

TCC COURSE ON OUTER SPACE AND AUTOMORPHISMS OF FREE GROUPS

EXERCISES 2

Whitehead's algorithm

- (1) In the proof of the cut vertex lemma in the online notes, what happens if $\phi(A)$ doesn't intersect A, B or C ? Can you still find a cut vertex?
- (2) In the online notes there is a sketch of a basepointed version of Whitehead's cut vertex lemma. Use this version to check that $a \mapsto c, b \mapsto a^{-1}bc^{-1}acb^{-1}a$ is not an automorphism of F_3 .
- (3) Use Whitehead's algorithm to decide whether each of the following is an automorphism of F_3 :
 - (a) $a \mapsto ab, b \mapsto ba^{-1}, c \mapsto cab$.
 - (b) $a \mapsto ab, b \mapsto bcab, c \mapsto b^{-1}a^{-1}c$.
 - (c) $a \mapsto u, b \mapsto v, c \mapsto w$ for three words u, v, w of your choosing.
- (4) (**) Suppose u_1, \dots, u_k and w_1, \dots, w_k are any words in F_n . Extend Whitehead's algorithm to decide whether there is an automorphism of F_n sending u_i to w_i for all i .

Stallings folds

- (1) Prove that if a graph morphism f from a graph X to a rose Y is a homotopy equivalence which is also locally injective, then it is a homeomorphism. (This is sketched in the online notes when Y is a rose).
- (2) Use Stallings folds to factor the automorphism $a \mapsto cba, b \mapsto a^{-1}c^{-1}ba, c \mapsto ca$ into a product of ρ_{xy} 's and λ_{xy} 's