TCC COURSE ON OUTER SPACE AND AUTOMORPHISMS OF FREE GROUPS

EXERCISES 4-5

1. Lecture 4

(1) Show that the sphere system which results from surgery on a simple sphere system is again simple.
(2) We defined a deformation retraction from CV_n to K_n by saying what it does to a single open simplex τ. If τ' is a face of τ show that the retraction of τ extends continuously to the retraction of τ'.

2. Lecture 5

(1) Complete the proof that the upper link $lk_{>\mathcal{S}}$ is a wedge of spheres. What remained was to prove that the subcomplex S_1 deformation retracts to S_0 by adding spheres in order of decreasing size. (See the notes online for the definitions and proof outline)
(2) If G is a connected graph, let $t(G)$ be the number of spheres in the geometric realization of the poset $F(G)$ of forests in G.
   (a) If e is an edge of G which is non-separating, show that $t(e) = t(G-e)+t(G_e)$.
   (b) Use this to calculate the number of spheres in $|F(G)$ for a particular graph of your choice.
   (c) Give an upper bound on the number of spheres in $F(G)$ for a trivalent graph G of rank n.