AAT Workshop List of Abstracts

- Eric Babson, *TBA*.

- Pavle Blagojevic, *"On the extended Vassiliev conjecture"*.

  We present new upper bounds for the height of elements in the cohomology of the unordered configuration space $H^*(\mathrm{Conf}_n(\mathbb{R}^d)/\mathfrak{S}_n;\mathbb{F}_p)$ with coefficients in the field $\mathbb{F}_p$. In the special case when d is a power of 2 and $p = 2$ we settle the original Vassiliev conjecture by proving that $\mathrm{height}(H^*(\mathrm{Conf}_n(\mathbb{R}^d)/\mathfrak{S}_n;\mathbb{F}_2)) = d$.

  As applications of these results we obtain new lower bounds for the existence of complex k-regular maps as well as for complex ℓ-skew maps $\mathbb{C}^d \to \mathbb{C}^N$.

  (This is joint work with F. Cohen, W. Lück, G. M. Ziegler)

- Alexander Dranishnikov, *"The LS-category of the product of lens spaces"*.

  It is well-known that the Lusternik-Schnirelmann category of an n-dimensional lens space L_p equals n. What is the category of the product of two n-dimensional lens spaces L_p and L_q with relatively prime p and q?

  The cup-length gives $n+1$ for the lower bound. Is $n+1$ the upper bound as well? We connect this question to the following:

  CONJECTURE (Yu. Rudyak) A degree one map between closed manifolds cannot raise the category.

- J.M. García Calcines, *"Fibrewise sectional category"*.

  We introduce the notion of fibrewise sectional category via a Whitehead-Ganea construction. Fibrewise sectional category is the analogue of the ordinary sectional category in the fibrewise setting and also the natural generalization of the fibrewise unpointed LS category in the sense of Iwase-Sakai, and therefore of the topological complexity of Farber. On the other hand the fibrewise pointed version is the generalization of the fibrewise pointed LS category in the sense of James-Morris, and therefore of monoidal topological complexity of Iwase-Sakai. Fibrewise pointed sectional category is also an extension of the relative category in the sense of Doeraene-El Haouari. After giving the main properties for the pointed and unpointed fibrewise sectional category we also establish a comparison between such two versions. We remark a theorem that gives sufficient conditions so that the unpointed and pointed versions agree. As corollaries we obtain the known corresponding results for topological complexity and the monoidal topological complexity given by Dranishnikov; also for sectional category and relative category given by Doeraene-El Haouari.

- Robert Ghrist, *"Poincaré Duality in Network Flow Optimization"*.

One of the classical cornerstones of optimization theory is LP (linear programming) duality, and one of its simplest applications is to the classical max-flow-min-cut theorem, which expresses a duality between optimal network flow values and optimal cut capacities. This talk argues that flow-cut duality is really topological in nature – an expression of Poincaré duality. A recent proof by S. Krishnan shows that Poincaré duality on sheaves of semimodules implies a sheaf-theoretic extension of the classical max-flow-min-cut theorem. This greatly expands the types of problems to which flow-cut dualities can be applied, as well as prompting computational challenges. Details and several examples will be given.

- Marek Golasínski, *"Free actions of discrete groups"*.

  Let $G \times \Sigma(1) \to \Sigma(1)$ be a free, properly discontinuous and cellular action of a group G on a finite dimensional CW-complex $\Sigma(1)$ that has the homotopy type of the circle. We determine all virtually cyclic groups G that act on $\Sigma(1)$ together with the induced action $G \to \mathrm{Aut}(H^1(\Sigma(1), \mathbb{Z}))$, and we classify the orbit spaces $\Sigma(1)/G$.

  Then, we study the same questions for certain families of groups. First, we consider the family of groups with vcd ≤ 1 which includes semi-direct products $\mathbb{Z}_n \rtimes F$ and $F \rtimes \mathbb{Z}_n$ and amalgamated products of finite groups with bounded orders since these groups have vcd $= 1$. Next, we study locally cyclic groups consisting of subgroups of the rationals $\mathbb{Q}$ with vcd ≤ 2 and subgroups of the quotient $\mathbb{Q}/\mathbb{Z}$ with vcd $= \infty$. The results obtained depend upon the subfamily in question. In particular, for an action of any subgroup of $\mathbb{Q}/\mathbb{Z}$ there is only one orbit space up to homotopy and the induced action on $H^1(\Sigma(1), \mathbb{Z})$ is trivial.

- Jesus González, *"Hopf invariants for sectional category with an application to topological robotics"*.

  We develop a Hopf-invariant viewpoint for studying the sectional category of arbitrary maps. The theory is applied in the study of Farber's topological complexity (TC) of 2-cell complexes. We provide an explanation, based on Hopf invariants, of the equality between $\mathrm{TC}(X)$ and the Lusternik-Schnirelmann category of the cofiber of the diagonal map $X \to X \times X$ when X is either a sphere or the cone of the Blakers-Massey map $S^6 \to S^3$. This is joint work with Mark Grant.

- Rocio González-Diaz, *"Real-time human activity monitoring using persistent homology"*.

  The powerful tool known as persistent homology is adapted to deal with human recognition and action detection. The algorithm developed to obtain all this information starts with a stack of human silhouettes, extracted by background subtraction and thresholding from a video sequence. These stacks are glued through their gravity centers, forming a 3D digital binary image I. The 3D image is then converted into a cubical complex $K(I)$. Different filtrations (orderings of the cells of $K(I)$) are then considered which capture relations among the parts of the human body when walking/running. The measure cosine is used to give a similarity value between topological signatures. This measure is invariant to the number of steps considered to built I and provides a fast and effective way to compare the signatures. We will show several experiments done for gait-based recognition (recognition of a person "by the way he/she walks"), gender classification, carrying detection and walking/running action detection.

- MARK GRANT, *"A mapping theorem for topological complexity"*.

  This talk will discuss new lower bounds for the topological complexity, obtained in joint work with Greg Lupton and John Oprea. If $f : Y \to X$ and $g : Z \to X$ are maps into X such that the induced homomorphisms of homotopy groups are injective and have complementary images, then $\mathsf{cat}(Y \times Z) \leq \mathsf{TC}(X)$. We obtain a similar lower bound for the rational topological complexity, in which the assumption of complementary images can be weakened to trivial intersection. Our results can be used to deduce consequences for the global rational homotopy structure of finite hyperbolic complexes, leading to some new cases of the Avramov–F' elix conjecture.

  In the setting of rational sectional category, our method of proof leads to a direct generalization of the mapping theorem of Félix and Halperin, which accounts for the title of the talk.

- MATTHEW KAHLE, *"The threshold for integer homology"*.

  Linial and Meshulam introduced the topological study of random simplicial complexes, and found the vanishing threshold for homology with field coefficients. In new work, Hoffman, Paquette, and I show that the vanishing threshold for integer coefficients is the same, at least up to a constant factor. I will discuss these new techniques and mention a few open problems.

- GIORGI KHIMSHIASHVILI, *TBA*.

- VITALIY KURLIN, *"A fast and robust algorithm to count topologically persistent holes in noisy clouds"*.

  Preprocessing a 2D image often produces a noisy cloud of interest points. We study the problem of counting holes in noisy clouds in the plane. The holes in a given cloud are quantified by the topological persistence of their boundary contours when the cloud is analyzed at all possible scales. We design the algorithm to count holes that are most persistent in the filtration of offsets (neighborhoods) around given points. The input is a cloud of n points in the plane without any user-defined parameters. The algorithm has a near linear time and a linear space. The output is the array (number of holes, relative persistence in the filtration). We prove theoretical guarantees when the algorithm finds the correct number of holes (components in the complement) of an unknown shape approximated by a cloud.

- Roy Meshulam, *"Uncertainty principles and sum complexes"*.

  Uncertainty type inequalities reflect quantitative aspects of the general principle that a nonzero function and its Fourier transform cannot both be sharply localized. In this talk we'll describe a link between discrete uncertainty inequalities on the cyclic group $\mathbb{C}_p$ and the topology of certain arithmetically defined simplicial complexes called sum complexes. The main ingredient in the proofs is the determination of the homology of sum complexes with arbitrary field coefficients. The computation depends on some properties of generalized Vandermonde determinants over the modular group algebra $\mathbb{F}_p[C_p^k]$ and involves Schur functions.

- Tahl Nowik, *"Complexity of curves and knot diagrams"*.

  We will be interested in the number of singular moves required for passing from one planar or spherical curve to another. Similarly, we will be interested in the number of Reidemeister moves required for passing from one knot diagram to another, or for splitting a diagram of a split link. We present invariants of curves, of knot diagrams, and of link diagrams, with which we establish lower bounds for the number of such moves. Upper bounds are established, on the other hand, by presenting explicit sequences of moves.

- Petar Pavesic, *"Incremental stability of persistence modules"*.

  We associate to every persistent module M a diagram of increments that provides a group valued measure of the variation of M in the neighborhood of its critical points. We then prove general stability theorems for increments of interleaved persistence modules.

- Dirk Schütz , *"Intersection homology of linkage spaces in odd dimensional Euclidean space"*.

  We consider the moduli spaces $\mathcal{M}_d(\ell)$ of a closed linkage with n links and prescribed lengths $\ell \in \mathbb{R}^n$ in d-dimensional Euclidean space. For $d > 3$ these spaces are no longer manifolds generically, but they have the structure of a pseudomanifold.

  We use intersection homology to assign a ring to these spaces that can be used to distinguish the homeomorphism types of $\mathcal{M}_d(\ell)$ for a large class of length vectors. These rings behave rather differently depending on whether d is even or odd. In this talk we will focus on the case where $d \geq 5$ is odd, with the main difference being the existence of an extra generator in the ring which can be thought of as an Euler class of a stratified bundle. We will also highlight the similarities to the case $d = 3$, which has been known for some time.

- LUCILE VANDEMBROUCQ, *"On the topological complexity of a two-cell complex "*.

  It is well-known that the Farber topological complexity of a space X is less than or equal to the Lusternik-Schnirelmann category of the product $X \times X$ and that the strict inequality can occur. In this talk, I will give a sufficient condition for a space of the form $X = S^p \cup_\alpha e^q$ to satisfy TC(X) < cat(X × X). This condition will be given in terms of a Hopf invariant of the attaching map α.

- SERGEY YUZVINSKY, *"Higher topological complexity of hyperplane arrangement complements"*.

  Topological complexity $\mathrm{TC}(X)$ of a topological space X was defined by M. Farber about 10 years ago as a specialization of the Schwarz genus. About 5 years ago, Yu. Rudyak extended Farber's definition to higher (s^{th}) topological complexity $\mathrm{TC}_s(X)$ which coincides with $\mathrm{TC}(X)$ for $s = 2$. One of common features of these invariants is a lower bound (depending on s) determined by $H^*(X)$.

  For X which is the complement of a complex arrangement of hyperplanes there were previous attempts to calculate $\mathrm{TC}(X)$. They turned out to be successful for some particular classes of arrangements such as Coxeter infinite series (Farber and Yu) and general position arrangements (D. Cohen, Yu). These examples prompted Conjecture that for all arrangement complements TC coincides with the cohomological low bound. In a recent paper by J. Gonzalez and M. Grant, TC_s was computed for every s for the Coxeter series of type A.

  In the talk we will give a simple combinatorial condition that allows us to compute TC_s for a wide class of arrangements (including all complex reflection arrangements and general position arrangements). In all arrangements of this class the value of TC_s coincides with the cohomological low bound.

- RADE T. ŽIVALJEVIĆ, *"Configuration spaces in applied and computational topology"*.

  One of the central paradigms for applying topological methods in discrete geometry and combinatorics is based on the so called Configuration Space/Test Map scheme (see R.T. Živaljević. Topological methods. Chapter 14 in *Handbook of Discrete and Computational Geometry*, J.E. Goodman, J. O'Rourke, eds, Chapman & Hall/CRC 2004). Configuration spaces arising in these applications are typically arrangements of geometric objects (points, lines, flags, convex polytopes), combinatorial objects (trees, graphs, partitions), or even more frequently spaces parameterizing configurations of mixed, geometric and combinatorial type (e.g. partitions convex polytopes with allocation functions, Lovász graph complexes, etc.). A good illustration is the configuration space of all binary trees of height d, with $2^d - 1$ internal nodes labelled by oriented hyperplanes in $\mathbb{R}^n$, used by Gromov for the proof of his Borsuk-Ulam type theorem, as a topological tool needed for the proof of his celebrated Waist of the Sphere theorem. Another example is the configuration space of 'Voronoi polyhedral partitions', associated to classical Fadell-Neuwirth configuration spaces, introduced by Aronov and Hubard and used (Aronov, Hubard, Soberon, Karasev, Blagojević, Matschke, Ziegler) for proofs of far reaching polyhedral equipartitions of measurable sets (measures).

We review some of the more recent constructions of configurations spaces including the *illumination complexes* and new generalizations of the polyhedral product functor. It is demonstrated how the illumination complexes and their relatives can be used as configuration spaces, leading to new 'fair division theorems'. Among the highlights is the 'polyhedral curtain theorem' (R. Živaljević, arXiv:1307.5138 [math.MG]) which is a relative of both the 'ham sandwich theorem' and the 'splitting necklaces theorem'.