

The Assouad dimension of self-similar sets with overlaps

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Abstract

Historically, the Assouad dimension has been important in the study of quasi-conformal mappings and embedding problems, but recently it has also been gaining some notoriety in the realm of fractal geometry. In general it is an upper bound for the Hausdorff dimension, but it is easily seen that these dimensions coincide for Euclidean self-similar sets which satisfy the open set condition. In 2011 Olsen asked if this remained true even if the defining iterated function system had non-trivial overlaps. In my PhD thesis I answered this question in the negative by constructing a self-similar subset of the unit interval which had Assouad dimension one, but whose Hausdorff dimension could be made arbitrarily small. In more recent work in collaboration with Henderson (Reno, Nevada), Olson (Reno, Nevada) and Robinson (Warwick), we have been able to provide the following precise dichotomy. Consider a self-similar set in the line. Either the weak separation condition of Zerner is satisfied, in which case the Hausdorff and Assouad dimensions coincide, or the weak separation condition is not satisfied, in which case the Assouad dimension is one, independent of the Hausdorff dimension. In this talk I will discuss this result and its proof and also some examples in higher dimensions which exhibit different types of behaviour.

References

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