TCC COURSE ON OUTER SPACE AND
AUTOMORPHISMS OF FREE GROUPS

EXERCISES

(1) In class we showed that the $\phi \in Out(F_3)$ sending $x \mapsto y, y \mapsto z, z \mapsto xy$ cannot be
realized on the surface $S_{1,2}$ of genus 1 with 2 punctures.
   (a) Show it also can't be realized on $S_{0,4}$ or on either non-orientable surface with
       fundamental group F_3.
   (b) Show that the eigenvalues of a surface homeomorphism are either 1 or come
       in pairs (λ, λ^{-1}) with $|\lambda| > 1$. This is a little tricky, to see how to do it look
       at Stallings' article on "Unrealizable automorphisms..."

(2) Let X be a finite graph.
   (a) Show that the group $\pi_0(Homeo(X))$ of homotopy classes of homeomorphisms
       of X is a finite group.
   (b) Show that the group $\pi_0(HE(X))$ of homotopy classes of homotopy equiva-
       lences of X is isomorphic to $Out(F_n)$.