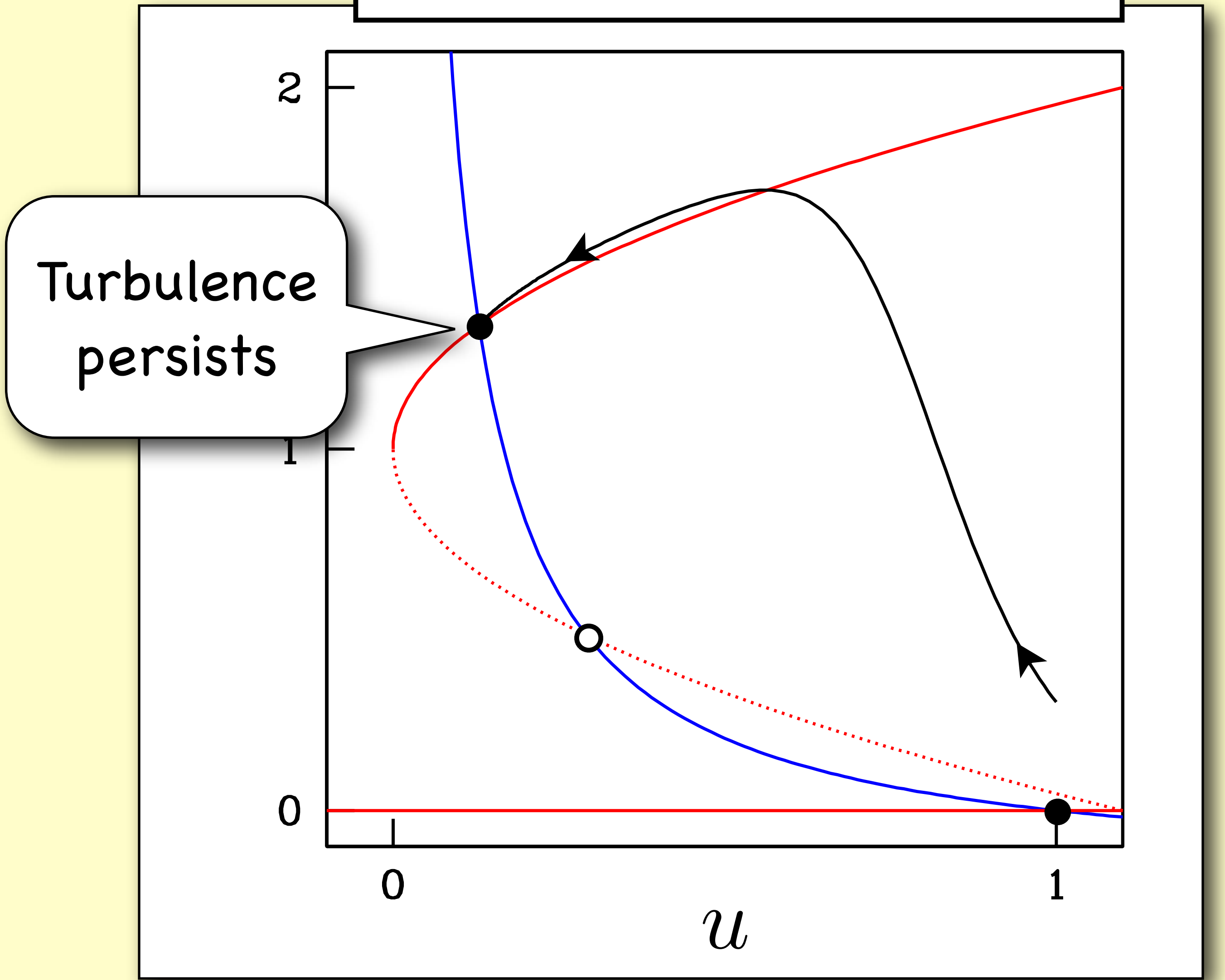
$$\dot{q} = q\left(u + r - 1 - (r+\delta)(q-1)^2\right)$$
$$\dot{u} = \epsilon_1(1-u) - \epsilon_2 uq$$

Two cases:

Excitable $r < r_c$



Bistable $r > r_c$



PDE Model

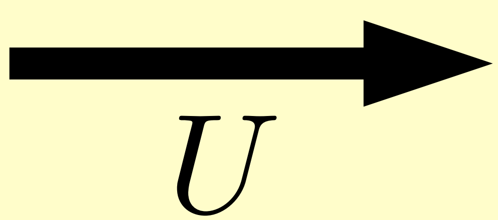
$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$
$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 uq - \partial_x u$$

Returning to the full model,
consider the role of the spatial derivatives

PDE Model

$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$
$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 uq - \partial_x u$$

Downstream advection by mean flow (parameter *U*)



PDE Model

$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$

$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 uq - \partial_x u$$

Diffusive coupling of the turbulent field (turbulence excites adjacent laminar flow)

PDE Model

$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$
$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 uq - \partial_x u$$

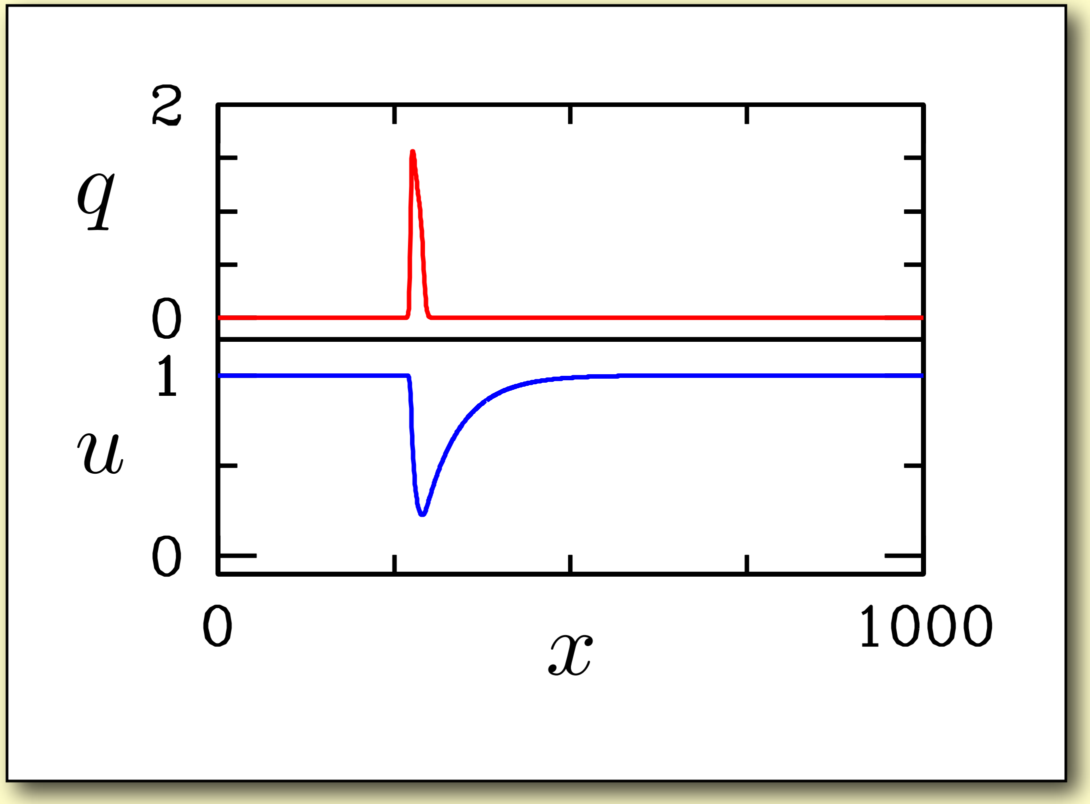
Left-Right symmetry breaking (other forms possible, but this is simplest)

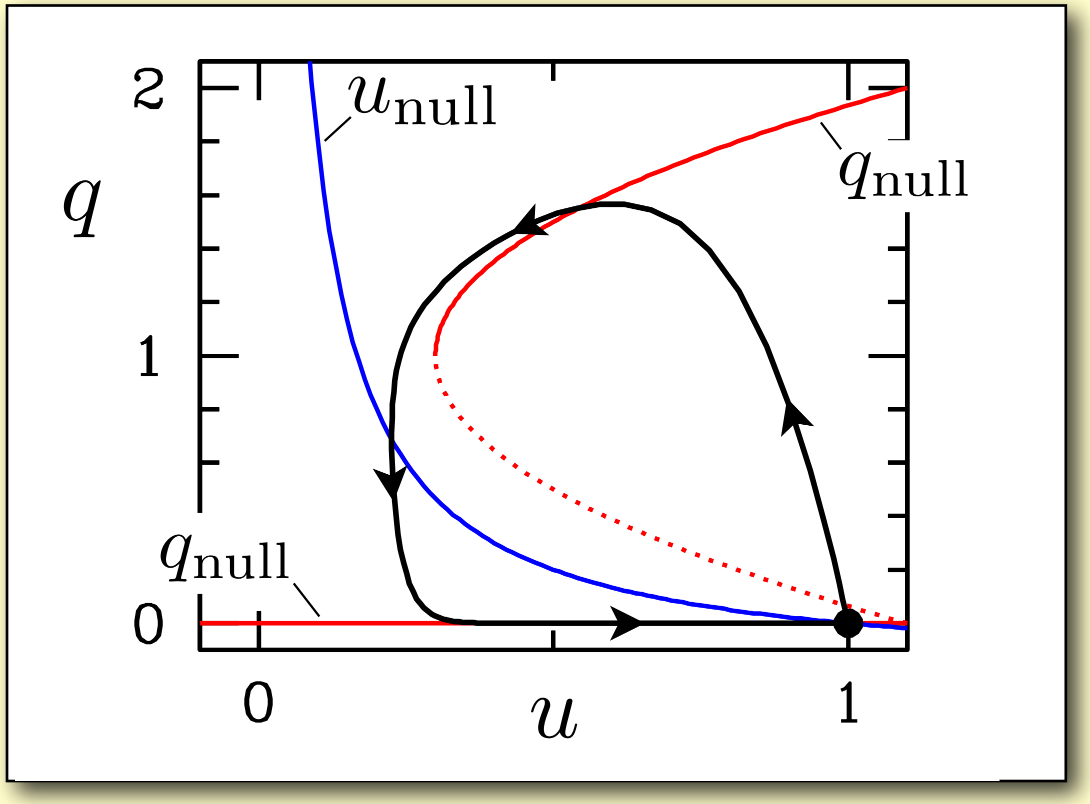
PDE Model

$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$
$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 u q - \partial_x u$$

Puffs corresponds to excitability

$$r < r_c$$



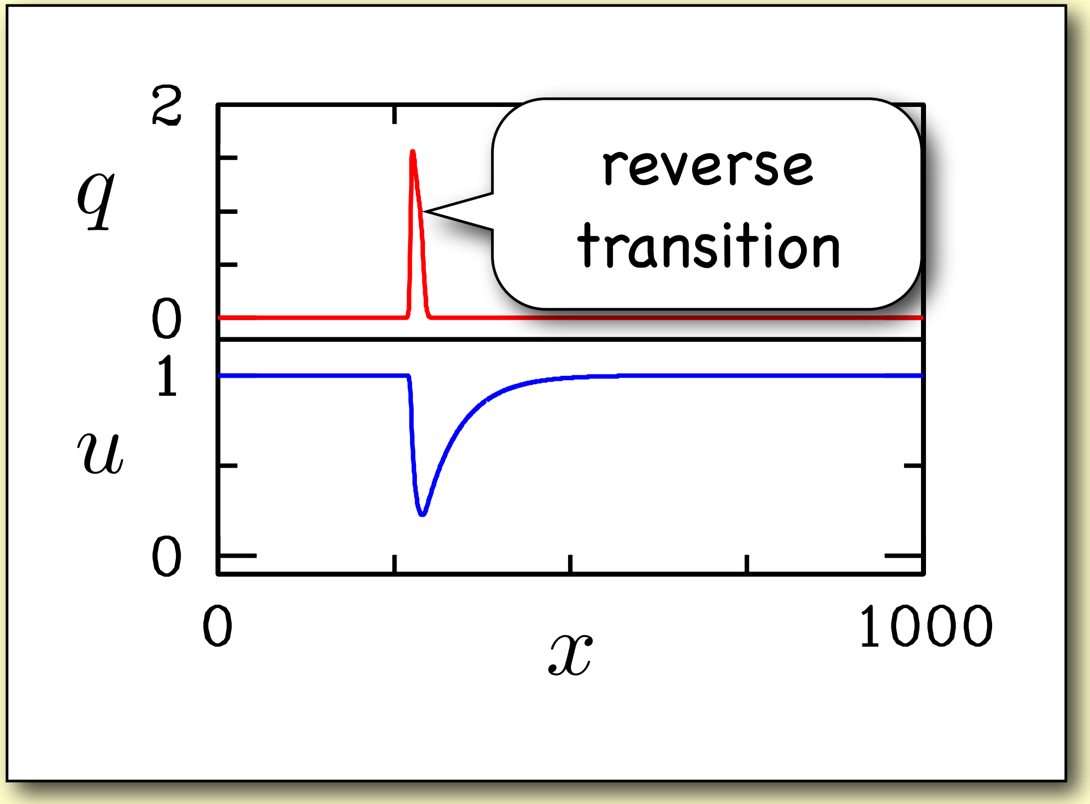


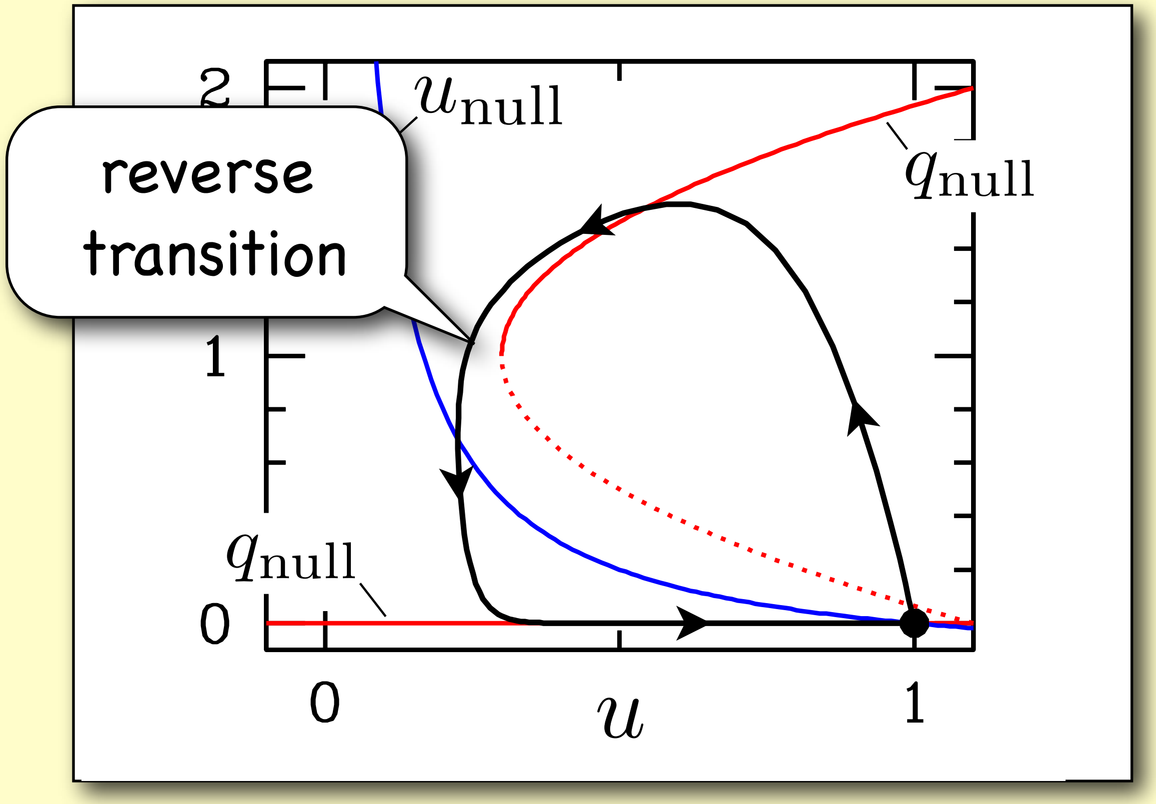
PDE Model

$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$
$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 uq - \partial_x u$$

Puffs corresponds to excitability

$$r < r_c$$



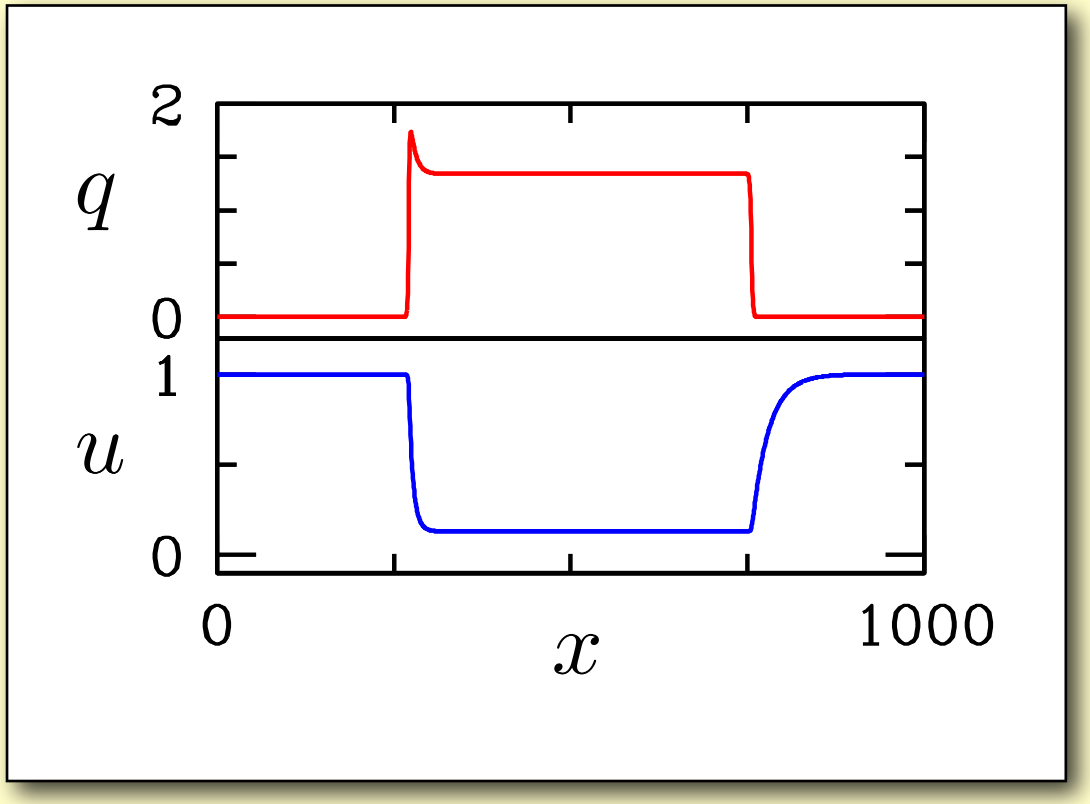


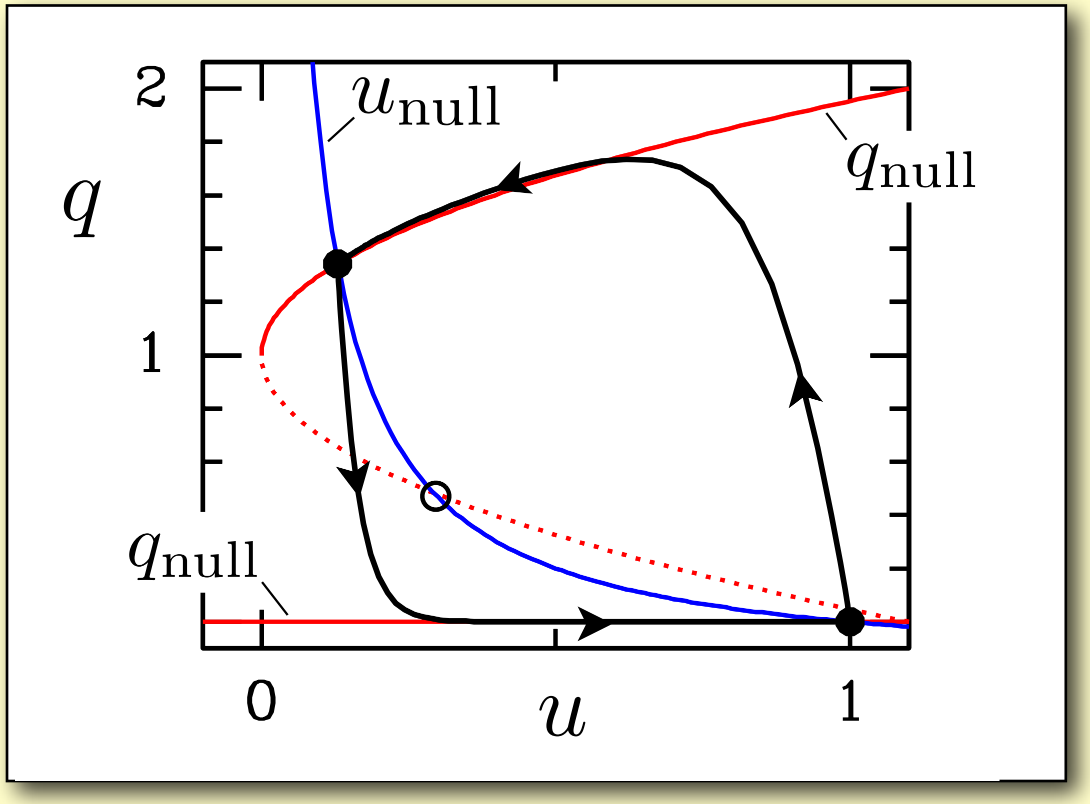
PDE Model

$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$
$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 uq - \partial_x u$$

Slugs corresponds to bistability

$$r > r_c$$



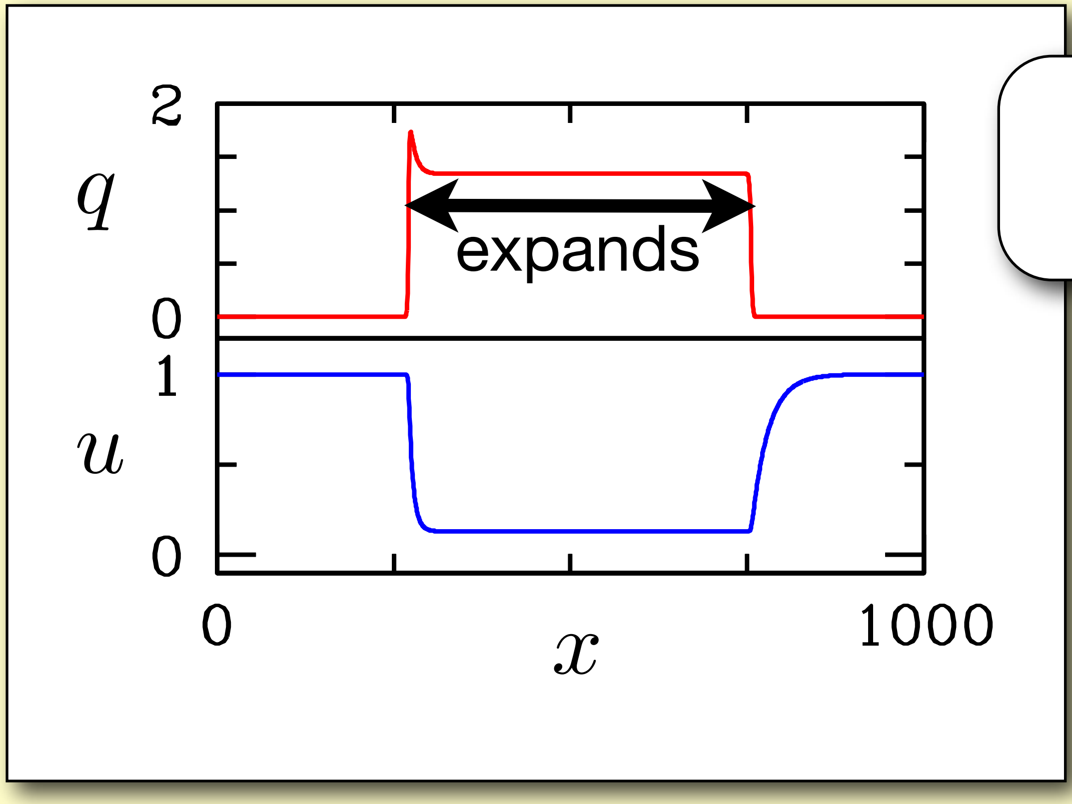


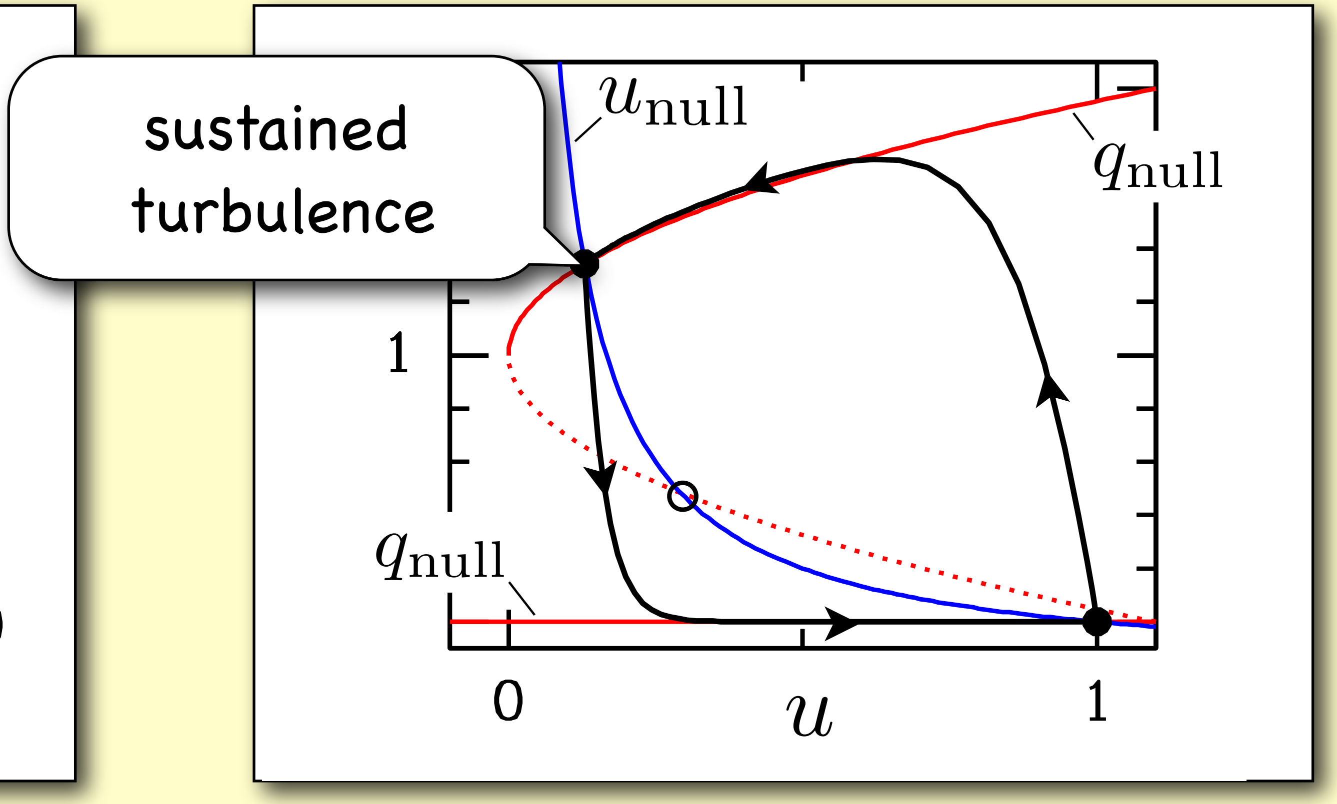
PDE Model

$$\partial_t q + U\partial_x q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$
$$\partial_t u + U\partial_x u = \epsilon_1(1-u) - \epsilon_2 uq - \partial_x u$$

Slugs corresponds to bistability

$$r > r_c$$

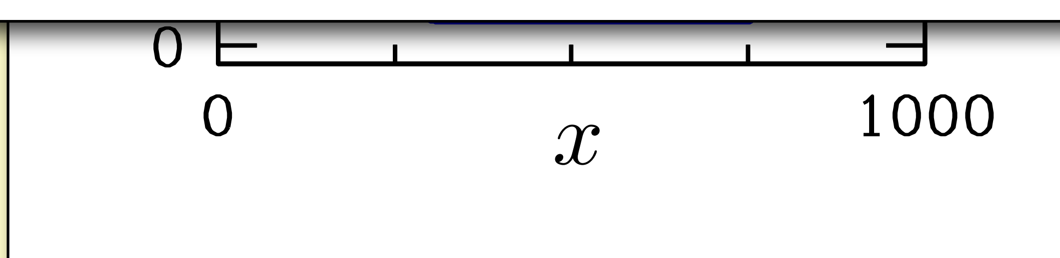


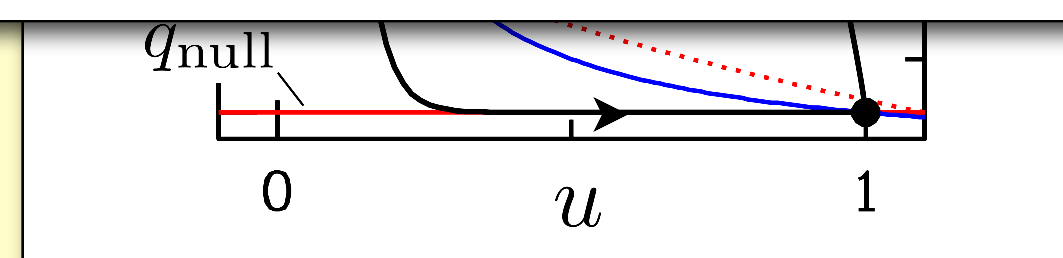


PDE Model

Homework

1) verify that the PDE model has all physical properties, except last.
2) show puffs correspond to excitability and slugs to bistability.





PDE model captures essence of puff-slug transition, but turbulence is too simplistic. Need Complex and Locally Transient Turbulence.

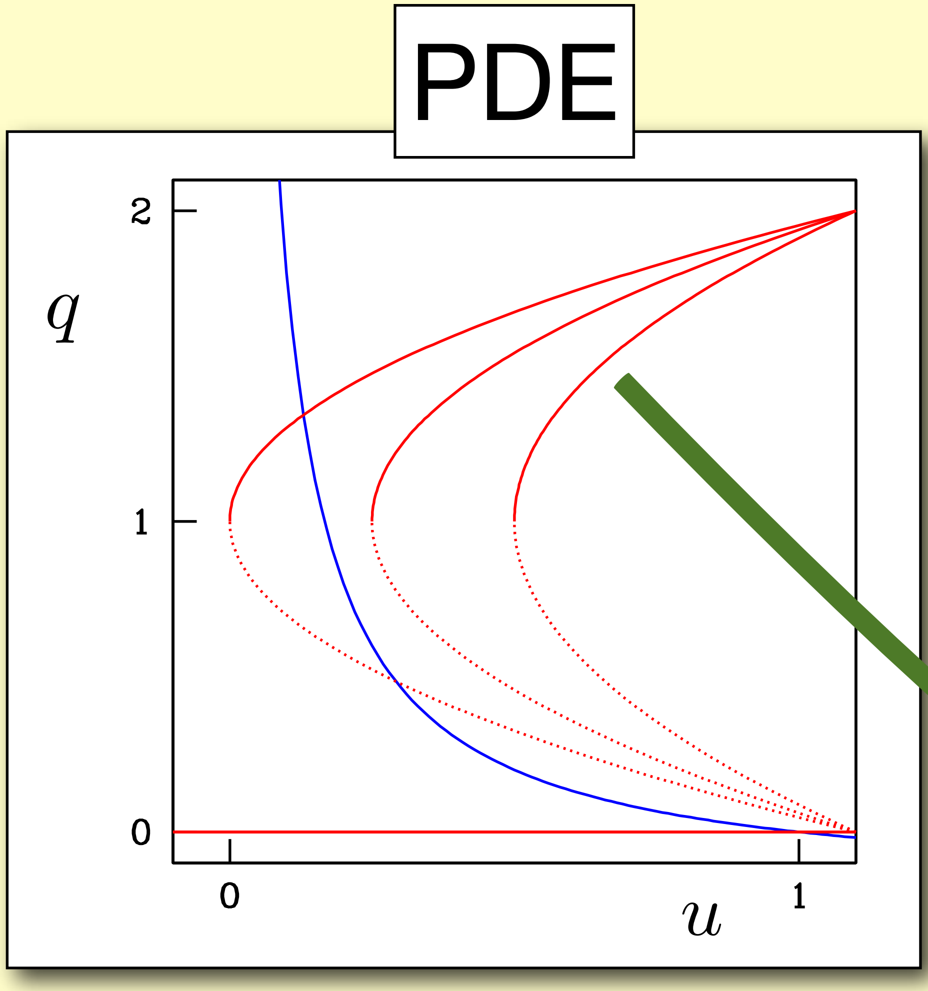
PDE model captures essence of puff-slug transition, but turbulence is too simplistic. Need Complex and Locally Transient Turbulence.

so

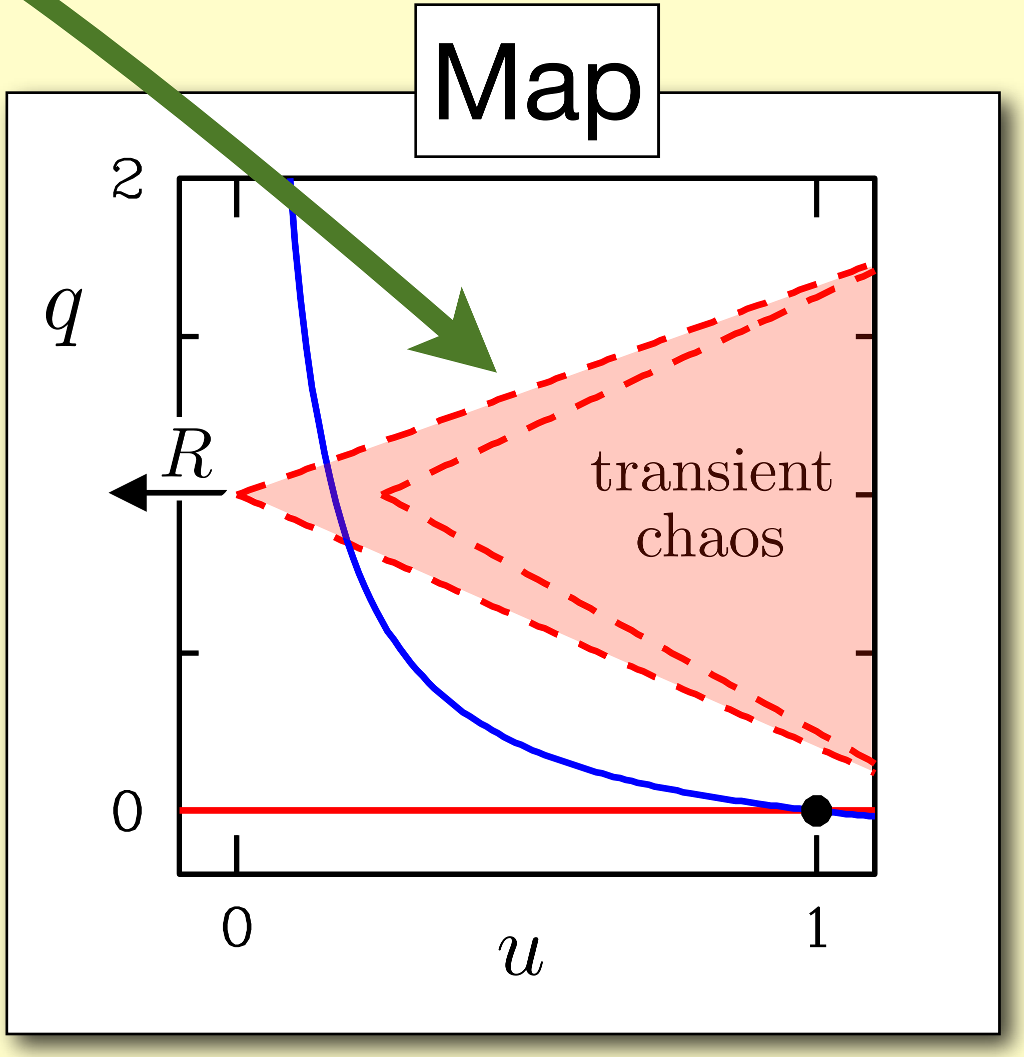
Model Turbulence with Chaotic Map

Model Turbulence with Noise

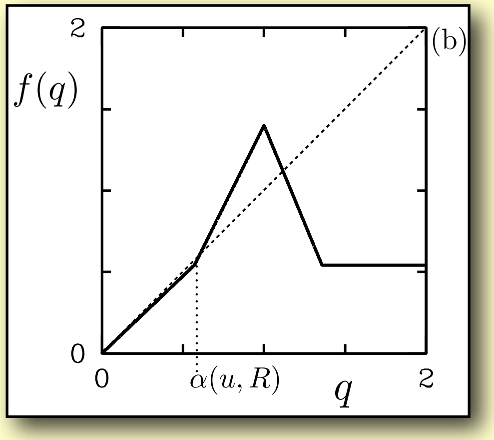
Map Model



Replace upper turbulent branch in PDE with region of transient chaos

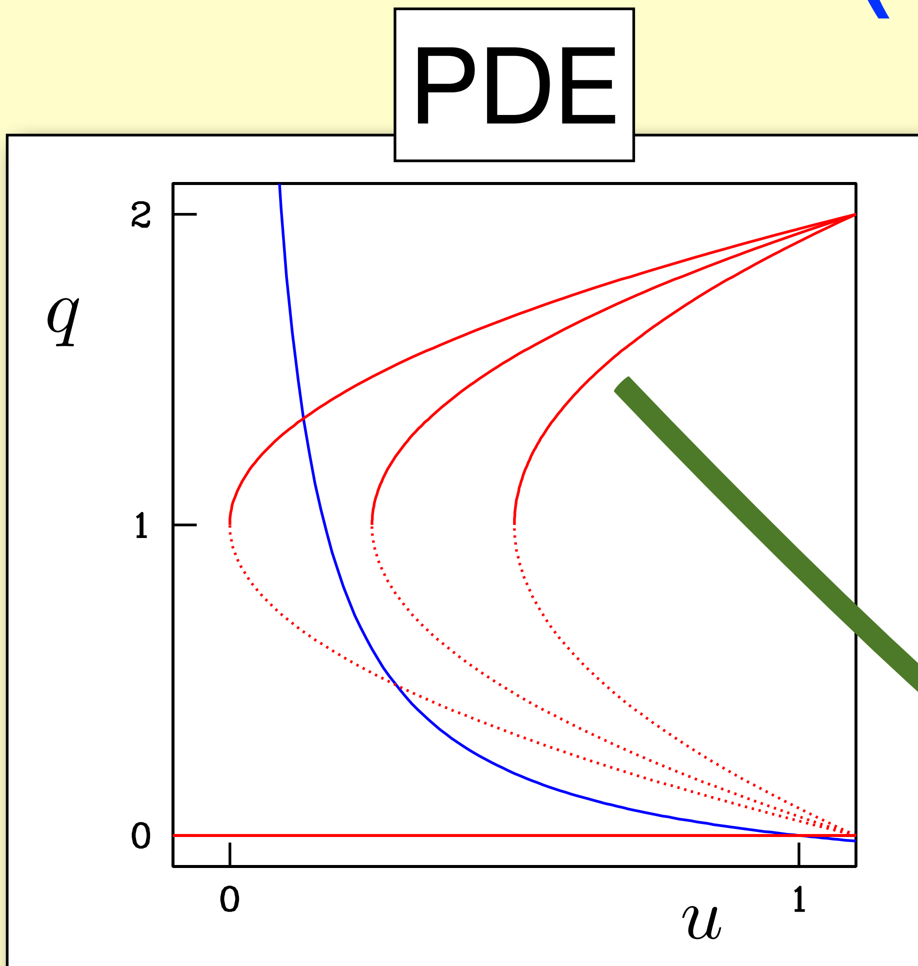


Transient chaos generated with tent map



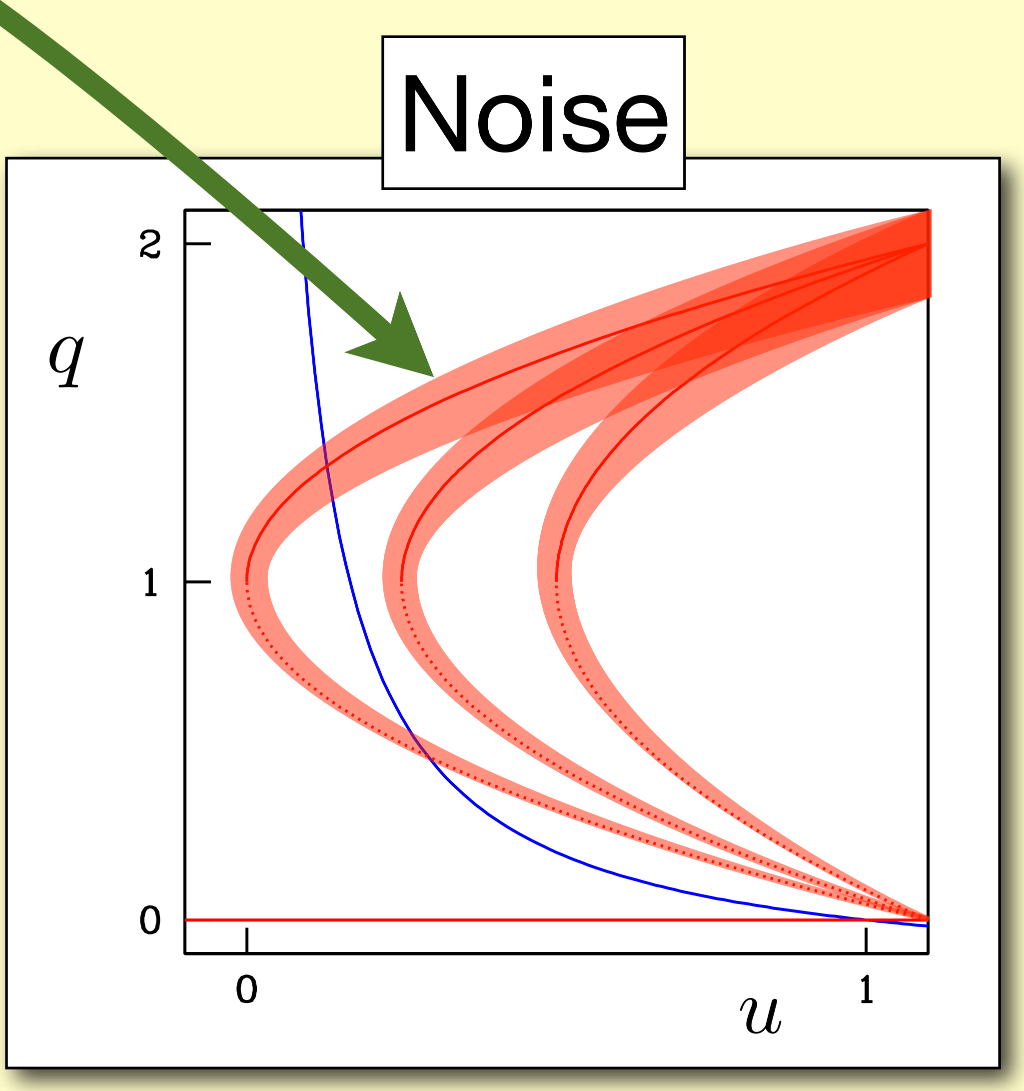
(c.f. Chate, Manneville *et al.*, Vollmer *et al.*)

Noise (SPDE) Model



Add multiplicative noise

$$+\sigma q \eta$$



Stochastic PDE (SPDE)

Barkley, ETC13 (to appear)

$$\begin{aligned} \partial_t q + U\partial_x q &= q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q + \sigma q \eta, \\ \partial_t u + U\partial_x u &= \epsilon_1(1-u) - \epsilon_2 u q - \partial_x u. \end{aligned}$$

η is space-time white Gaussian noise

Simulations of Map Model

Summary of Models

* **PDE:** Simple, yet contains most physical features. Captures essence of puff-slug transition.
* **Map:** Deterministic, low-dimensional dynamics. Local turbulence explicitly chaotic saddle. Discrete space and time.
* **SPDE (Noise):** Infinite-dimensional, but random dynamics. Connected to PDE.

Comparison with Reality

Comparison with Reality

Reality

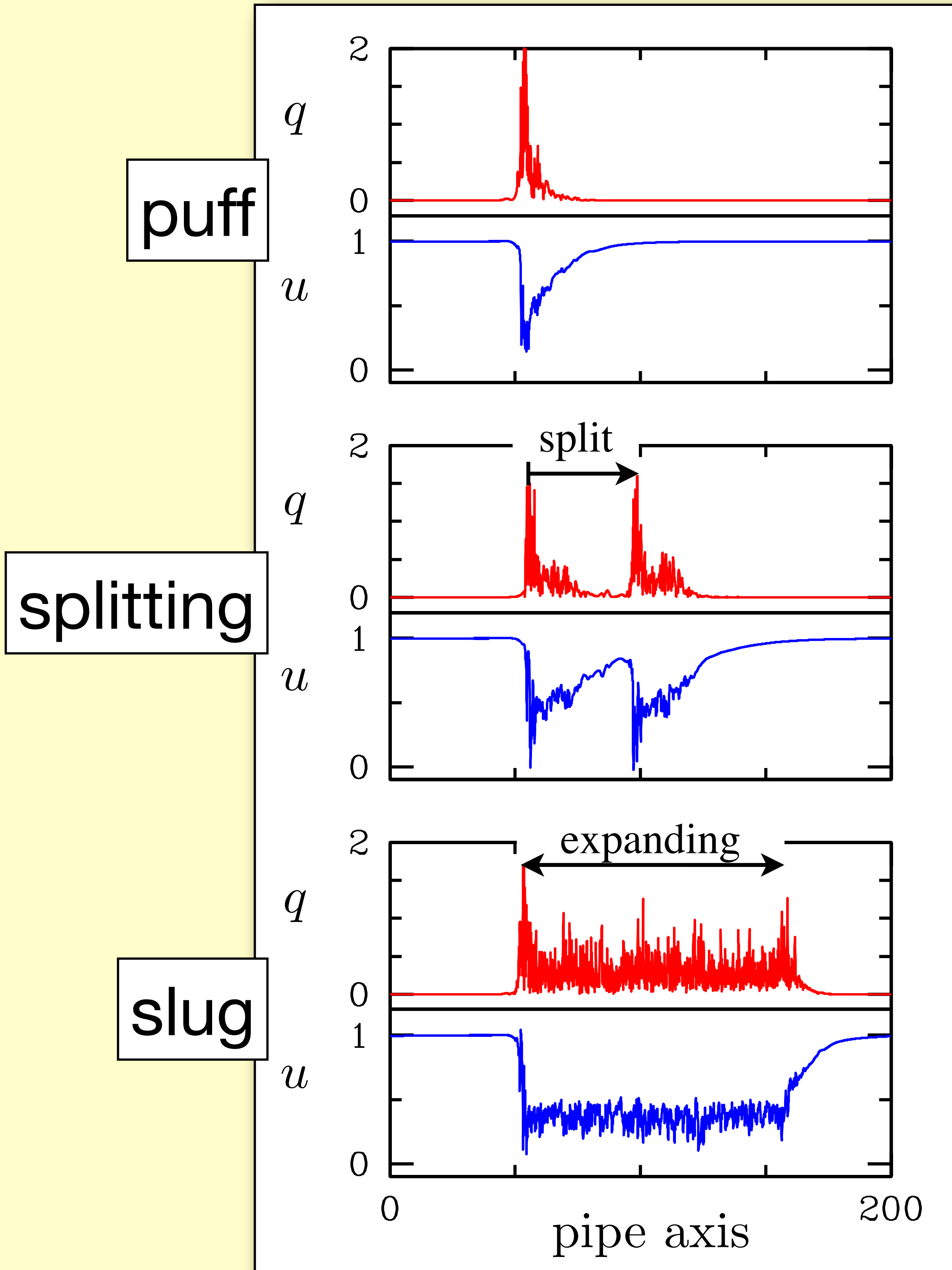
experiment or
direct numerical simulation
(various sources)

Model

PDE, MAP, or SPDE
(replotted from published and to-be-published sources)

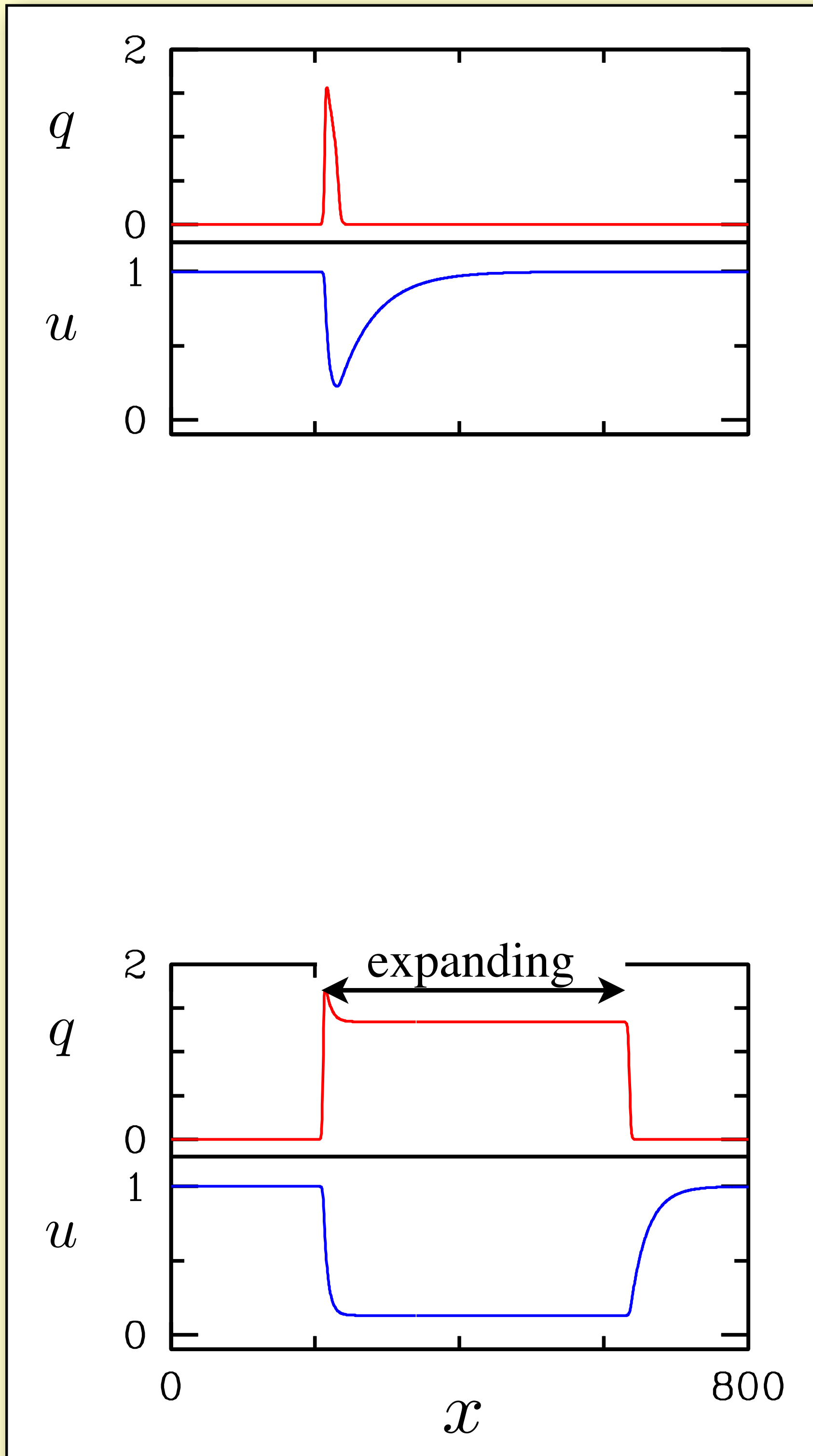
Direct Numerical Simulation

Barkley, *Phys. Rev. E* **84,** 016309 (2011)



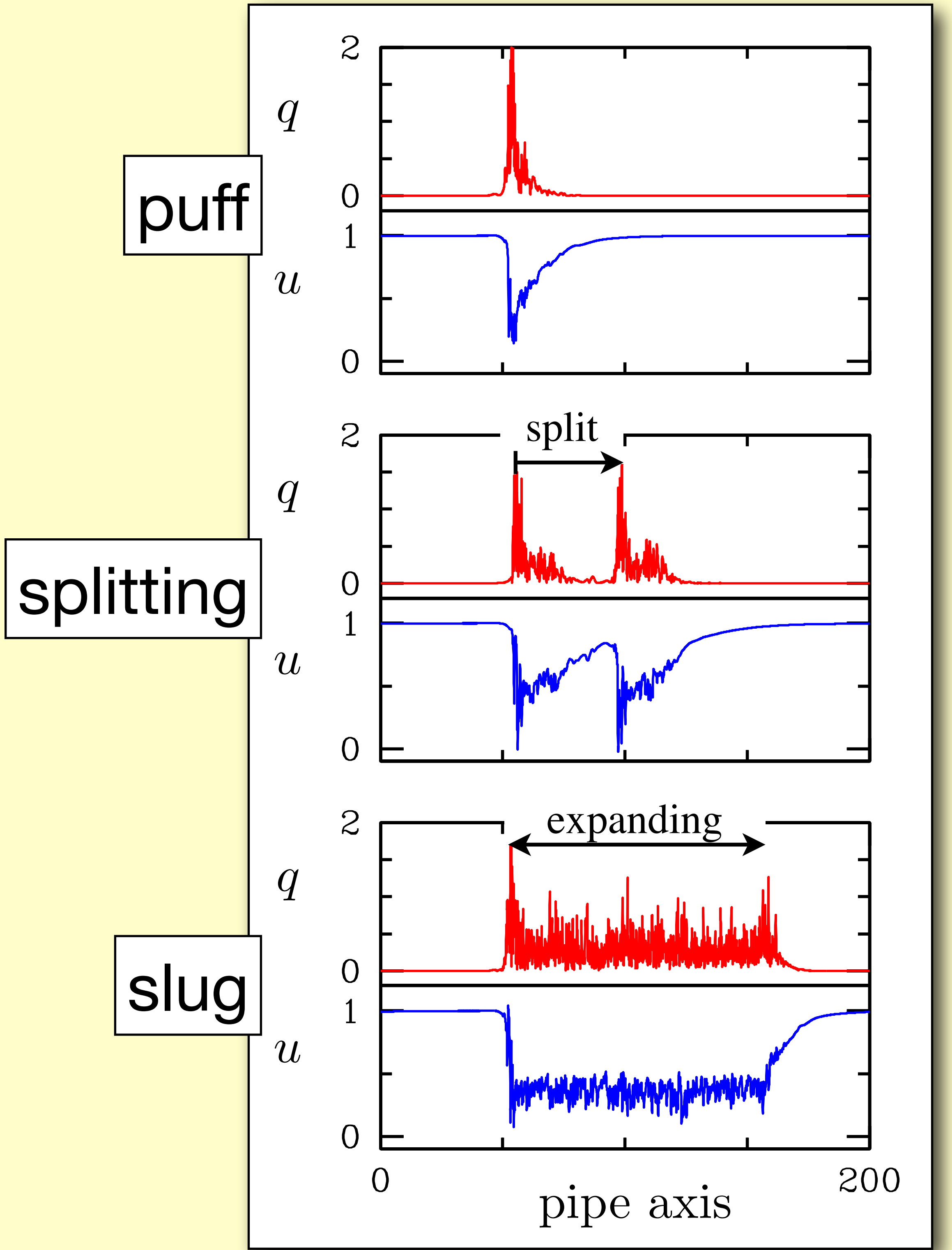
PDE Model

Barkley, *Phys. Rev. E* **84,** 016309 (2011)



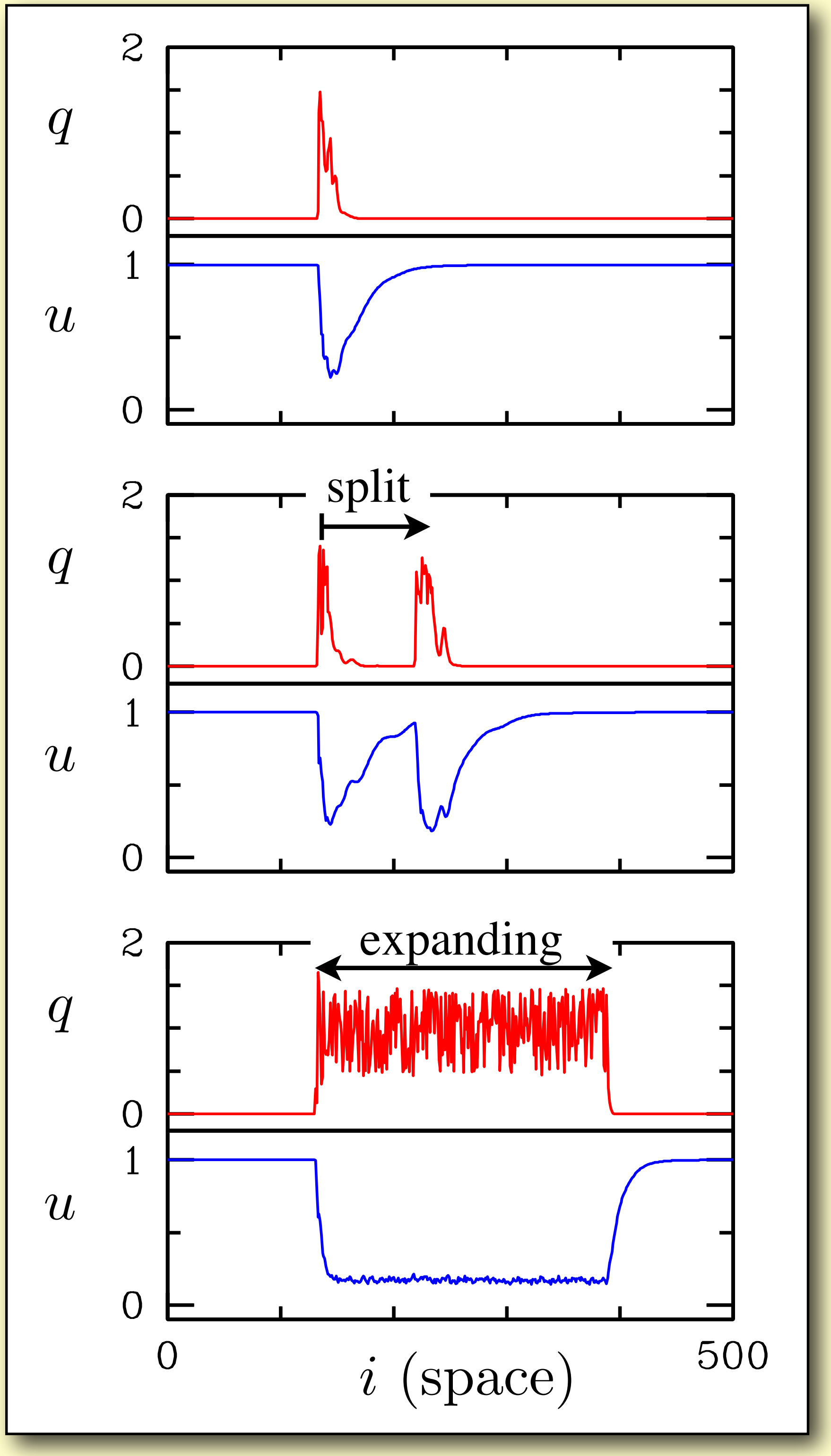
Direct Numerical Simulation

Barkley, *Phys. Rev. E* **84,** 016309 (2011)



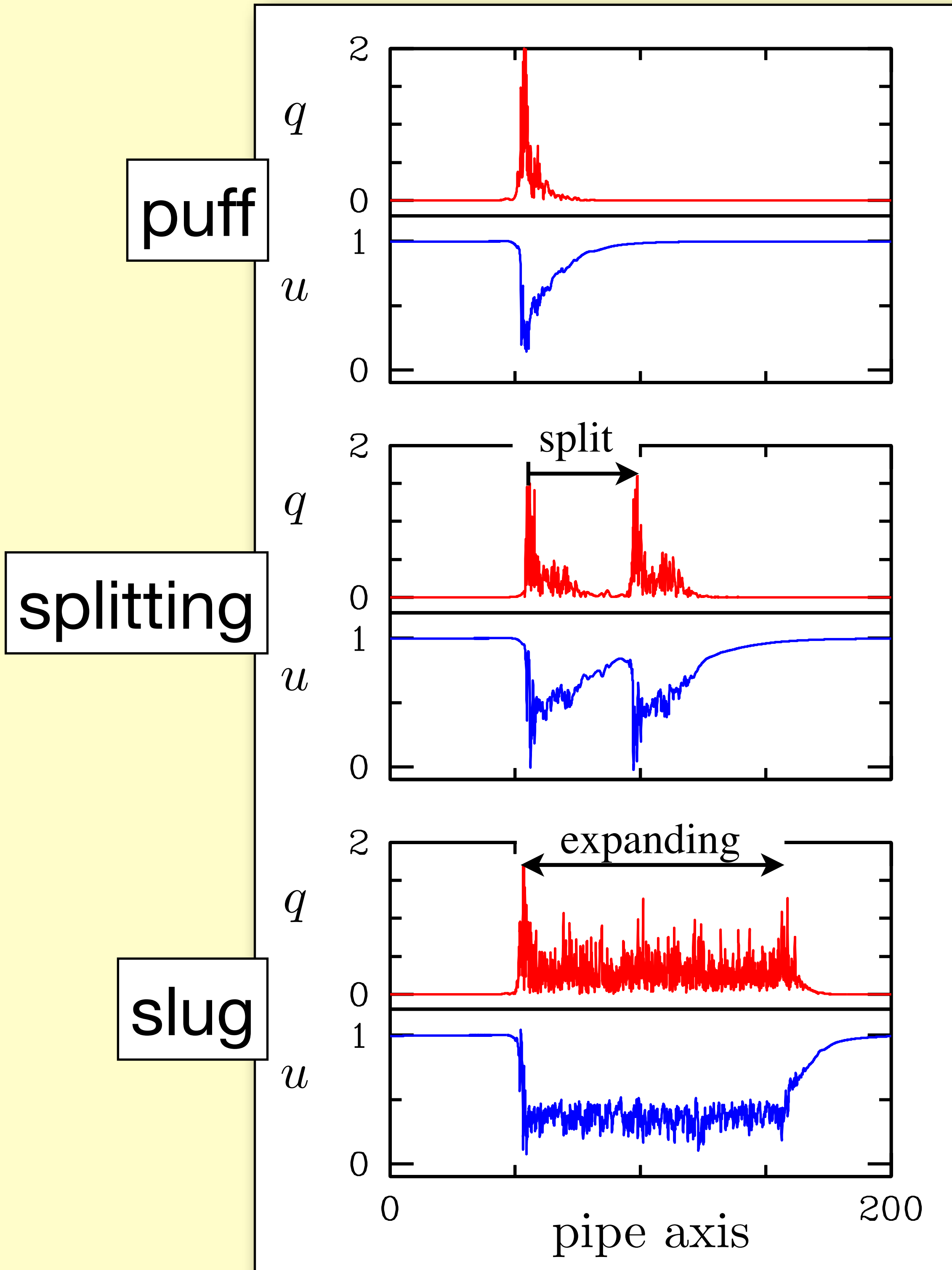
MAP Model
(chaos)

Barkley, *Phys. Rev. E* **84,** 016309 (2011)



Direct Numerical Simulation

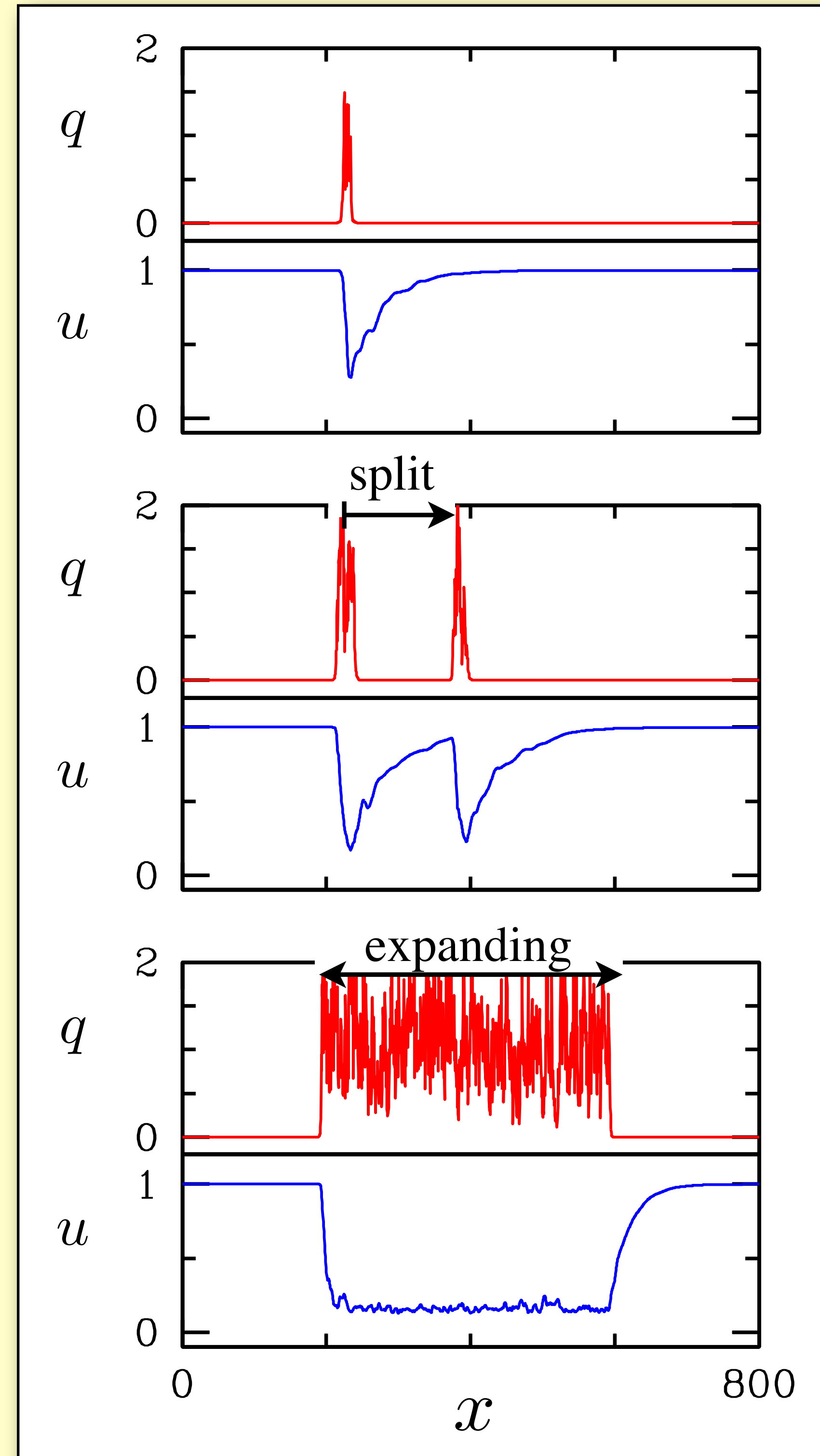
Barkley, *Phys. Rev. E* **84,** 016309 (2011)



SPDE Model

(noise)

Barkley, ETC13 (to appear)

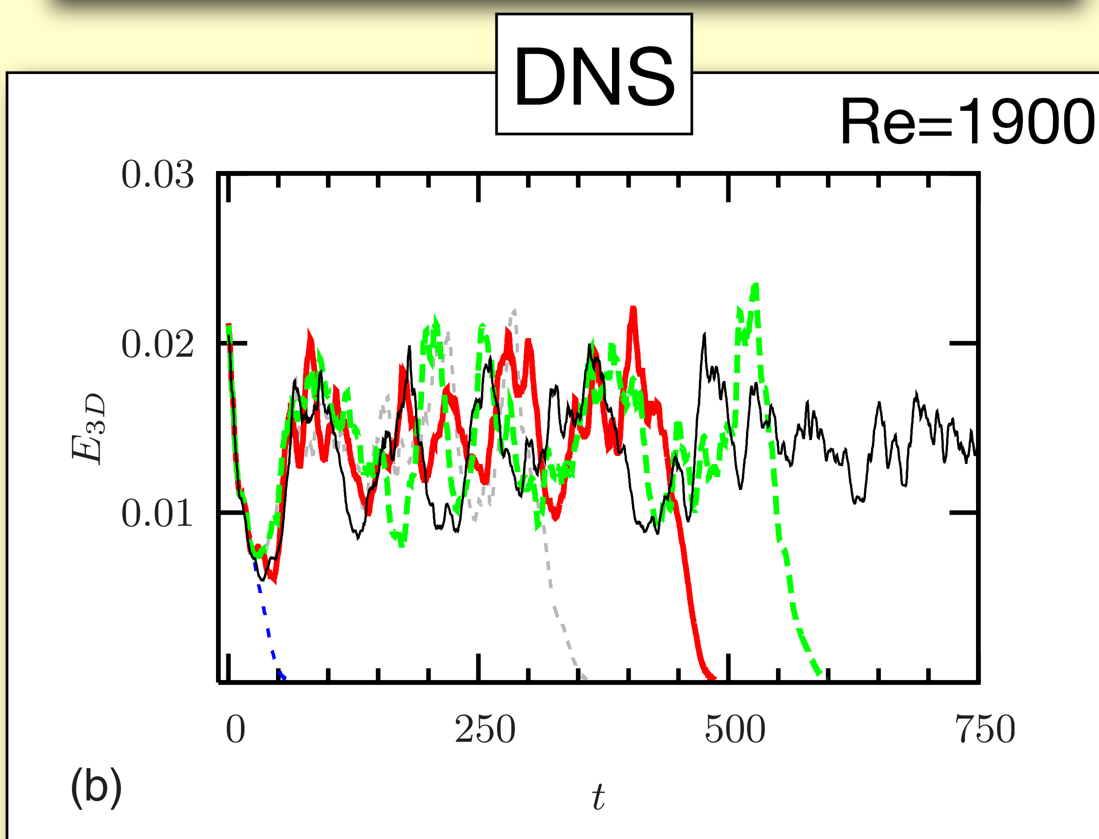


Unpredictable Decay of Turbulence

At low Re turbulence is transient.
Minute changes in initial conditions results in wildly different decay times.

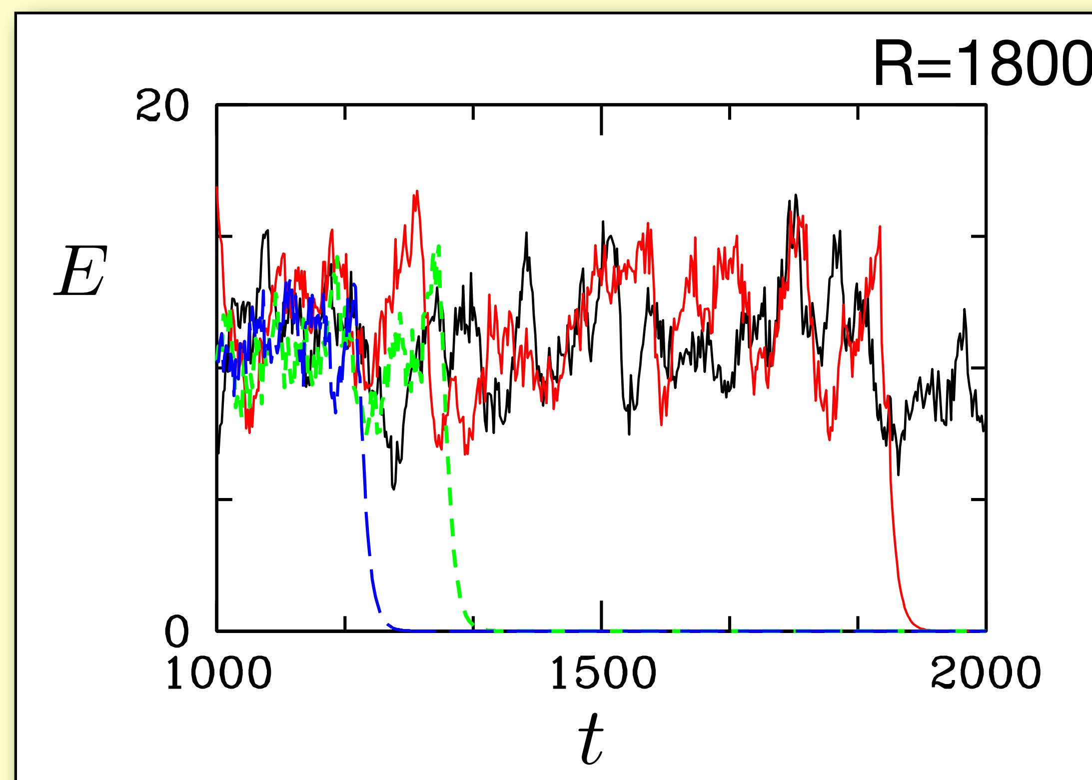
Lifetime statistics in transitional pipe flow

Tobias M. Schneider[1,*] and Bruno Eckhardt[1,†]



Map model

Barkley, *Phys. Rev. E* **84,** 016309 (2011)



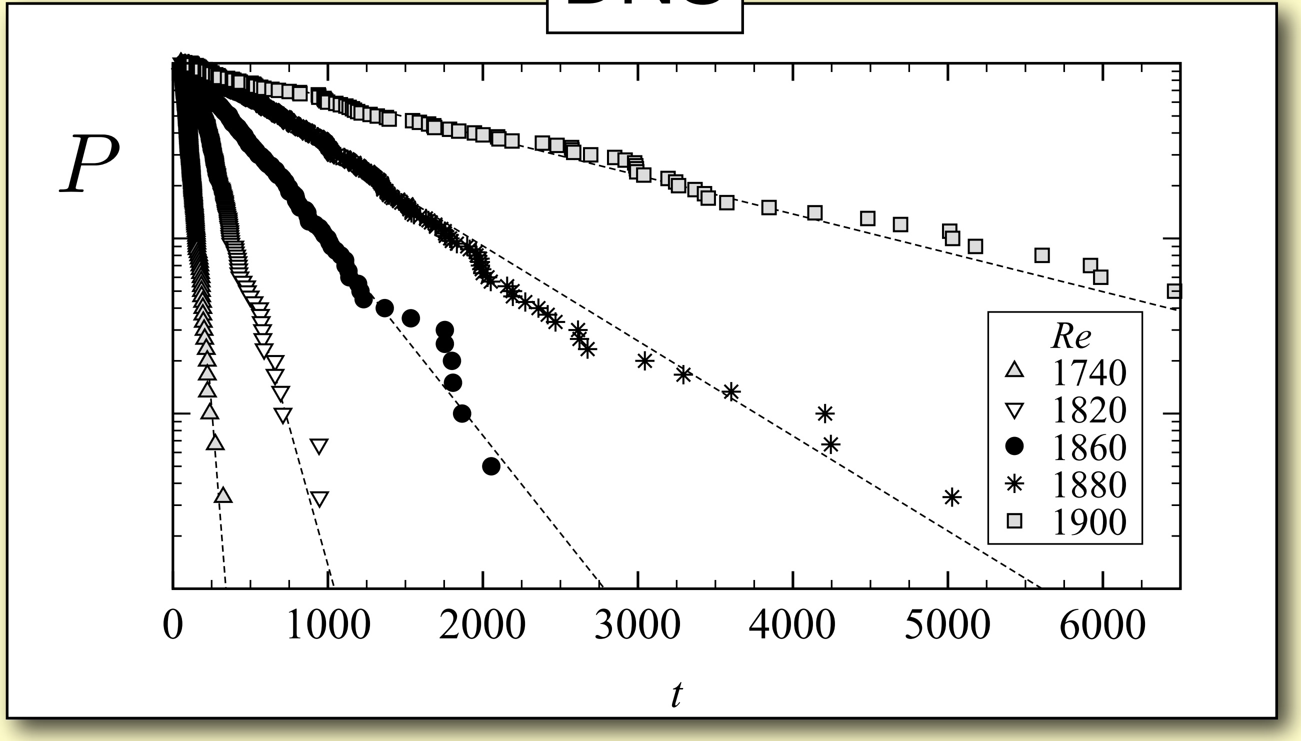
Decay is Memoryless

Giving rise to exponential lifetime distributions

On the transient nature of localized pipe flow turbulence

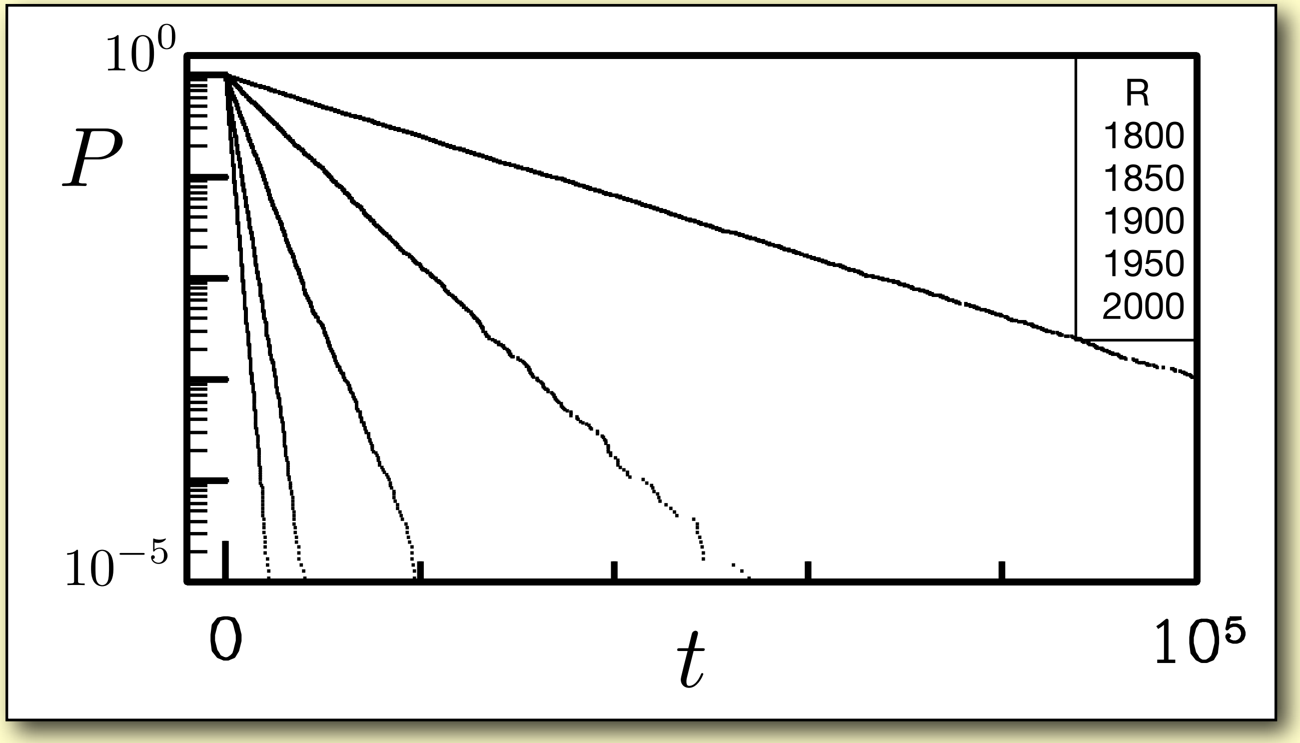
MARC AVILA[1]†, ASHLEY P. WILLIS[2] AND BJÖRN HOF[1]

DNS



Map model

Barkley, *Phys. Rev. E* **84,** 016309 (2011)



Puff Splitting

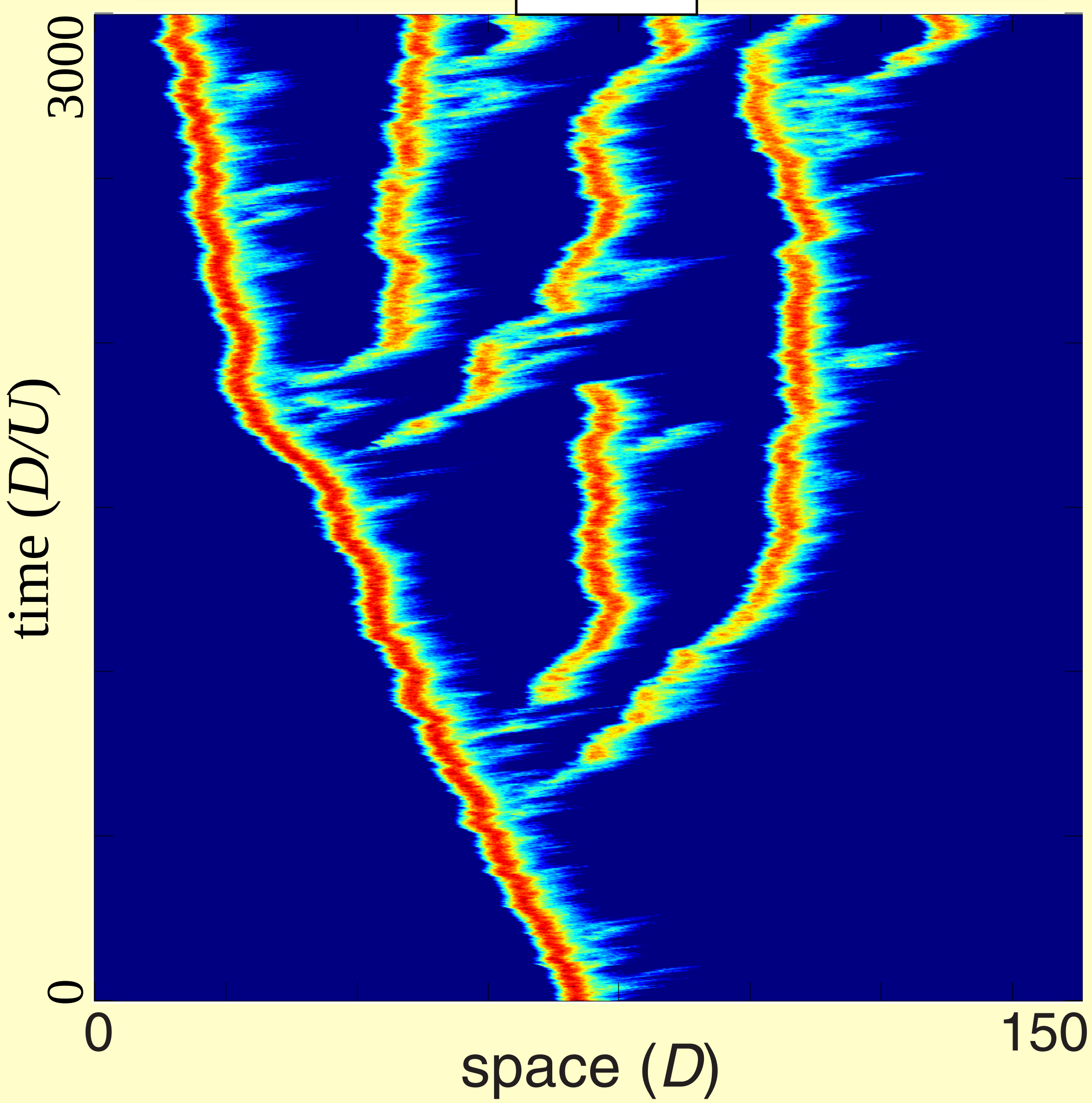
New puffs randomly split from downstream side

The Onset of Turbulence in Pipe Flow

Kerstin Avila,[1]* David Moxey,[2] Alberto de Lozar,[1] Marc Avila,[1] Dwight Barkley,[2,3] Björn Hof[1]*

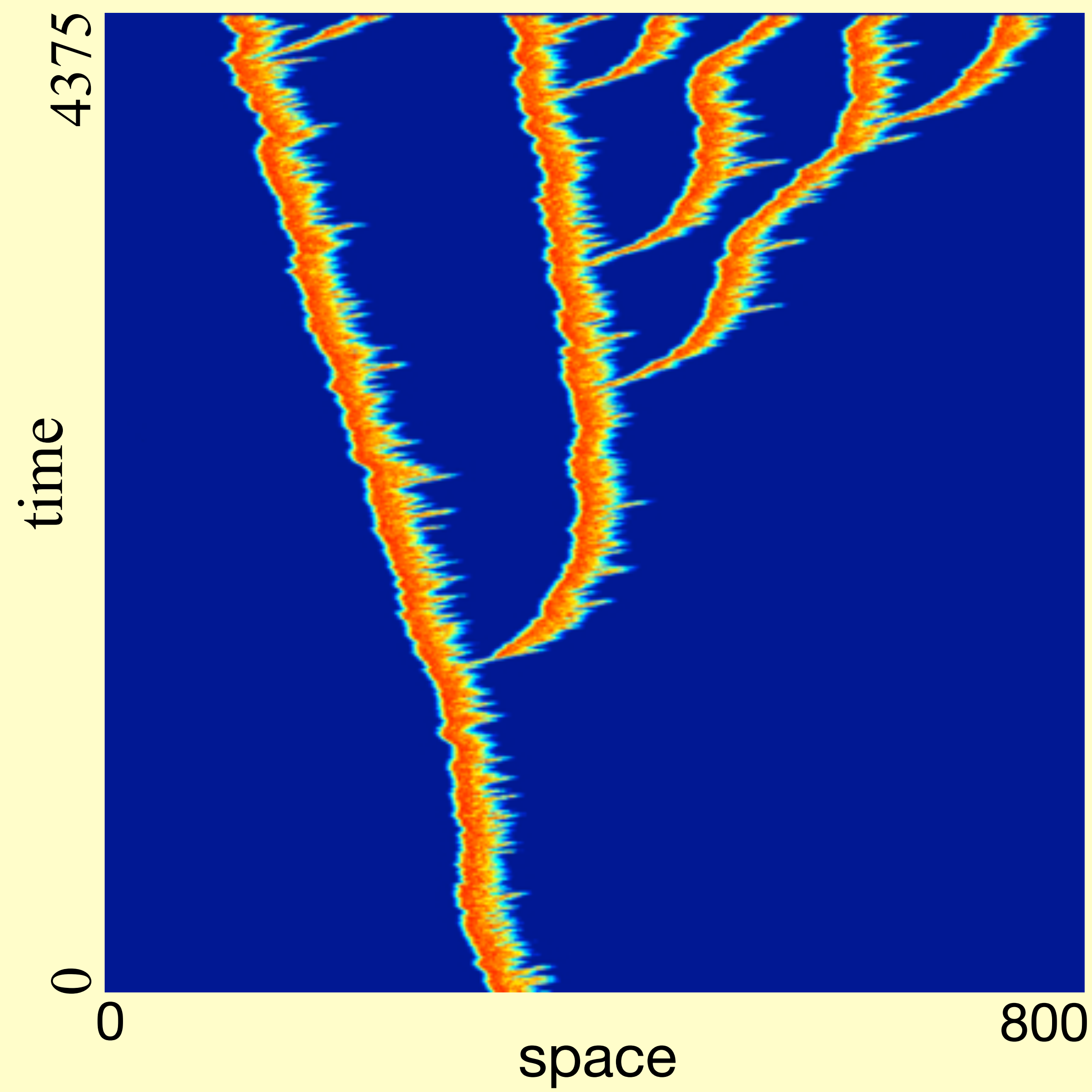
8 JULY 2011 VOL 33 ...w.sciencemag.org

DNS



SPDE model

(Barkley, ETC 13)



(co-moving frame, log scale)

Puff Splitting is Memoryless

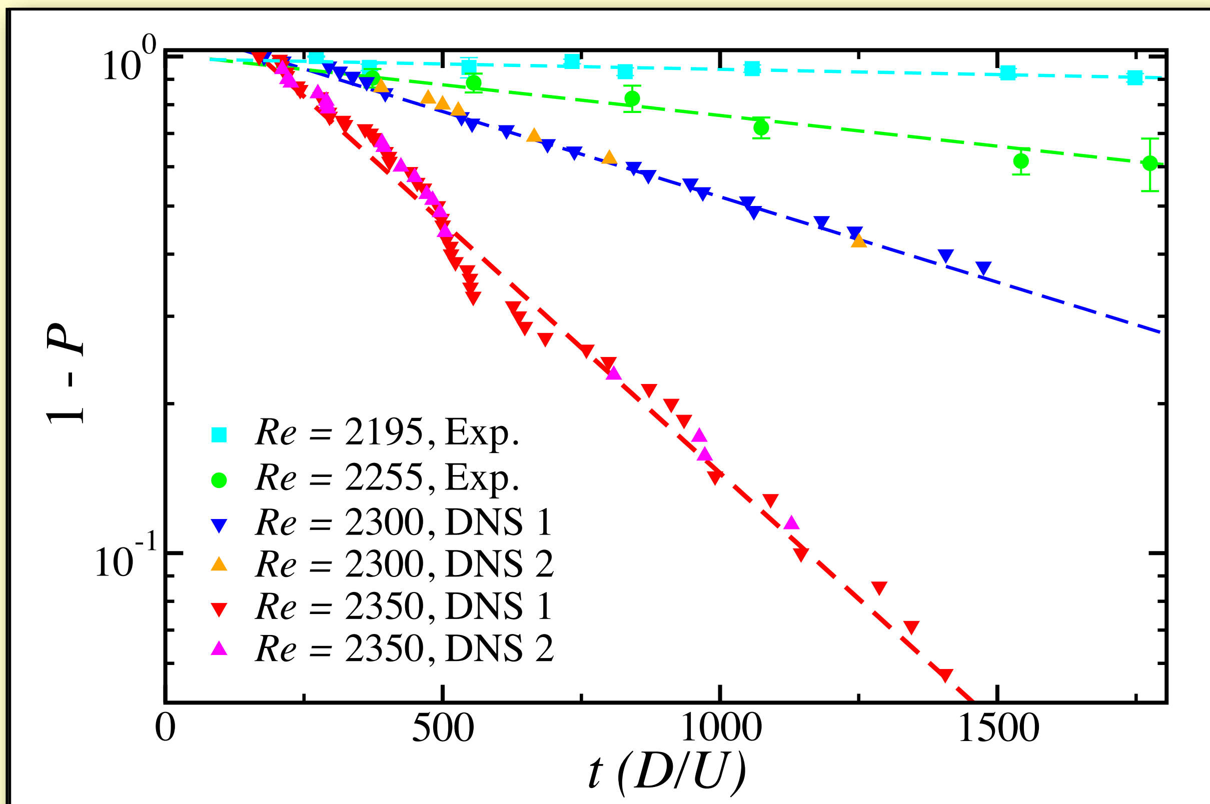
Giving rise to exponential lifetime distributions

The Onset of Turbulence in Pipe Flow

Kerstin Avila,[1*] David Moxey,[2] Alberto de Lozar,[1] Marc Avila,[1] Dwight Barkley,[2,3] Björn Hof[1*]

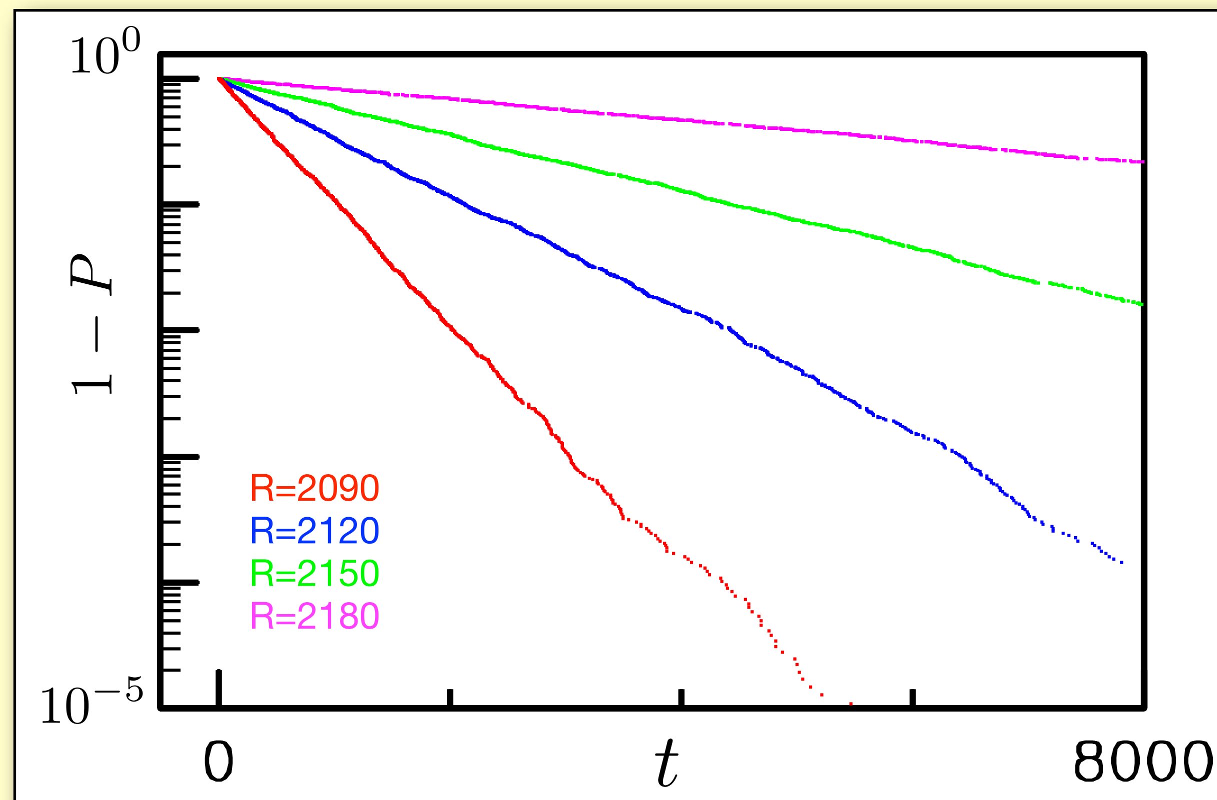
8 JULY 2011 VOL 333 SCIENCE www.sciencemag.org

Experiment and DNS



Map model

Barkley, *Phys. Rev. E* **84,** 016309 (2011)



Critical Point

Decay and splitting lifetimes cross giving rise to a critical point

The Onset of Turbulence in Pipe Flow

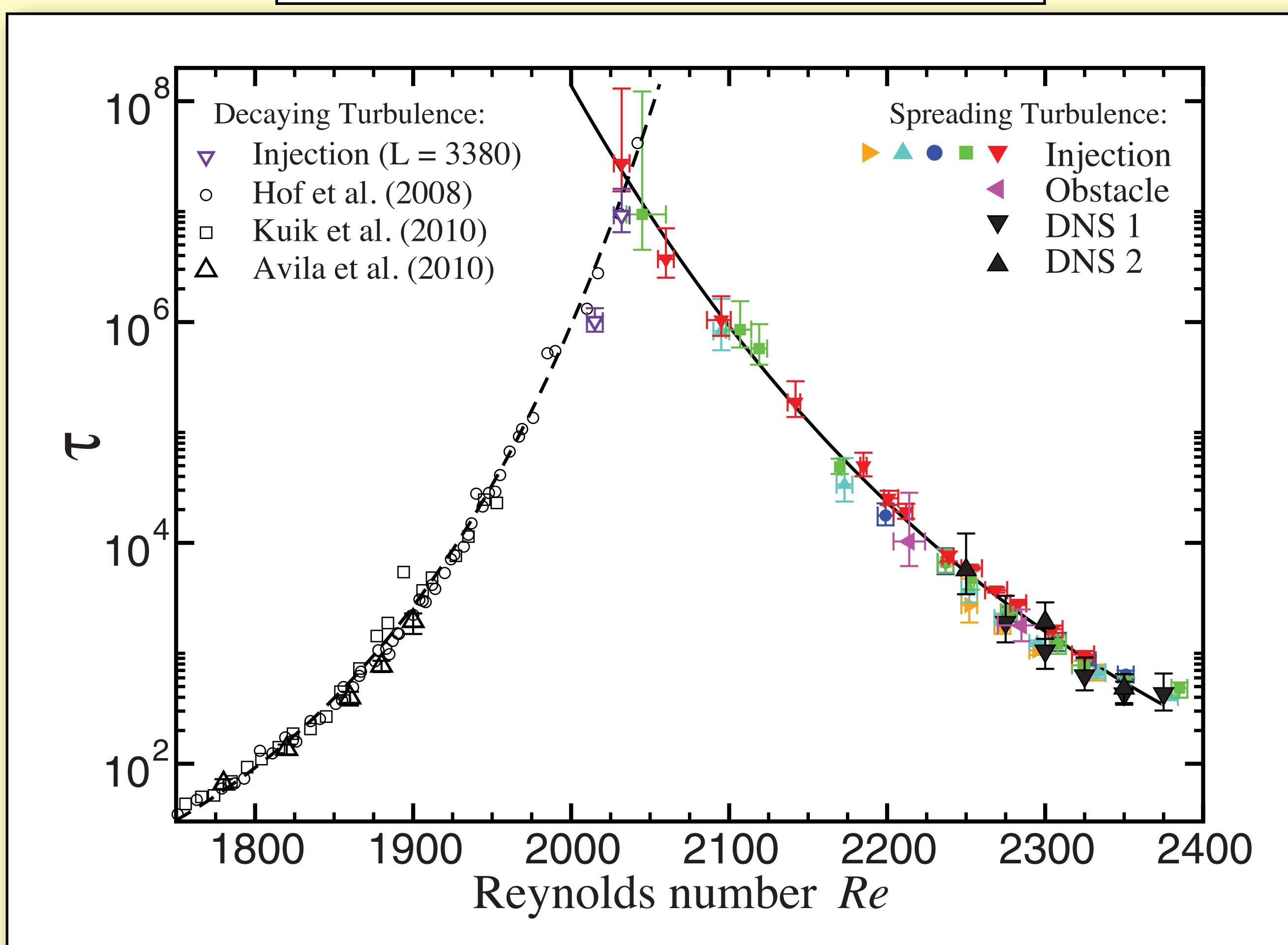
Kerstin Avila,[1*] David Moxey,[2] Alberto de Lozar,[1] Marc Avila,[1] Dwight Barkley,[2,3] Björn Hof[1*]

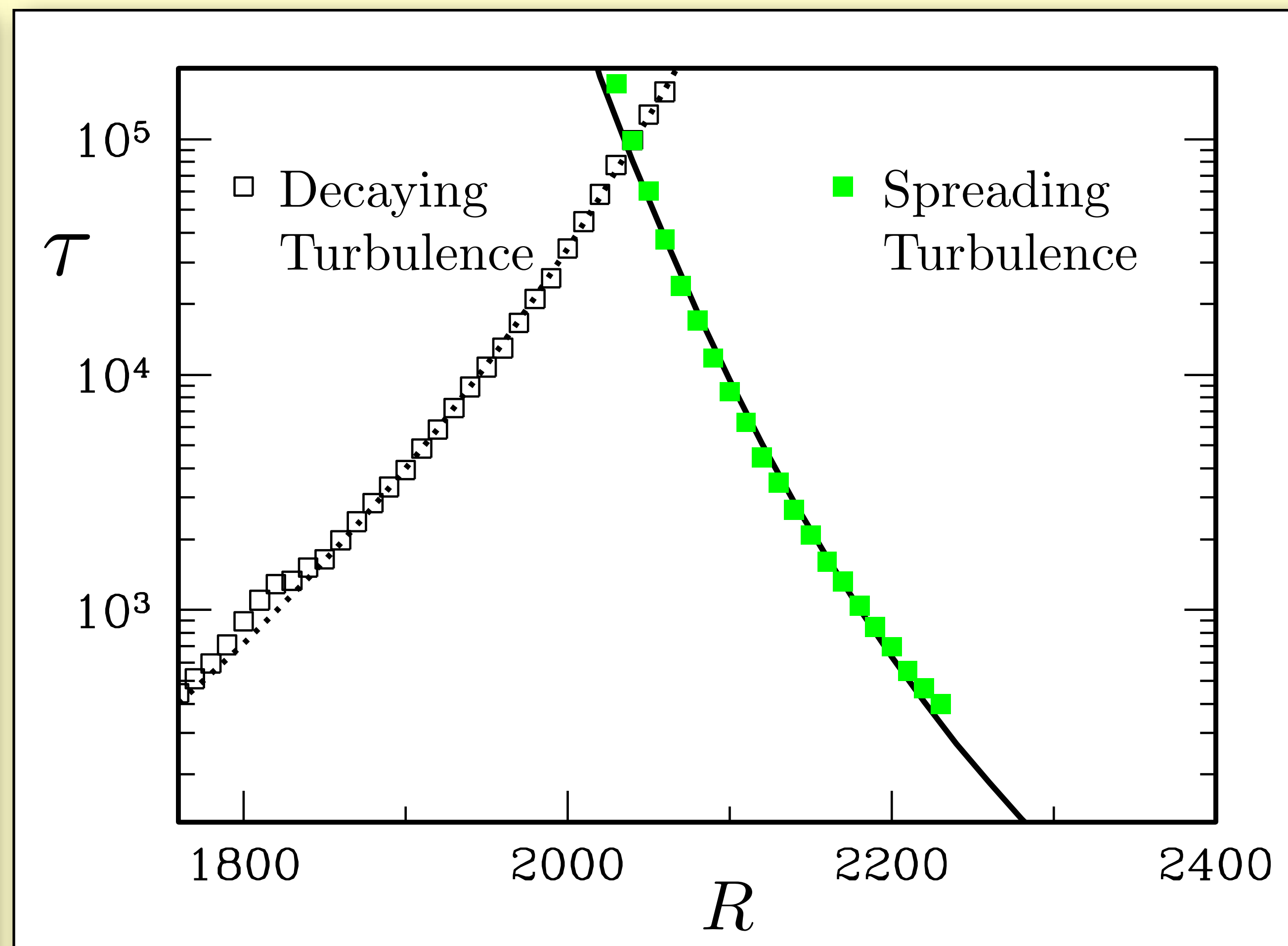
8 JULY 2011 VOL 333 **SCIENCE** www.sciencemag.org

Map model

Barkley, *Phys. Rev. E* **84,** 016309 (2011)

Experiment and DNS





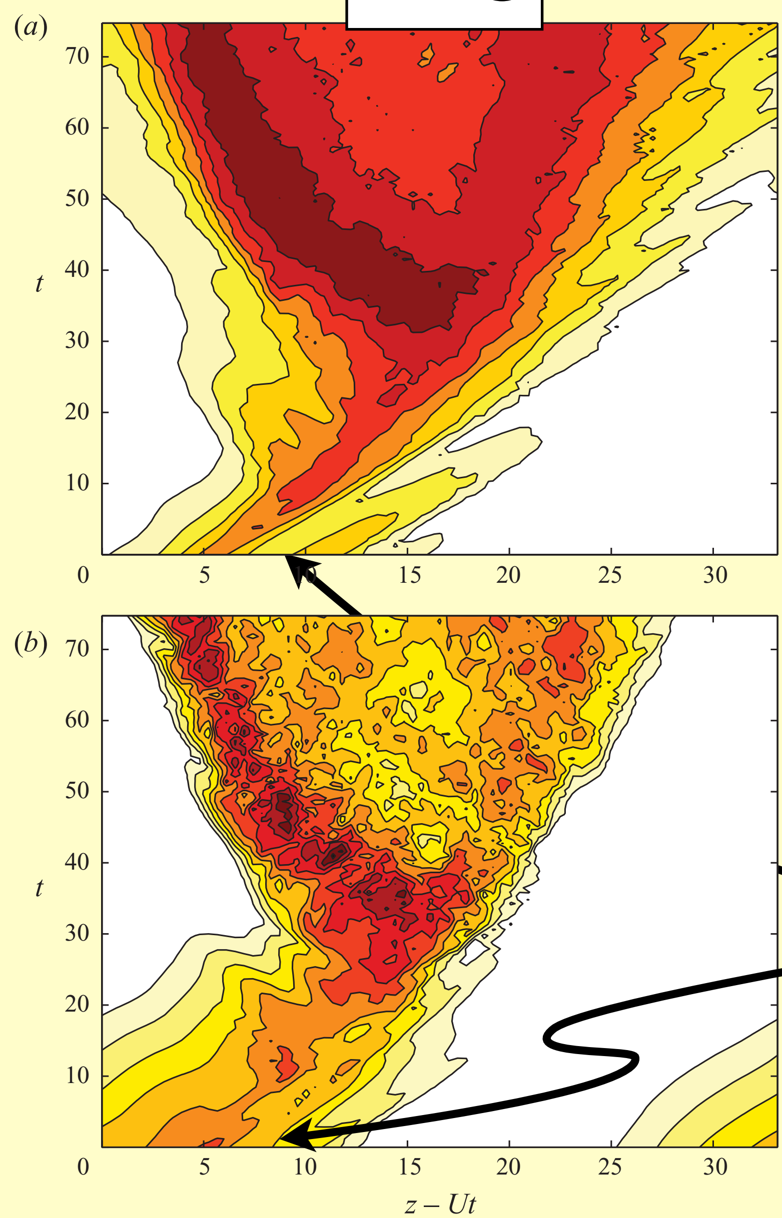
Slug Formation from Edge State

Space-time plots of Energy (co-moving frame, log scale)

Slug genesis in cylindrical pipe flow

Y. DUGUET,[1,2,4]† A. P. WILLIS [1,3] AND R. R. KERSWELL[1]

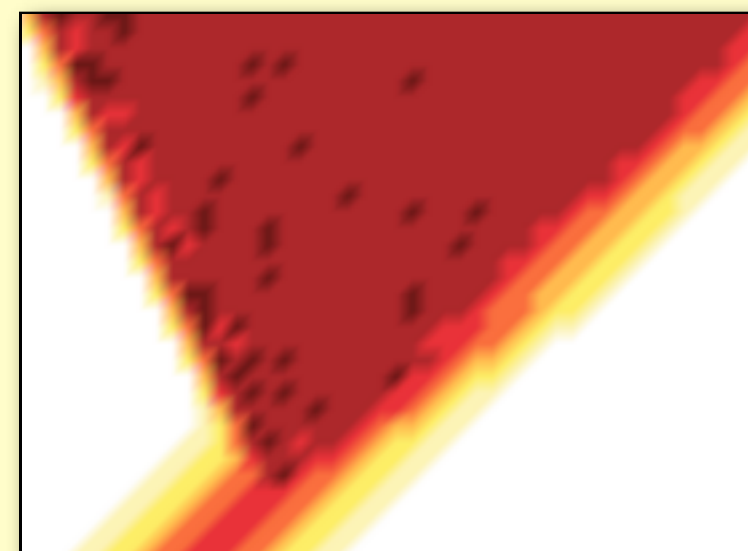
DNS



(Colour online) Genesis of a slug from the edge state at $Re = 3000$. (a) E_{roll} (b) E_{streak} (scales as in figure 12).

Map model

Barkley, *Phys. Rev. E* **84,** 016309 (2011)



R=3000

Edge state

(low amplitude, localized)

You get the point by now

You get the point by now

These models do not capture:

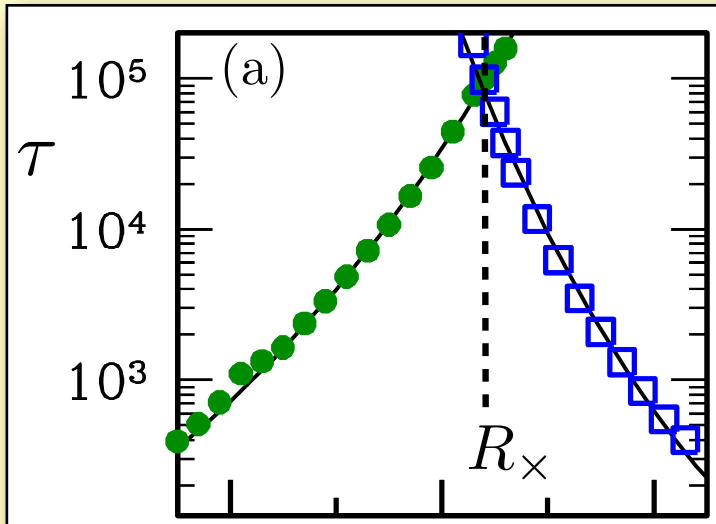
☒ Fractal Basin Boundaries

3 Things We Can Learn from Models

not easily accessible to Experiment or DNS

1) Sustained Turbulence

Sustained Model Turbulence

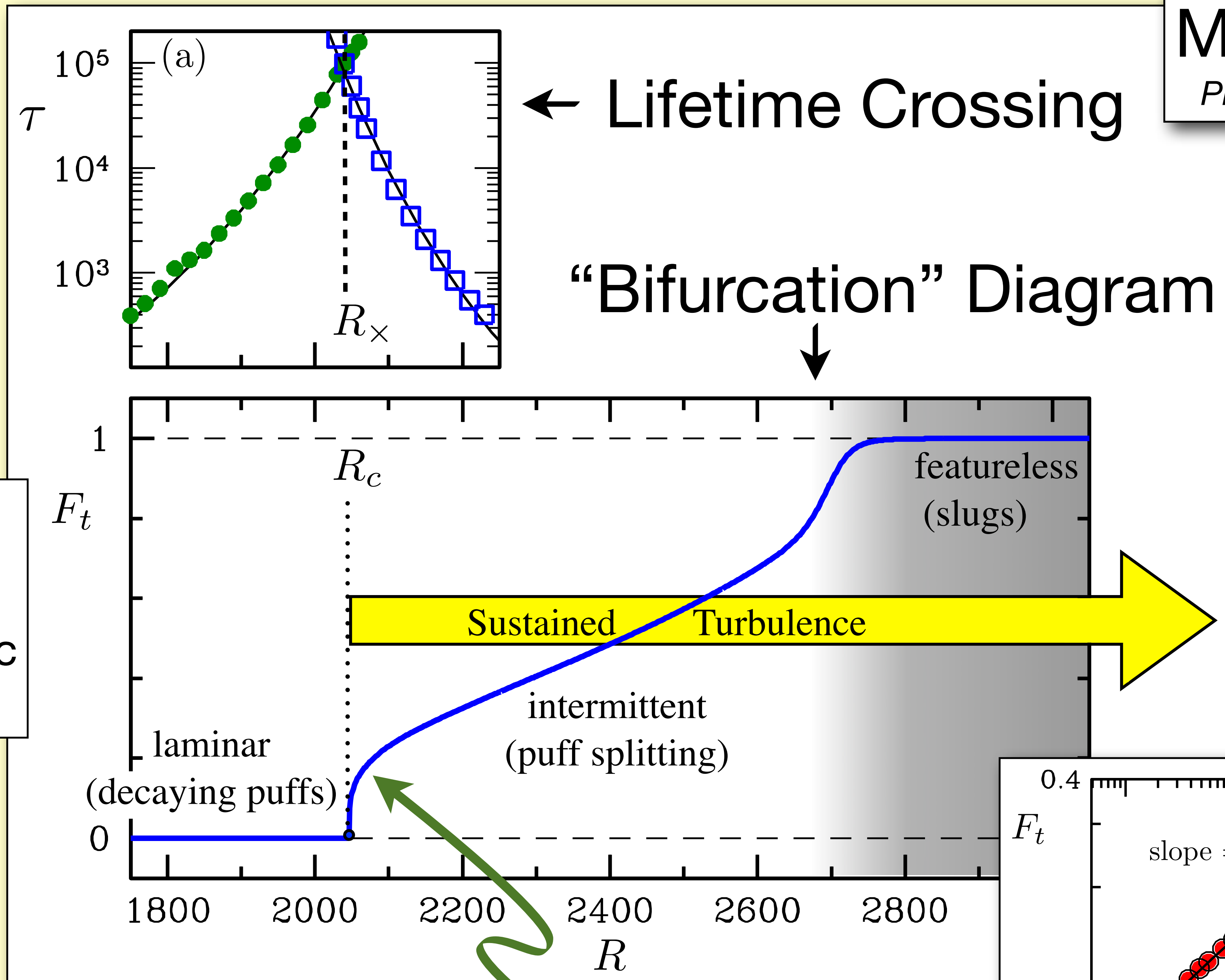


← Lifetime Crossing

Map model
Phys. Rev. E (2011)

Sustained Model Turbulence

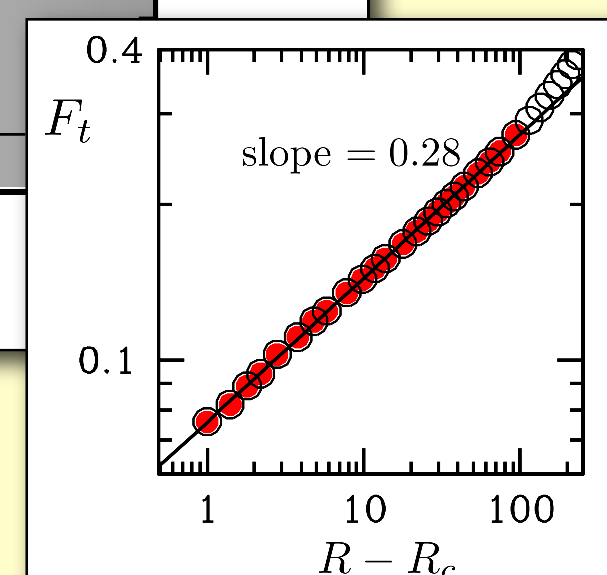
Map model
Phys. Rev. E (2011)



Turbulence Fraction (thermodynamic limit)

$Re_c = Re_x$ to within 0.3%

Continuous transition to sustained turbulence



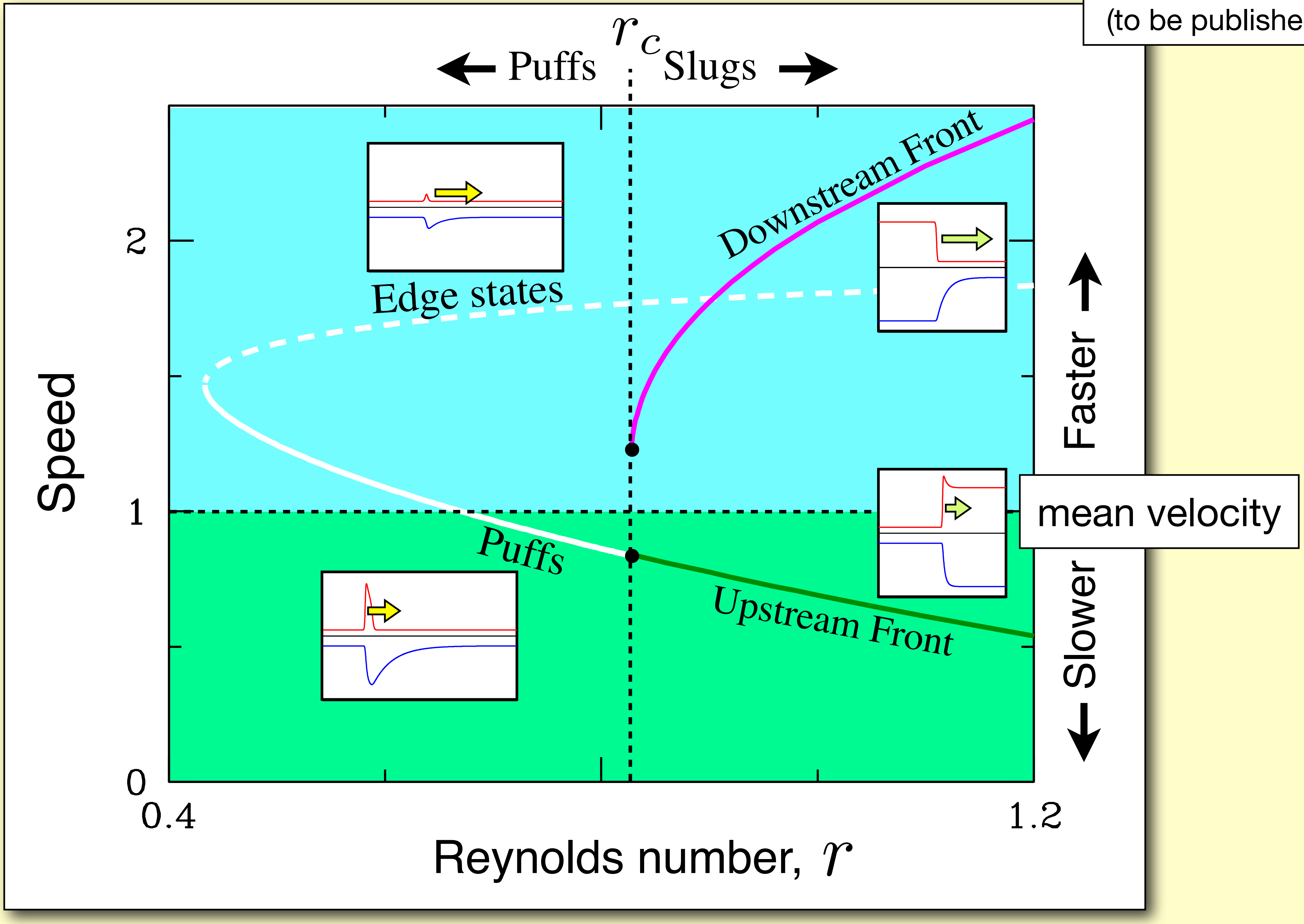
Directed Percolation in (1+1)D

2) Speed of pulses and fronts

Speed of Pulses and Fronts

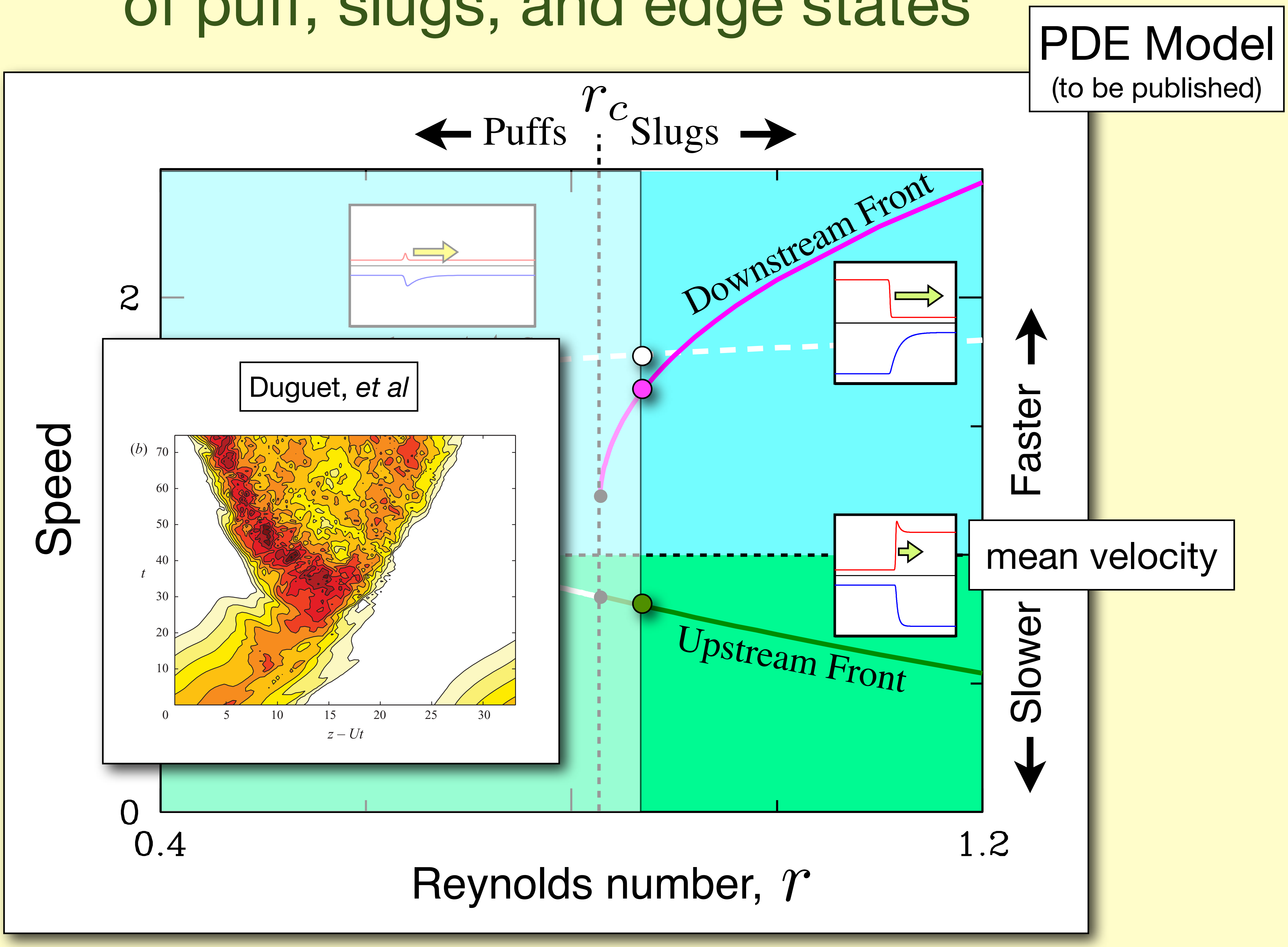
This analysis provides understanding of many features of puff, slugs, and edge states

PDE Model
(to be published)



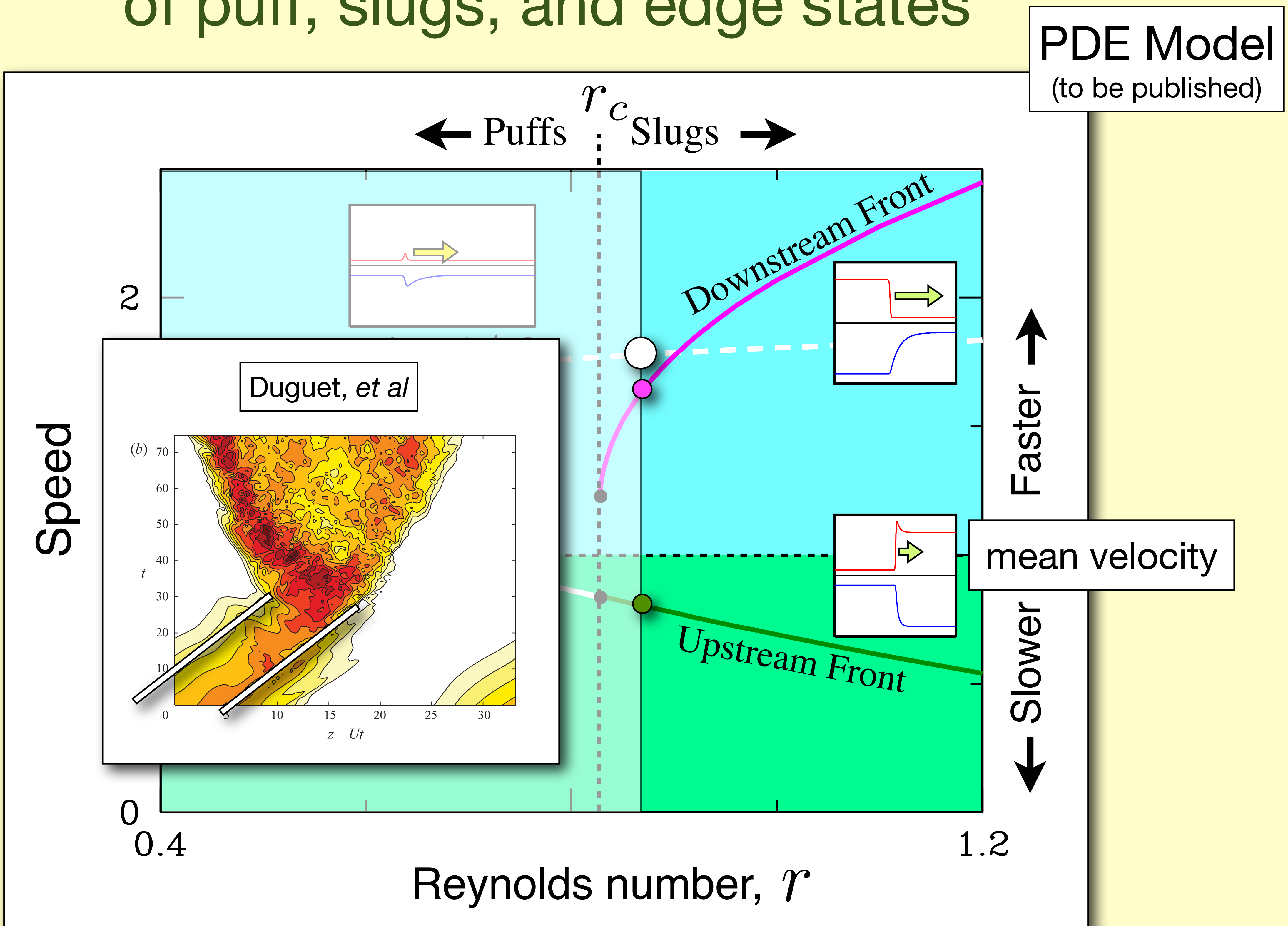
Speed of Pulses and Fronts

This analysis provides understanding of many features of puff, slugs, and edge states



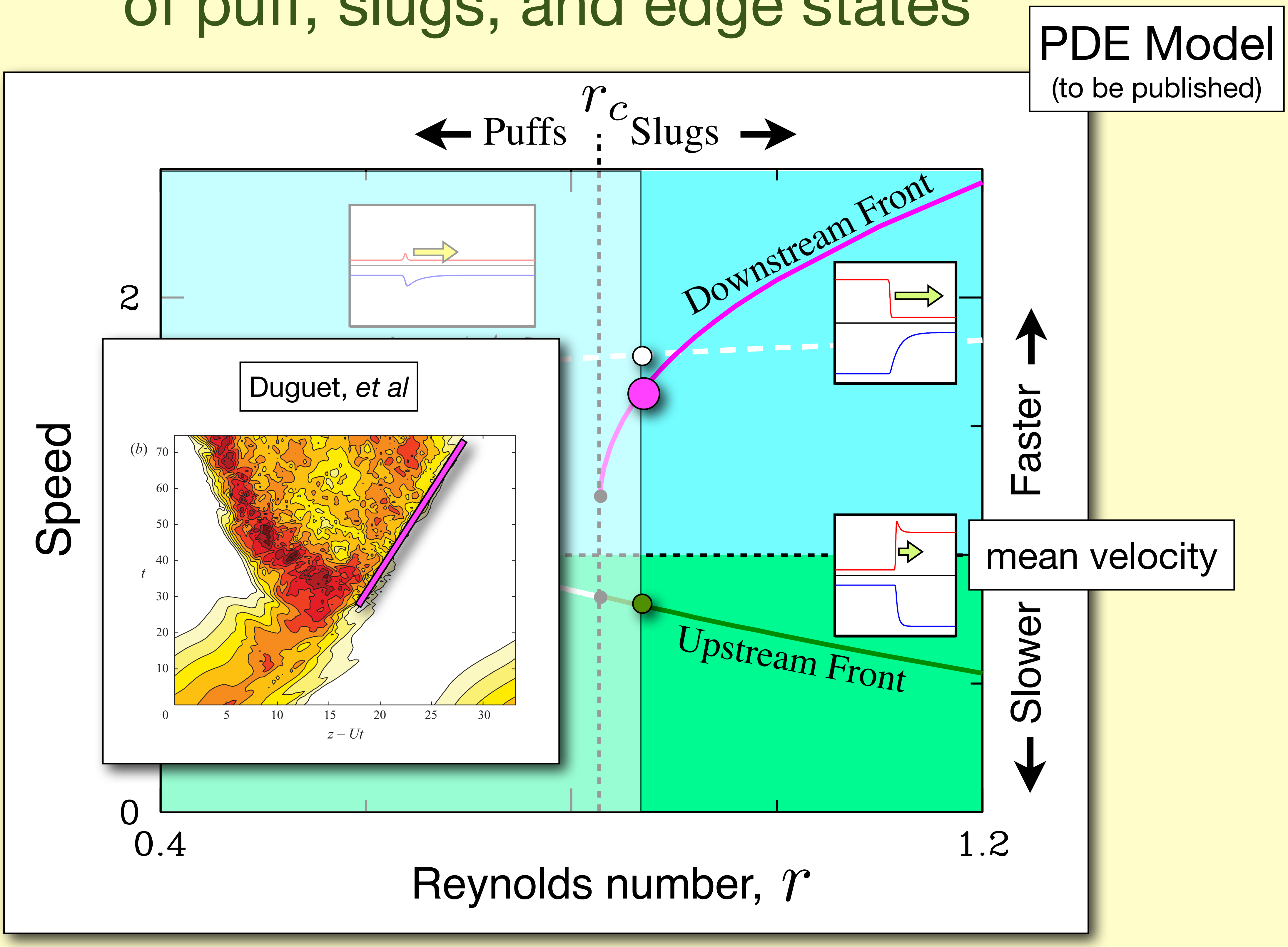
Speed of Pulses and Fronts

This analysis provides understanding of many features of puff, slugs, and edge states



Speed of Pulses and Fronts

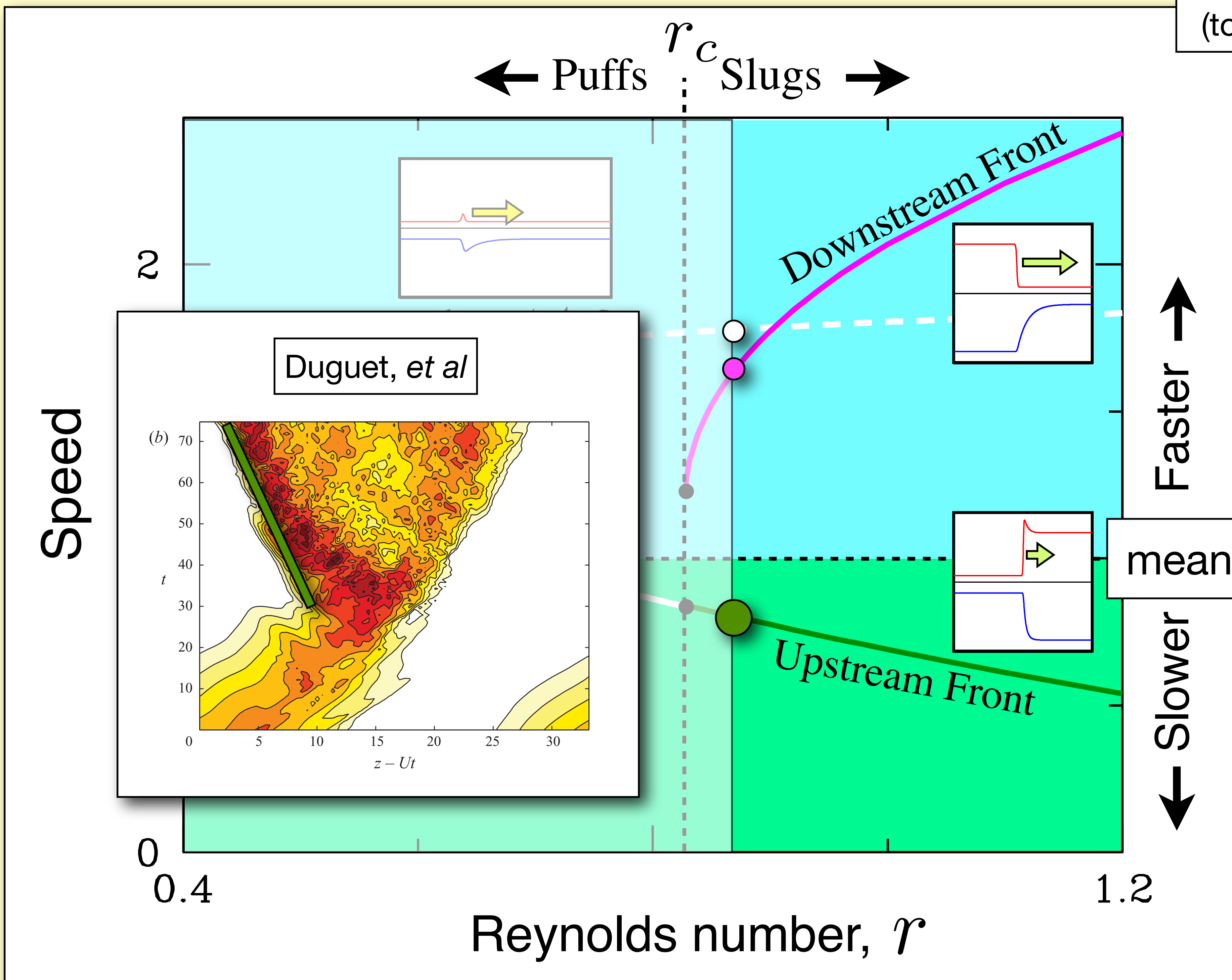
This analysis provides understanding of many features of puff, slugs, and edge states



Speed of Pulses and Fronts

This analysis provides understanding of many features of puff, slugs, and edge states

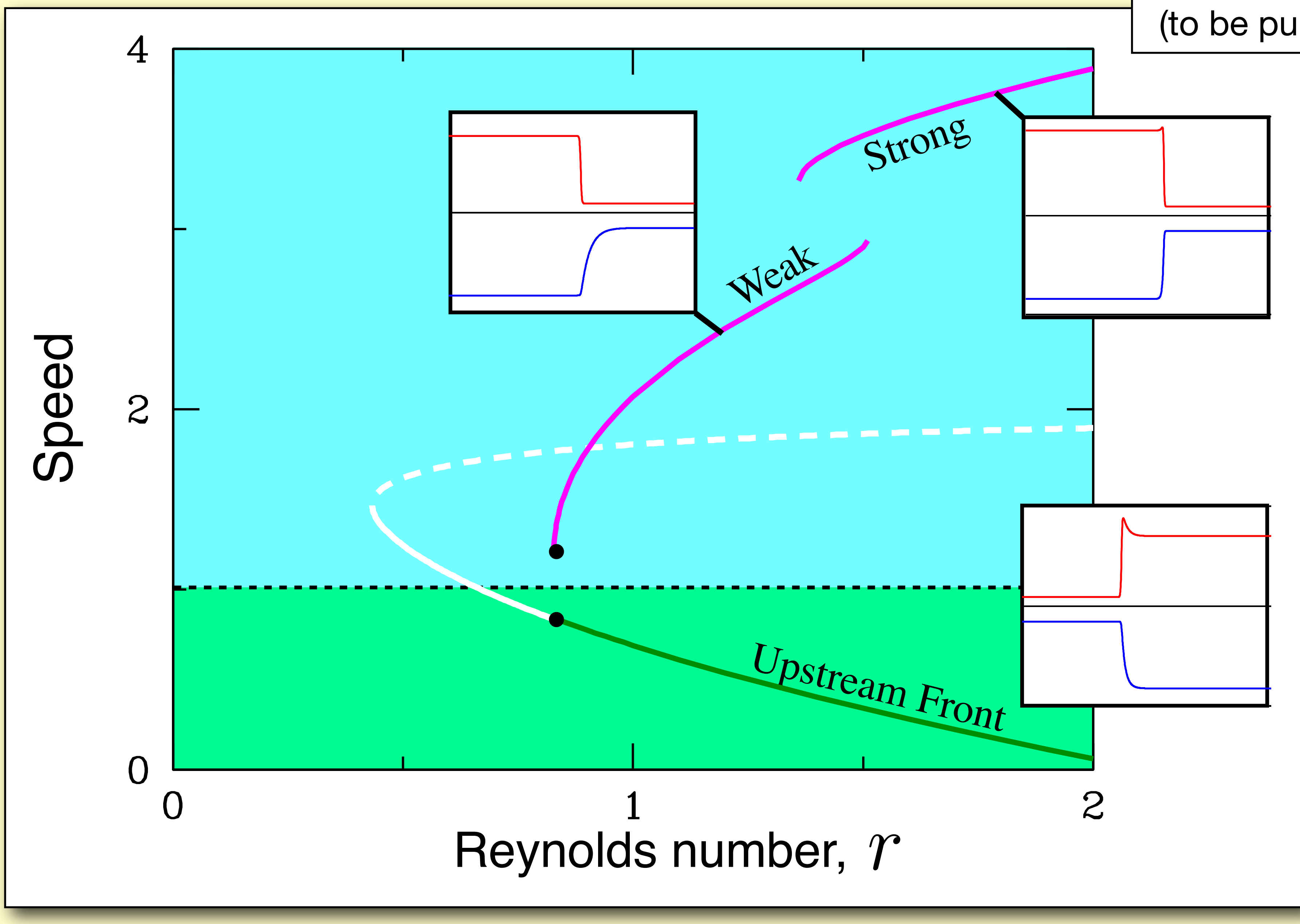
PDE Model
(to be published)



2 Types of Slugs: Weak & Strong

The strong front is like the upstream front,
the weak one is not

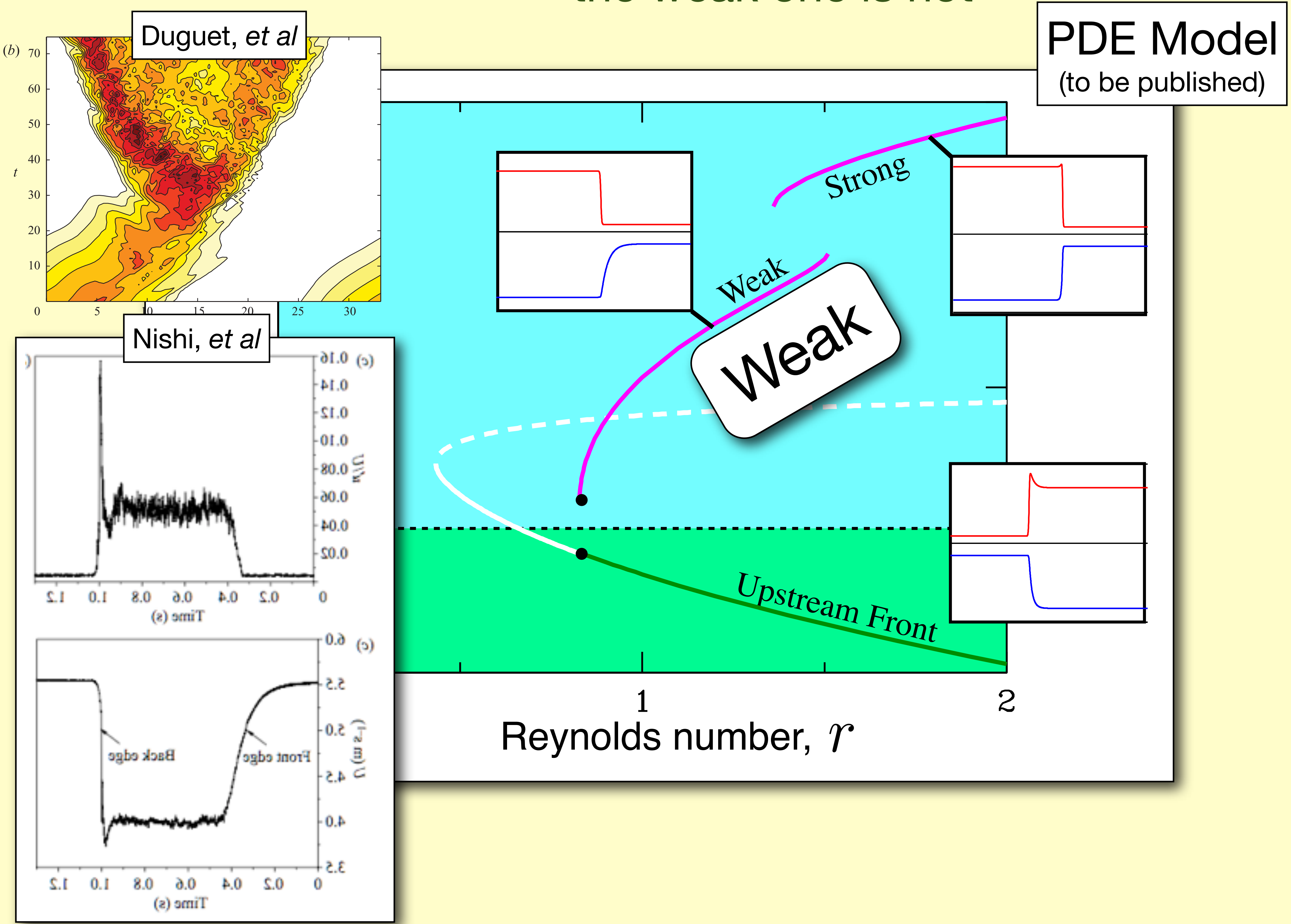
PDE Model
(to be published)



2 Types of Slugs: Weak & Strong

Weak

The strong front is like the upstream front, the weak one is not

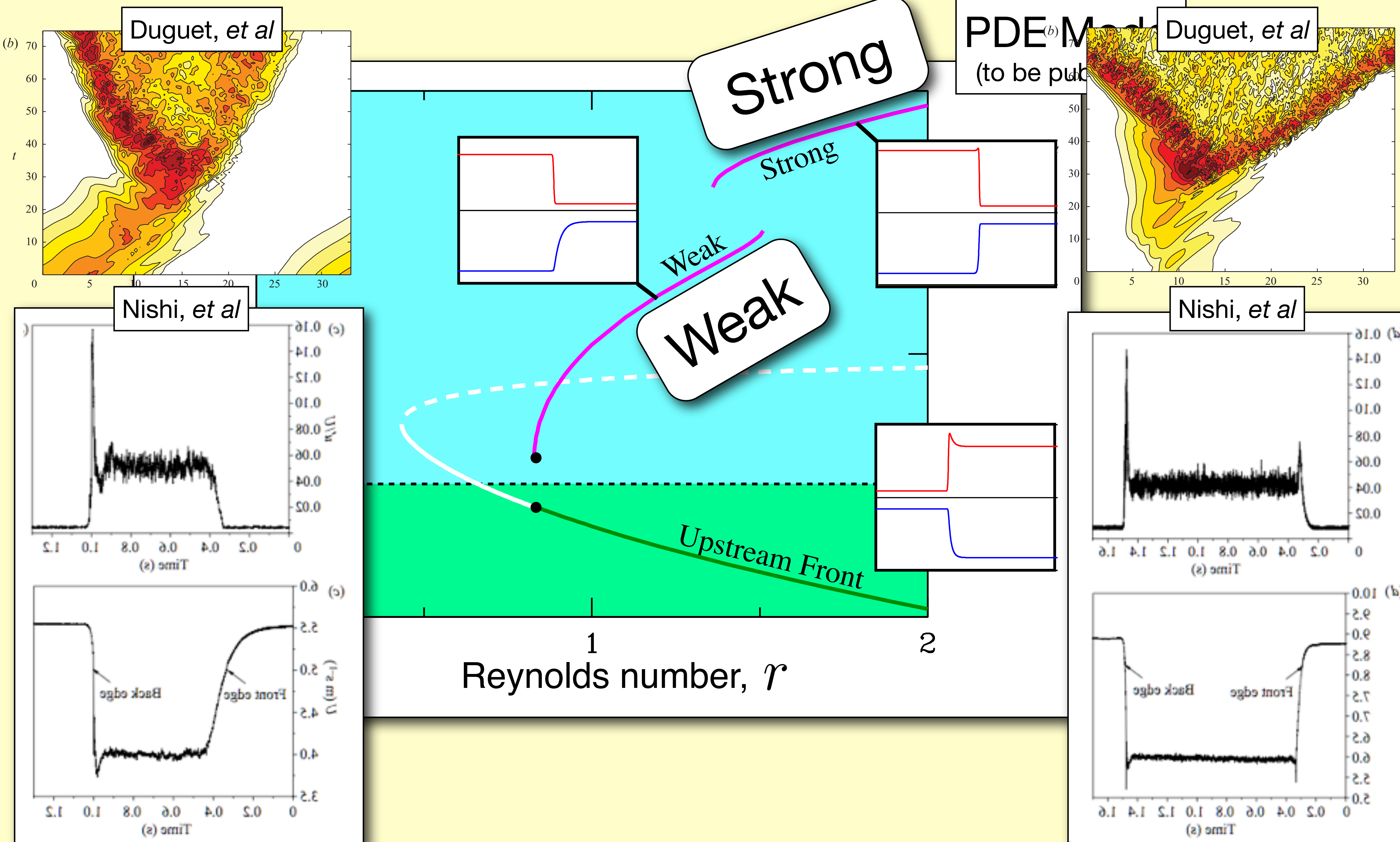


2 Types of Slugs: Weak & Strong

Weak

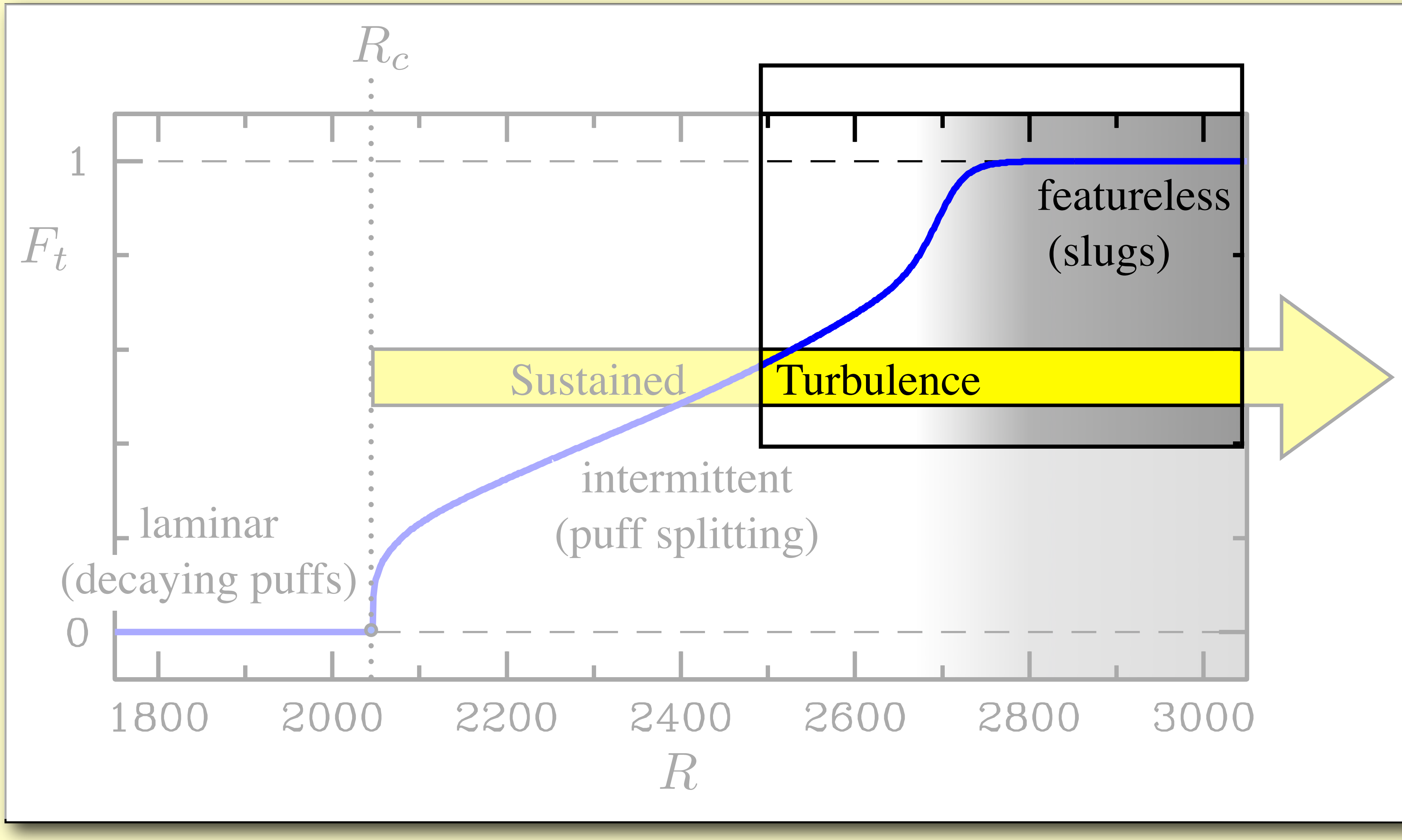
The strong front is like the upstream front,
the weak one is not

Strong



3) Transition to Uniform, Featureless Turbulence

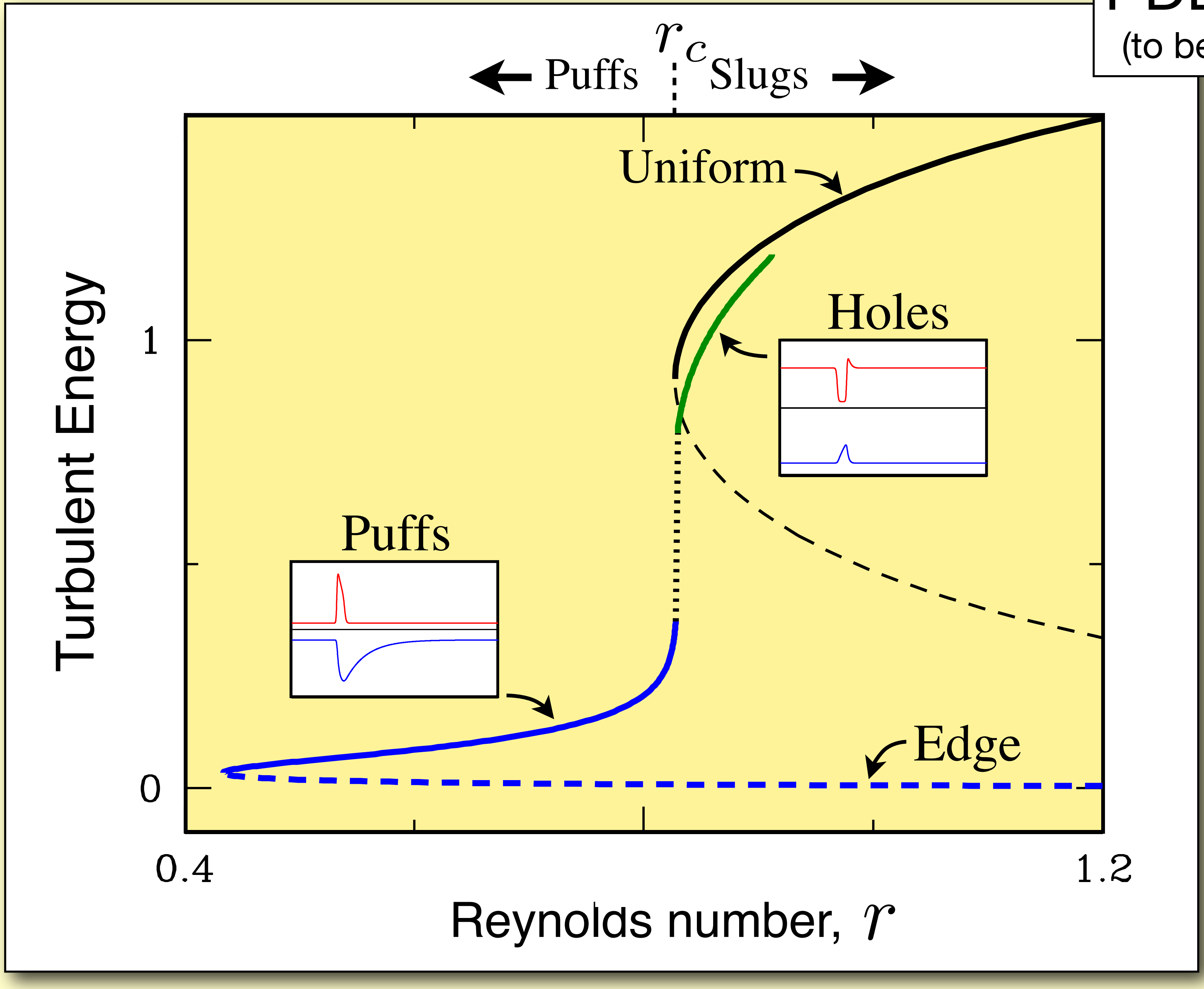
Transition to Uniform Turbulence



Transition to Uniform Turbulence

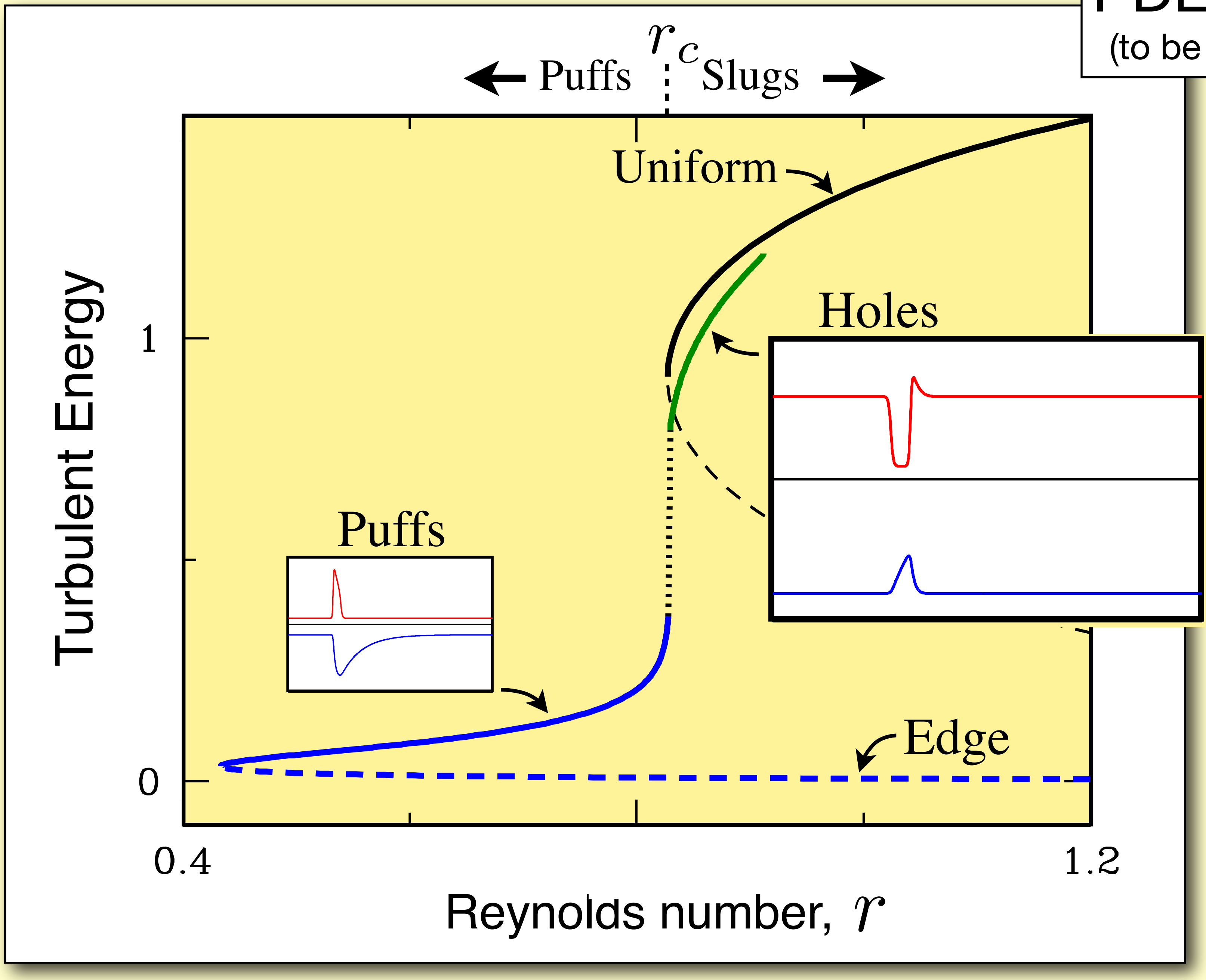
Bifurcation Diagram

PDE Model
(to be published)



Transition to Uniform Turbulence

Bifurcation Diagram



PDE Model
(to be published)

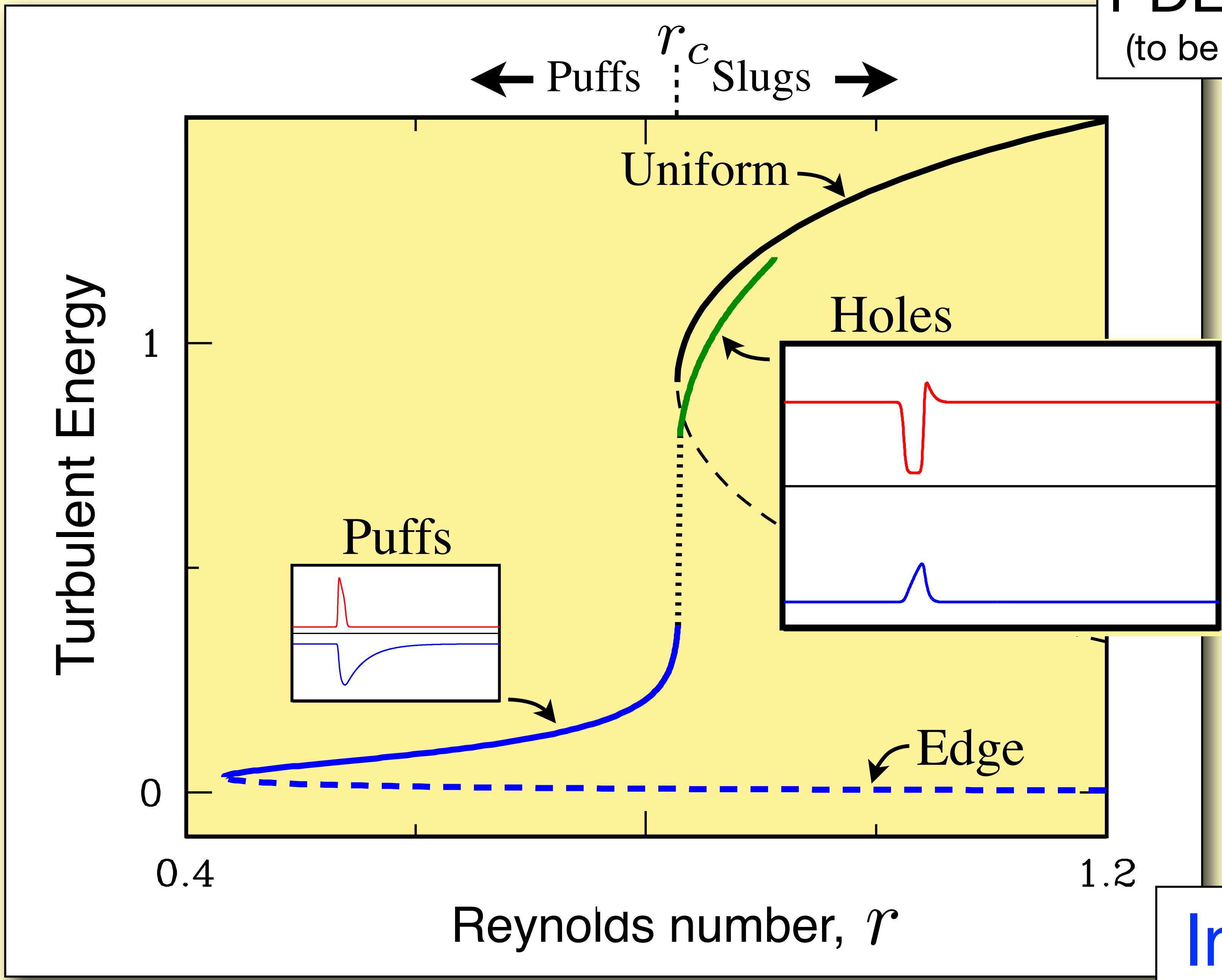
Localized laminar patch in turbulent background

somewhat like anti-puff but selection principle is different

Transition to Uniform Turbulence

Bifurcation Diagram

PDE Model
(to be published)



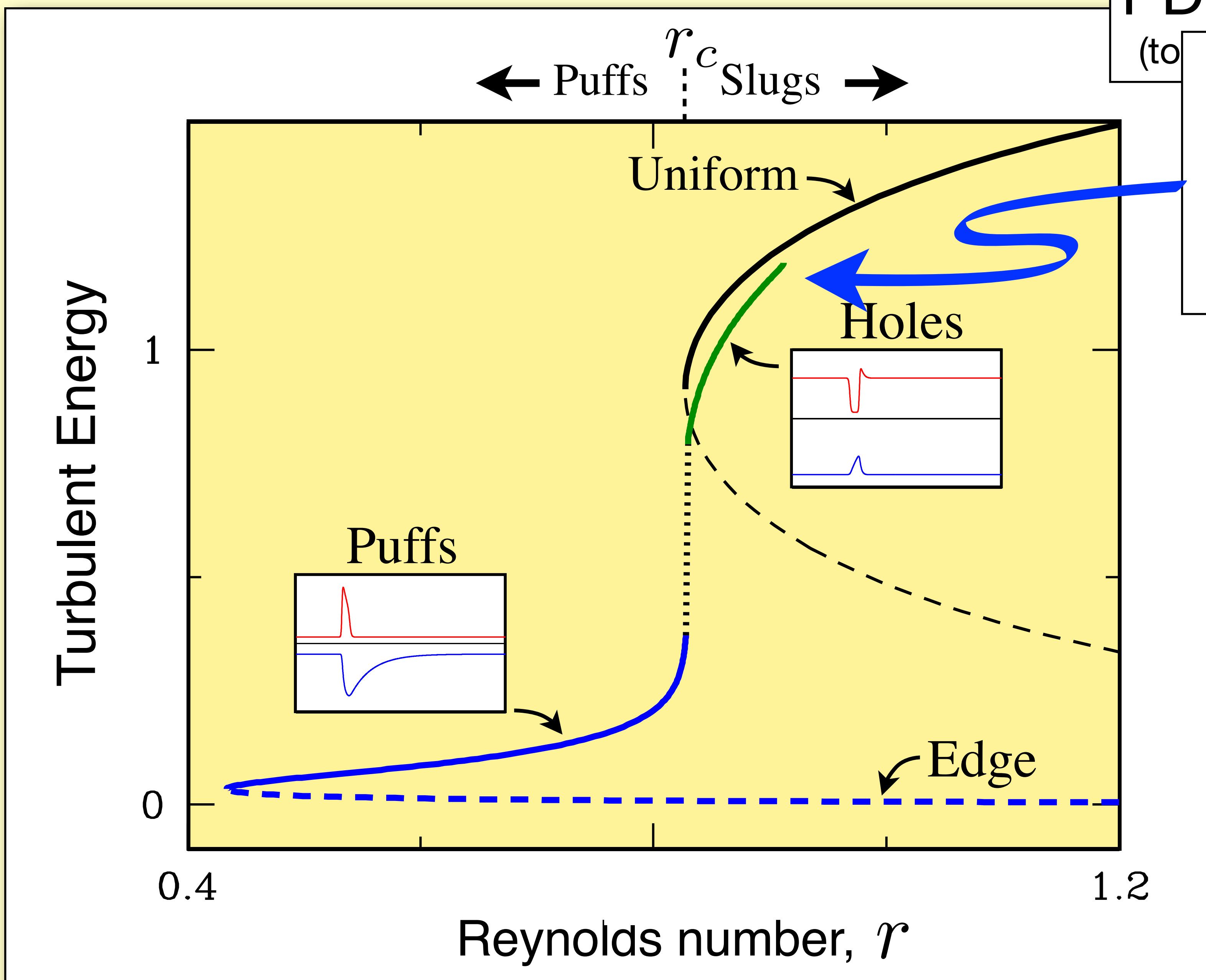
Localized laminar patch in turbulent background

somewhat like anti-puff but selection principle is different

Interaction distance of Hof, *et al*

Transition to Uniform Turbulence

Bifurcation Diagram



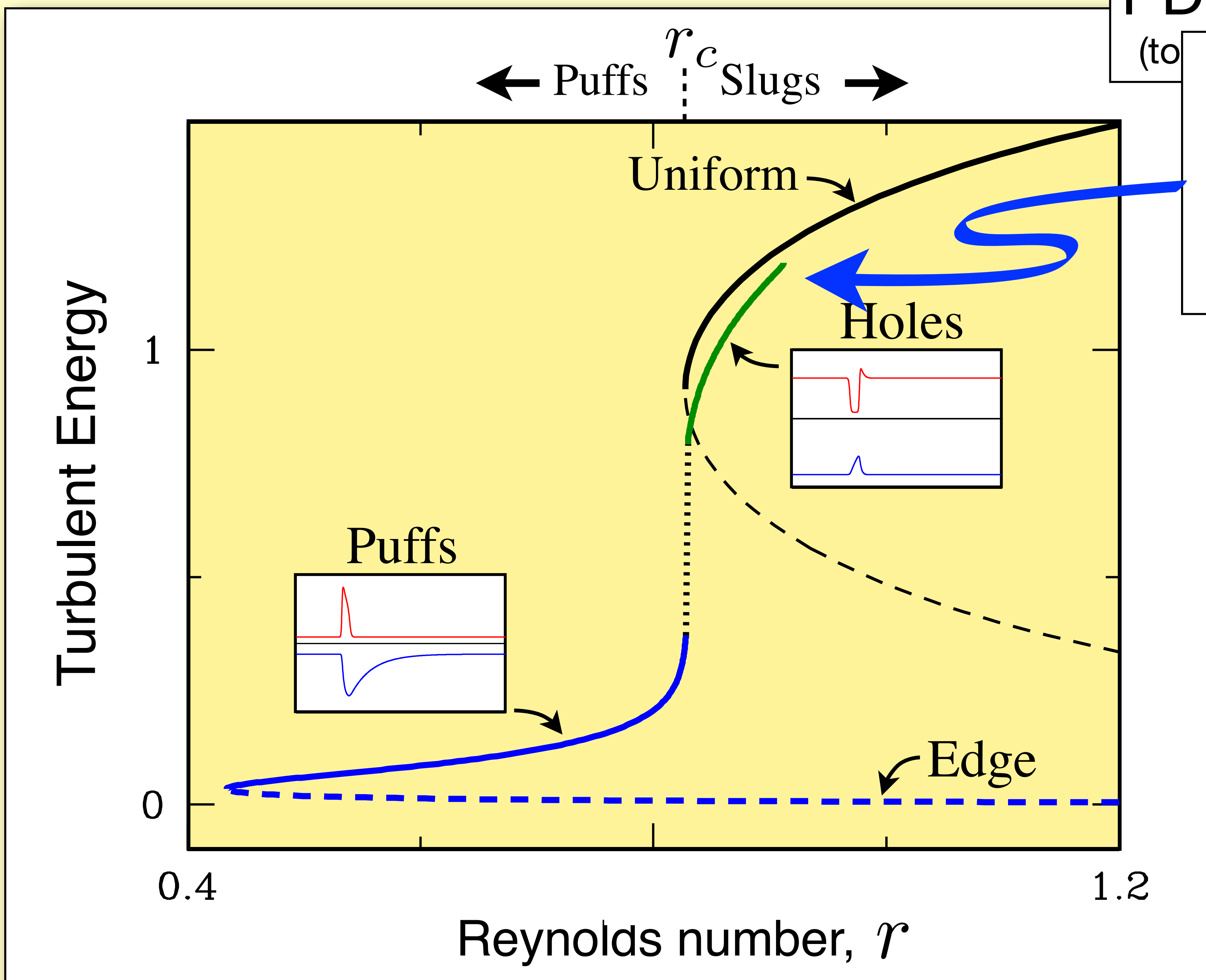
PDE Model

(to

This branch sets transition to uniform turbulence

Transition to Uniform Turbulence

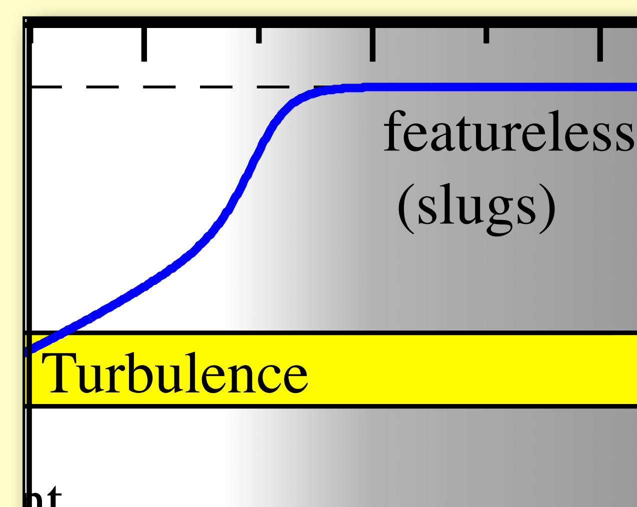
Bifurcation Diagram



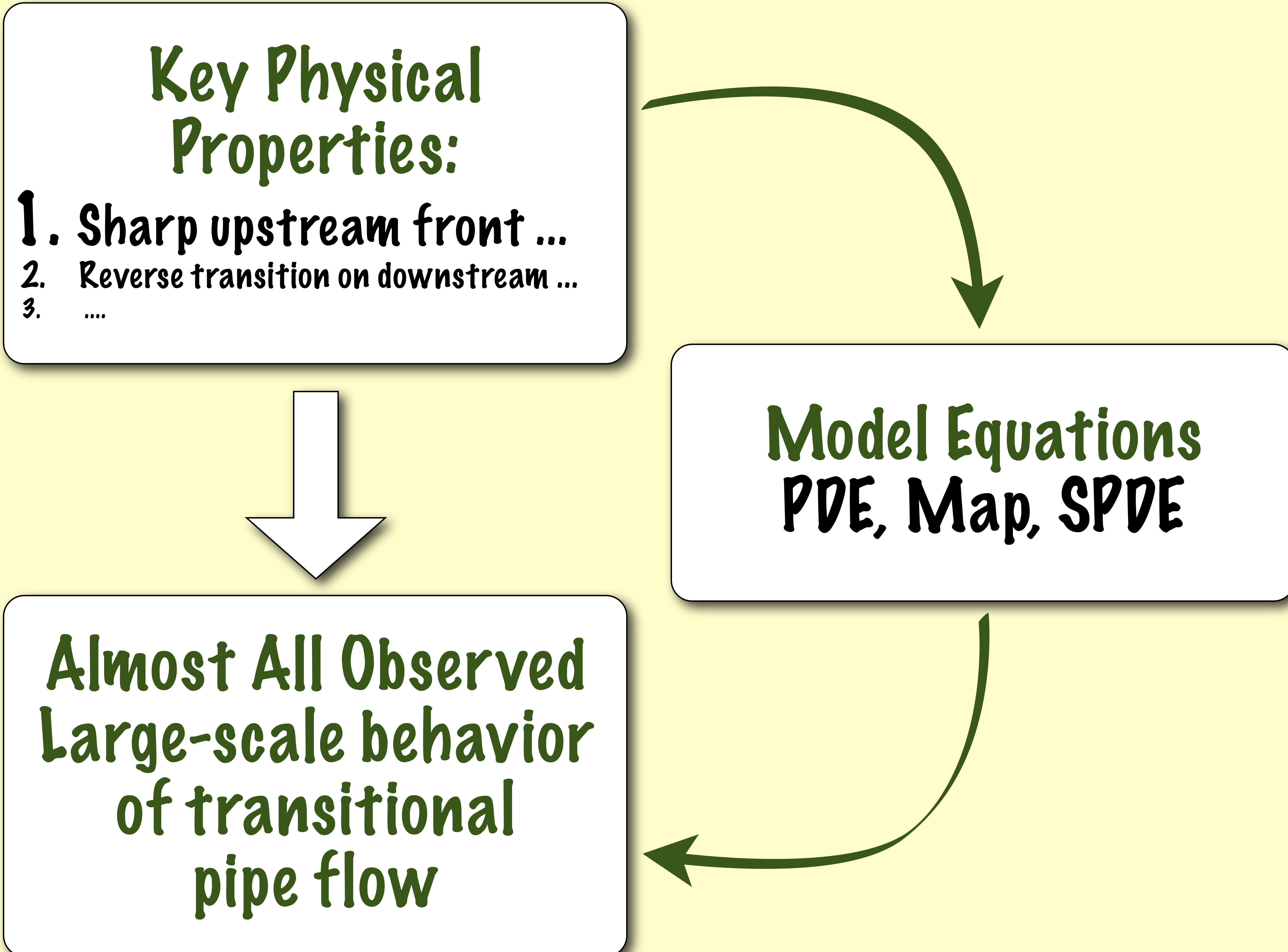
PDE Model
(to

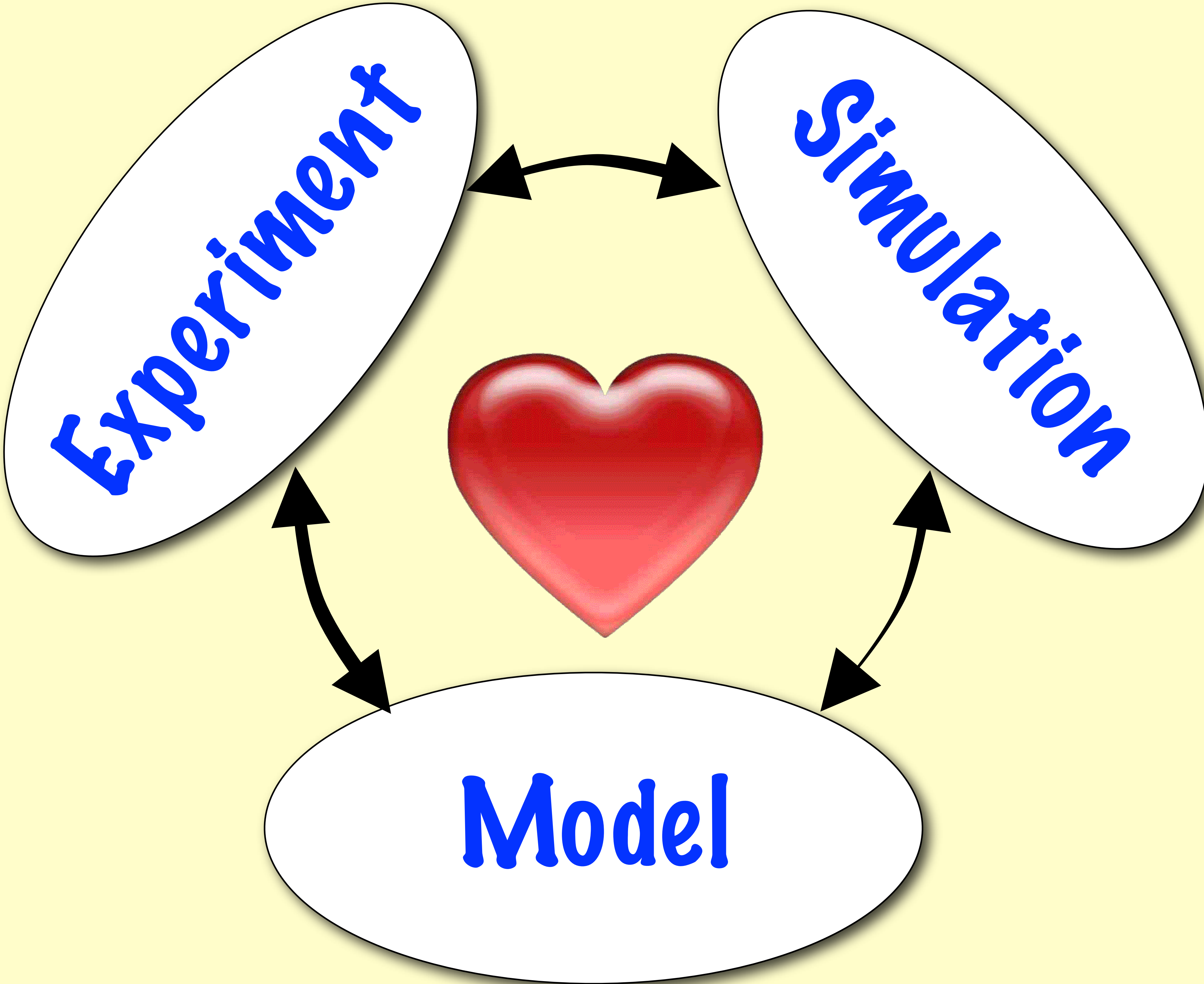
This branch sets transition to uniform turbulence

For complex turbulence bistabilty goes away and becomes continuous transition to uniform turbulence



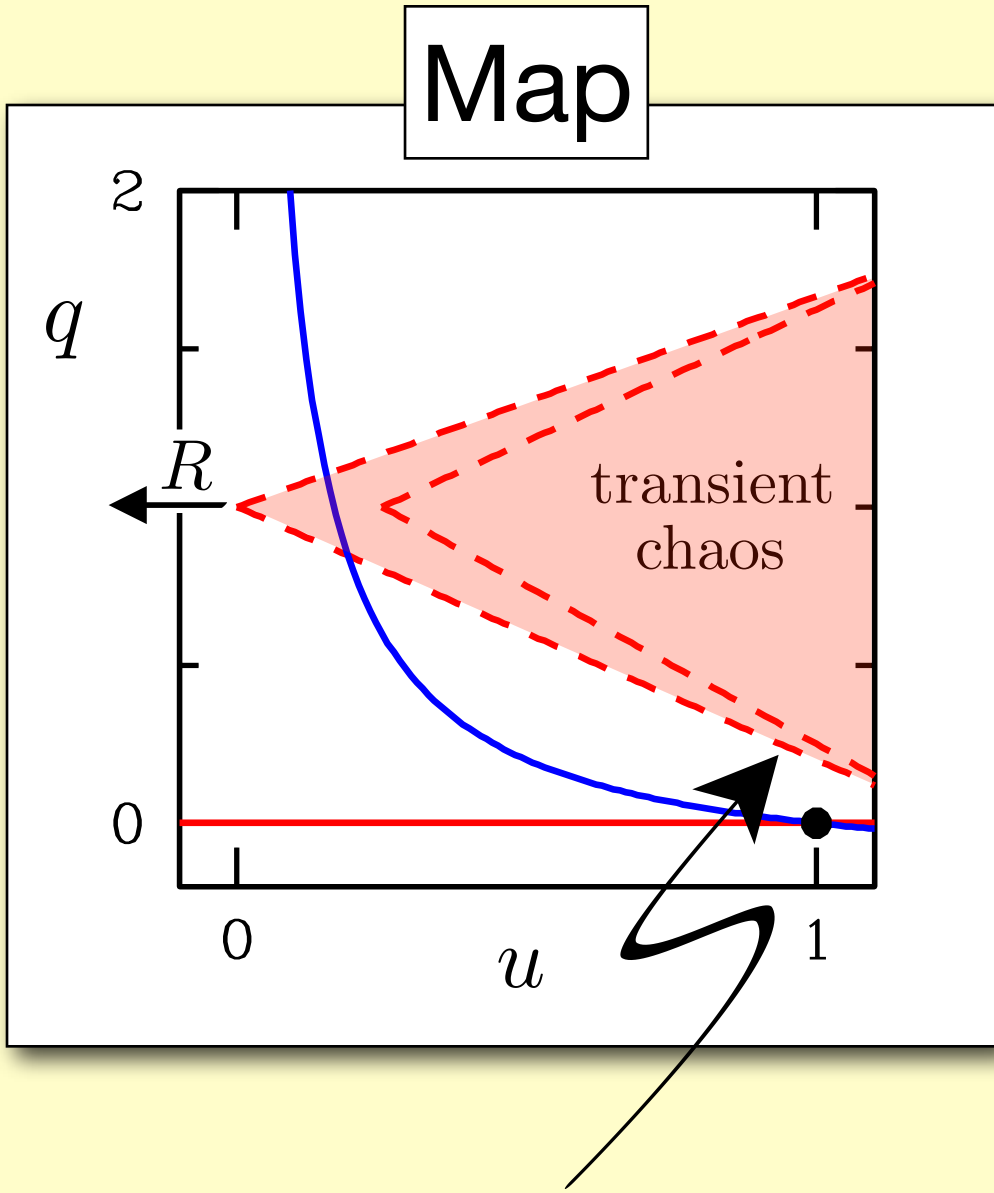
Concluding Remarks


Key Physical Properties:
1. Sharp upstream front ...
2. Reverse transition on downstream ...
3. ....
Model Equations
PDE, Map, SPDE
Almost All Observed Large-scale behavior of transitional pipe flow


Experiment
Simulation
Model

Fractal Basin Boundary

Not enough variables in current model to (naturally) get a fractal basin boundary

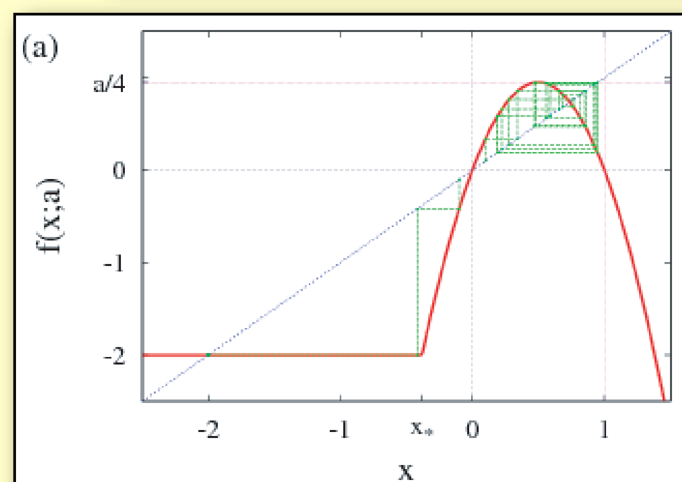


Smooth, not fractal

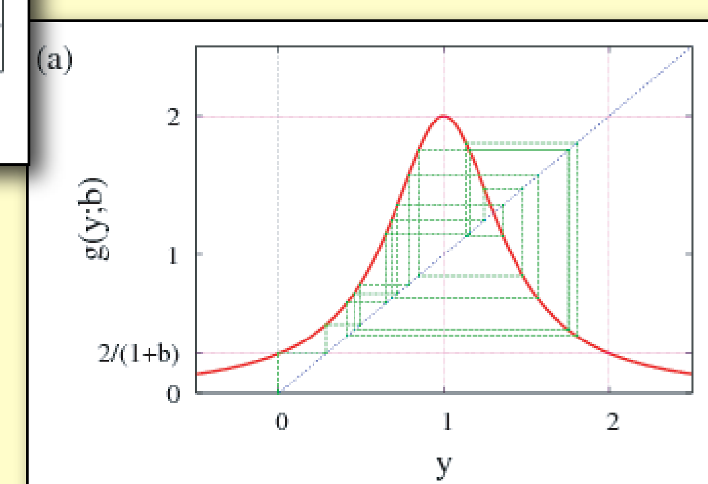
Resolve this by adding second turbulent variable, as in Vollmer *et al*.

Basin boundary, edge of chaos and edge state in a two-dimensional model

Jürgen Vollmer[1,2,3], Tobias M Schneider[2] and Bruno Eckhardt[2]

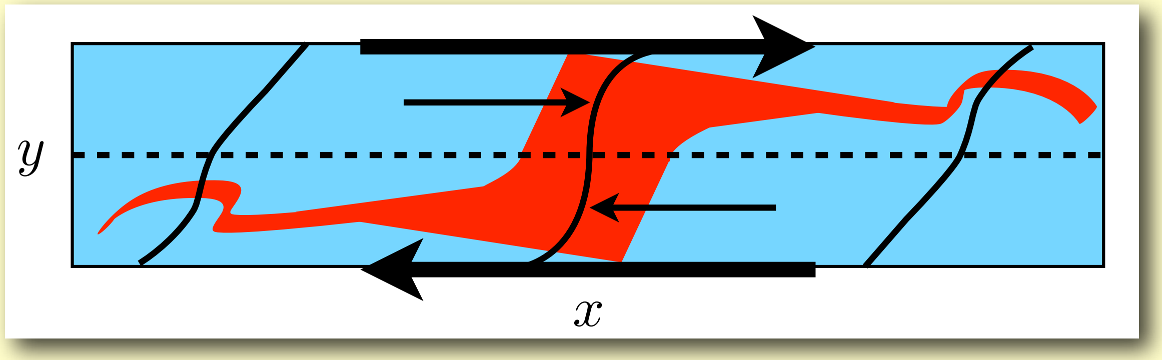


2 variables for turbulence

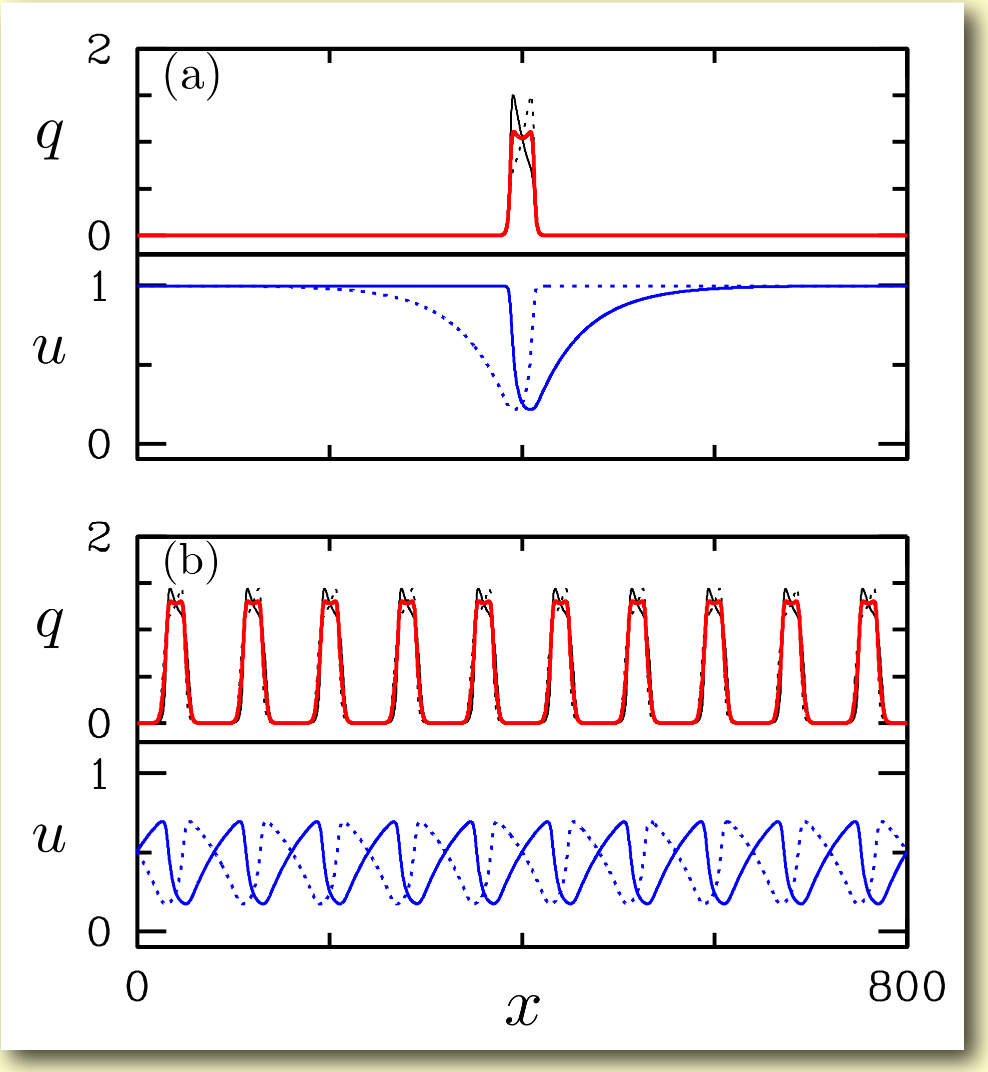


Extension to Other Shear Flows

Limited model of plane Couette flow



Localize and Spatially Periodic Turbulent-Laminar Patterns (See ETC13 Proceedings)



Thanks:

PMMH (Physiqe et Mecanique des Milieux Heterogenes)
L. Tuckerman (PMMH),
Dave Moxey (Warwick),
K. Avila, M. Avila, A. de Lozar, B. Hof (Gottingen)

Other Approaches:

C. Marschler and J. Vollmer (Gottingen)
M. Sipos, N. Goldenfeld (UIUC)
Allhoff, Eckhardt (Marburg)
Alexander Morozov (Edinburgh)

Available Publications (see my web page):

- Moxey and Barkley, PNAS **107**, 8091 (2010)
- Avila, *et al*, Science **333**, 192 (2011)
- Barkley, Phys. Rev. E **84**, 016309 (2011)
- Barkley, proceedings of ETC13
- EZ-Pipe v0.3
- This talk ←