Averaging and Homogenization in Deterministic and Stochastic Systems. CIRM 11–15 May

Abstracts

- BAILLEUL Ismaël (Université Rennes 1)

  Rough flows and homogenization of fast-slow systems

  Abstract. I will explain in this talk how the recently introduced machinery of approximate flows allows to set up a framework, called "rough flows", where an easy integration theory for time-dependent vector fields is available. This framework encompasses the theory of rough differential equations and stochastic flows, and provides a unifying setting where to prove a number of homogenization results for fast-slow dynamics.

- BALADI Viviane (ENS Paris)

  Linear response: a survey of rigorous results

  Abstract. Consider a one-parameter family of maps $t \mapsto F_t(x)$ so that F_t admits a physical (SRB) invariant probability measure m_t (for all – or many – t close to zero), one asks whether $t \mapsto m_t$ inherits the differentiability of $t \mapsto F_t$. In the affirmative, one would like to have a formula (called linear response formula) for the derivative of m_t as a function of the parameter. Ruelle solved the problem for smooth Anosov diffeomorphism almost 20 years ago, giving the hope that a linear response formula would hold when the SRB measure is connected to the fixed point of a transfer operator enjoying a spectral gap. We shall present various results from the past decade showing that the spectral gap is not sufficient to get linear response. Very recent joint work in progress with Michael Todd indicate that the spectral gap may not be necessary either.

- BASILE Giada (Sapienza Universita di Roma)

  Stationary temperature profile in one dimensional chains of oscillators

  Abstract. We consider a chain of harmonic oscillators whose dynamics is perturbed by a stochastic noise conserving energy and momentum, in contact with two stochastic Langevin heat baths at different temperatures. We look at the evolution of the energy profile under a proper rescaling of space and time. We will discuss our conjecture on the stationary profile. Joint work (in progress) with T. Komorowski and S. Olla.

- BERNARDIN Cedric (University of Nice)

3/4 fractional superdiffusion of energy in a harmonic chain with bulk noise

Abstract. We consider a harmonic chain perturbed by an energy conserving noise and show that after a space-time rescaling the energy-energy correlation function is given by the solution of a skew-fractional heat equation with exponent 3/4. We also investigate an interpolation microscopic model which makes the bridge between the heat equation and the skew-fractional heat equation.

- DE SIMOI Jacopo (University of Toronto)

  Fast-slow partially hyperbolic systems: beyond averaging

  Abstract. We obtain a local central limit theorem and large deviation estimates for fast-slow partially hyperbolic systems. We use these results to prove exponential decay of correlation for an open class of fast-slow partially hyperbolic systems. Such systems are perturbations of non-ergodic ones and the LCLT is of crucial importance to obtain a near-optimal bound on the rate of decay of correlation as the perturbation parameter tends to 0. Additionally, Large Deviations estimates allow to relate the deterministic dynamics to Freidlin Wentzell-type processes. This is a joint project with C. Liverani.

- DEMERS Mark (Fairfield University)

  Exponential mixing for Sinai billiard flows

  Abstract. While billiard maps for large classes of dispersing billiards are known to enjoy exponential decay of correlations, the corresponding flows have so far resisted such analysis. We describe recent results, based on the construction of function spaces on which the associated transfer operator has good spectral properties, which provide a description of the spectrum of the generator of the semi-group. This construction, together with a Dolgopyat-type cancellation argument to eliminate certain eigenvalues, prove that the generator has a spectral gap and that the flow has exponential decay of correlations. This is joint work with V. Baladi and C. Liverani.

- FRIZ Peter (TU and WIAS Berlin)

  Some examples of homogenization related to rough paths

- GUBINELLI Massimiliano (Université Paris Dauphine)

  Pathwise regularisation by noise in PDEs

- KELLY David (Courant Institute)

  Fast slow systems with chaotic noise

- KIFER Yuri (Hebrew University)

  An almost sure view of averaging

  Abstract. I will discuss some almost sure limit theorems in the averaging setup.

- KOMOROWSKI Tomasz (M. Curie Sklodowska University)

  Energy transport in an infinite chain of harmonic oscillators with a degenerate noise

  Abstract. We consider a one dimensional infinite chain of harmonic oscillators whose dynamics is perturbed by a stochastic term conserving energy and momentum (in the unpinned case). We prove that in the unpinned case the macroscopic evolution of the energy converges to a fractional diffusion governed by $-|\Delta|^{3/4}$. For a pinned system we prove that energy evolves diffusively, generalizing some of the earlier results. This is a joint work with S. Olla (CEREMADE, Univ. Paris-Dauphine) and M. Jara (IMPA, Rio de Janeiro).

- KORALOV Leonid (University of Maryland)

  Averaging, homogenization, and large deviation results for the study of randomly perturbed dynamical systems

- KOREPANOV Alexey (University of Warwick)

  Perturbed fast-slow systems

  Abstract: We consider a family of skew product fast-slow systems

  $$\begin{aligned} x_{n+1} &= x_n + \varepsilon f(x_n, y_n) \\ y_{n+1} &= T_\varepsilon y_n, \end{aligned}$$

  where T_ε is a family of close (in a certain sense) nonuniformly hyperbolic systems, $x_0 = \xi$ is fixed, and y_0 is distributed according to either an invariant measure ν_ϵ for T_ε, or a reference measure m. We provide a general condition for convergence of discrete trajectories $x_{\lfloor t\varepsilon^{-1} \rfloor}$ to an averaging limit: solution $X(t)$ of a differential equation $\dot{X} = \bar{f}(X)$, $X(0) = \xi$, where $\bar{f}(x) = \nu_0(f(x, y))$.

  Our method applies to families of logistic and Henon maps when parameters vary on the Benedicks-Carleson parameter sets, as well as Lorenz and Lorentz flows, chaotic billiards, Bunimovich stadiums, and other systems. This is a joint work with Zemer Kosloff and Ian Melbourne.

- NANDORI Peter (New York University)

  Local thermal equilibrium for certain stochastic models of heat transport

Abstract. This talk is about nonequilibrium steady states (NESS) of a class of stochastic models in which particles exchange energy with their 'local environments' rather than directly with one another. The physical domain of the system can be a bounded region of $\mathbb{R}^d$ for any dimension d. We assume that the temperature at the boundary of the domain is prescribed and is nonconstant, so that the system is forced out of equilibrium. Our main result is local thermal equilibrium in the infinite volume limit. We also prove that the mean energy profile of NESS satisfies Laplace's equation for the prescribed boundary condition. Our method of proof is duality: by reversing the sample paths of particle movements, we convert the problem of studying local marginal energy distributions at x to that of joint hitting distributions of certain random walks starting from x. This is a joint work with Yao Li and Lai-Sang Young.

- PARDOUX Étienne (Aix-Marseille Univ)

  Semilinear heat equation with highly oscillating random potential

  Abstract. In this talk, I will first present a joint work with M. Hairer, concerning a Wong-Zakai type of result for a semi linear heat equation on the one dimensional torus, driven by a mollified space-time white noise. Due to the fact that the Ito-Stratonovich correction is infinite, we need to renormalize the approximating equation (i.e. to subtract a term which tends to infinity). The proof is based upon the theory of Regularity Structures.

  If we modify the approximation so that the Ito-Stratonovich correction does not explode, we then have (without renormalization) convergence towards a deterministic PDE, which is the homogenized equation, thus generalizing a result of P.-Piatnitski AOP '12 (which was restricted to the bilinear case). We can then go one step further and obtain a Central Limit Theorem. The second part of the talk is work in progress with M. Hairer and A. Piatnitski.

- REY-BELLET Luc (University of Massachusetts)

  Some applications of irreversibility

- RYZHIK Lenya (Stanford University)

  Radiative transport and homogenization for the random Schroedinger equation

- SOUGANIDIS Panagiotis (University of Chicago)

  Scalar conservation laws with rough fluxes

  Abstract. I will discuss the recently developed theory of rough kinetic/entropy solutions for scalar conservation laws with nonlinear rough forcing. I will present a uniqueness result and discuss the long time behavior with rates, the existence

of invariant measures as well as stochastic regularizing effects when the rough paths are Brownian. I will also present convergence results for some numerical approximations with error estimates.

- TOTH Imre Péter (Budapest)

  Averaging in a heat conduction model with collisions of disks and pistons

  Abstract. We consider a Hamiltonian heat conduction model which consists of a lattice of confined billiard disk and pistons, so that disks can only interact with pistons and pistons can only interact with disks. This is motivated by the Gaspard-Gilbert model of heat conduction [1] (also Bunimovich-Liverani-Pellegrinotti-Suhov [2]). The approach is to understand heat conduction in two steps. First, take a a “rare interaction limit”, as geometric parameters are tuned so that most of the collisions are with the fixed scattering walls. In this limit, through an averaging procedure, one obtains a Markov interacting particle system governing the evolution of particle energies. Then, in a second step, the hydrodynamic limit of this interacting particle system can (hopefully) be studied with probabilistic methods.

  In this talk I present the simplest possible setup, in which the first step of this approach - averaging in the rare interaction limit - can be rigorously carried out. The result is a pure jump Markov process (in contrast with the diffusion process obtained in a “weak interaction limit” by Dolgopyat and Liverani [3]).

  [1] Gaspard P, Gilbert T. Heat conduction and Fourier’s law in a class of many particle dispersing billiards. New Journal of Physics 10 103004 (2008)

  [2] Bunimovich L; Liverani C; Pellegrinotti A; Suhov Y. Ergodic systems of n balls in a billiard table. Comm. Math. Phys. 146 (1992), no. 2, 357–396.

  [3] Dolgopyat D, Liverani C. Energy transfer in a fast-slow Hamiltonian system. Comm. Math. Phys. 308 (2011) 201-225.