AAT Workshop List of Abstracts

- Michał Adamaszek, *"Vietoris-Rips complexes of circular points"*.

  We study the Vietoris-Rips complexes of subsets of the circle with an arbitrary distance parameter, and we show that they are always homotopy equivalent to an odd sphere or a wedge of even spheres. In particular, this gives the homotopy types of all Vietoris-Rips complexes of S^1, the first such result for a manifold. Our technique is robust and allows us to study the stochastic evolution of the complex when points from S^1 are sampled at random.

  Joint work with Henry Adams, Corrine Previte and Christopher Peterson.

- Manuel Amann, *"Computational complexity of topological invariants"*. It is an interesting question to determine how complicated it is to actually compute topological invariants of certain spaces from a suitable algebraic model.

  In this talk I shall approach the following concrete problem of that kind: Given a simply-connected space X with both $H_*(X, \mathbb{Q})$ and $\pi_*(X) \otimes \mathbb{Q}$ being finite-dimensional, what is the computational complexity of an algorithm computing the cup-length and the rational Lusternik–Schnirelmann category of X?

  Basically, by a reduction from the decision problem whether a given graph is k-colourable for $k \geq 3$ it will be shown that even stricter versions of the problem above are **NP**-hard.

- Eric Babson, *"Random triangulations of the two sphere"*.

  We study the local geometry of random triangulations of the two sphere by constructing a foliated space with a measure so that the balls in its leaves are the balls in the triangulations with the measure recording their frequency. This construction distinguishes a class of differential operators for which we study a density of states.

  Joint with: Nathan Hannon and Jerome Kaminker.

- Ibai Basabe, *"Minimum Instructions for Robot Motion on Certain State Spaces"*.

  A topological approach to the problem of finding the minimum number of instructions for robot motion appeared in the early 2000s in the work of topologist Michael Farber. I will present ideas related to the concept of Topological Complexity of robot motion, outline some techniques used to estimate its value and find the value for some topological state spaces.

- Francisco Belchí, *"New soaps: A_∞ persistence"*.

  Persistent Homology can be seen as some soap you can use to clean the glasses you are wearing when looking at your data. In this talk, we will show some of our new soaps – namely, A_∞ persistence, a family of homological invariants that may detect noise beyond what persistent Betti numbers can, through the geometrical interpretation of some A_∞ structures. This allows us to get persistent information related to cup product, Massey products, linking number...

- Pavle Blagojevic, *"On the extended Vassiliev conjecture"*.

  We present new upper bounds for the height of elements in the cohomology of the unordered configuration space $H^*(\mathrm{Conf}_n(\mathbb{R}^d)/\mathfrak{S}_n; \mathbb{F}_p)$ with coefficients in the field $\mathbb{F}_p$. In the special case when d is a power of 2 and $p = 2$ we settle the original Vassiliev conjecture by proving that $\text{height}(H^*(\text{Conf}_n(\mathbb{R}^d)/\mathfrak{S}_n; \mathbb{F}_2)) = d$.

As applications of these results we obtain new lower bounds for the existence of complex k-regular maps as well as for complex ℓ-skew maps $\mathbb{C}^d \to \mathbb{C}^N$.

(This is joint work with F. Cohen, W. Lück, G. M. Ziegler)

- Barbara Di Fabio, *"A stable combinatorial distance for Reeb graphs of surfaces."*

  For every smooth closed manifold M, the Reeb graph labeled on the vertices is an invariant for the equivalence classes of simple Morse functions $f : M \to \mathbb{R}$ induced by the action of the set of self-diffeomorphisms of M.

  Reeb graphs have been used as an effective tool for shape analysis and description tasks since [4]. Their main characteristics are the one-dimensional structure, the high modularity, the low computational cost for their construction, the ability to discriminate shapes indistinguishable by other similar invariants.

  The increasing interest in Reeb graphs as topological shape descriptors has led to propose several methodologies for their comparison to estimate the similarity of the associated shapes. Only recently this problem has been tackled taking into account the property of robustness under function perturbations. As an example, in the preprint [1] a functional distortion distance between Reeb graphs has been proposed, with proven stable and discriminative properties.

  Generalizing to the case of surfaces the techniques developed in [2], we provide a dissimilarity measure for Reeb graphs, that is combinatorial. It is defined as the infimum cost we have to pay to transform a Reeb graph into another by edit operations.

  We prove that this metric is stable under function perturbations, and that, despite its combinatorial nature, coincides with the natural pseudo-distance [3]. Our distance is therefore more discriminative than the bottleneck distance of persistent homology and also than the functional distortion distance between Reeb graphs, when applicable.

  [1] Ulrich Bauer and Xiaoyin Ge and Yusu Wang. "Measuring Distance between Reeb Graphs", arXiv:1307.2839v1 (2013)

  [2] Di Fabio, B. and Landi, C., "Reeb graphs of curves are stable under function perturbations", Mathematical Methods in the Applied Sciences. (2012), **35** 1456–1471.

  [3] Donatini, P. and Frosini, P., "Natural pseudodistances between closed manifolds.", Forum Mathematicum. (2004) **16**, no 5 696–715.

  [4] Shinagawa, Y. and Kunii, T. L. and Kergosien, Y. L., "Surface coding based on Morse theory", IIEEE Computer Graphics and Applications (1991), **11**, no. 5, 66–78.

- Alexander Dranishnikov, *"The LS-category of the product of lens spaces"*.

  It is well-known that the Lusternik-Schnirelmann category of an n-dimensional lens space L_p equals n. What is the category of the product of two n-dimensional lens spaces L_p and L_q with relatively prime p and q?

  The cup-length gives $n + 1$ for the lower bound. Is $n + 1$ the upper bound as well? We connect this question to the following:

  CONJECTURE (Yu. Rudyak) A degree one map between closed manifolds cannot raise the category.

- Florian Frick, *"Tverberg plus constraints."*

  Tverberg type results, named after Helge Tverberg, are concerned with intersection patterns of faces in a simplicial complex when mapped to some Euclidean space. For example, maps of 1-dimensional complexes to the plane are the well-studied area of graph drawings and in particular graph planarity.

We will give completely elementary proofs of Tverberg type results that were believed to require advanced machinery from algebraic topology. Our simplification builds on a combinatorial reduction to the topological Tverberg Theorem, which we use as a "black box."

This is joint work with Pavle V. M. Blagojević and Günter M. Ziegler.

- J.M. García Calcines, *"Fibrewise sectional category"*.

  We introduce the notion of fibrewise sectional category via a Whitehead-Ganea construction. Fibrewise sectional category is the analogue of the ordinary sectional category in the fibrewise setting and also the natural generalization of the fibrewise unpointed LS category in the sense of Iwase-Sakai, and therefore of the topological complexity of Farber. On the other hand the fibrewise pointed version is the generalization of the fibrewise pointed LS category in the sense of James-Morris, and therefore of monoidal topological complexity of Iwase-Sakai. Fibrewise pointed sectional category is also an extension of the relative category in the sense of Doeraene-El Haouari. After giving the main properties for the pointed and unpointed fibrewise sectional category we also establish a comparison between such two versions. We remark a theorem that gives sufficient conditions so that the unpointed and pointed versions agree. As corollaries we obtain the known corresponding results for topological complexity and the monoidal topological complexity given by Dranishnikov; also for sectional category and relative category given by Doeraene-El Haouari.

- Robert Ghrist, *"Poincaré Duality in Network Flow Optimization"*.

  One of the classical cornerstones of optimization theory is LP (linear programming) duality, and one of its simplest applications is to the classical max-flow-min-cut theorem, which expresses a duality between optimal network flow values and optimal cut capacities. This talk argues that flow-cut duality is really topological in nature − an expression of Poincaré duality. A recent proof by S. Krishnan shows that Poincaré duality on sheaves of semimodules implies a sheaf-theoretic extension of the classical max-flow-min-cut theorem. This greatly expands the types of problems to which flow-cut dualities can be applied, as well as prompting computational challenges. Details and several examples will be given.

- Marek Golasiński, *"Free actions of discrete groups"*.

  Let $G \times \Sigma(1) \to \Sigma(1)$ be a free, properly discontinuous and cellular action of a group G on a finite dimensional CW-complex $\Sigma(1)$ that has the homotopy type of the circle. We determine all virtually cyclic groups G that act on $\Sigma(1)$ together with the induced action $G \to \mathrm{Aut}(H^1(\Sigma(1), \mathbb{Z}))$, and we classify the orbit spaces $\Sigma(1)/G$.

  Then, we study the same questions for certain families of groups. First, we consider the family of groups with vcd ≤ 1 which includes semi-direct products $\mathbb{Z}_n \rtimes F$ and $F \rtimes \mathbb{Z}_n$ and amalgamated products of finite groups with bounded orders since these groups have vcd $= 1$. Next, we study locally cyclic groups consisting of subgroups of the rationals $\mathbb{Q}$ with vcd ≤ 2 and subgroups of the quotient $\mathbb{Q}/\mathbb{Z}$ with vcd $= \infty$. The results obtained depend upon the subfamily in question. In particular, for an action of any subgroup of $\mathbb{Q}/\mathbb{Z}$ there is only one orbit space up to homotopy and the induced action on $H^1(\Sigma(1), \mathbb{Z})$ is trivial.

- Jesus González, *"Hopf invariants for sectional category with an application to topological robotics"*.

  We develop a Hopf-invariant viewpoint for studying the sectional category of arbitrary maps. The theory is applied in the study of Farber's topological complexity (TC) of 2-cell complexes. We provide an explanation, based on Hopf invariants, of the equality between $\mathrm{TC}(X)$ and the Lusternik-Schnirelmann category of the cofiber of the diagonal map $X \to X \times X$ when X is either a sphere or the cone of the Blakers-Massey map $S^6 \to S^3$. This is joint work with Mark Grant.

- Rocio González-Diaz, *"Real-time human activity monitoring using persistent homology"*.

  The powerful tool known as persistent homology is adapted to deal with human recognition and action detection. The algorithm developed to obtain all this information starts with a stack of human silhouettes, extracted by background subtraction and thresholding from a video sequence. These stacks are glued through their gravity centers, forming a 3D digital binary image I. The 3D image is then converted into a cubical complex $K(I)$. Different filtrations (orderings of the cells of $K(I)$) are then considered which capture relations among the parts of the human body when walking/running. The measure cosine is used to give a similarity value between topological signatures. This measure is invariant to the number of steps considered to built I and provides a fast and effective way to compare the signatures. We will show several experiments done for gait-based recognition (recognition of a person "by the way he/she walks"), gender classification, carrying detection and walking/running action detection.

- Mark Grant, *"A mapping theorem for topological complexity"*.

  This talk will discuss new lower bounds for the topological complexity, obtained in joint work with Greg Lupton and John Oprea. If $f : Y \to X$ and $g : Z \to X$ are maps into X such that the induced homomorphisms of homotopy groups are injective and have complementary images, then $\mathsf{cat}(Y \times Z) \leq \mathsf{TC}(X)$. We obtain a similar lower bound for the rational topological complexity, in which the assumption of complementary images can be weakened to trivial intersection. Our results can be used to deduce consequences for the global rational homotopy structure of finite hyperbolic complexes, leading to some new cases of the Avramov–F' elix conjecture.

  In the setting of rational sectional category, our method of proof leads to a direct generalization of the mapping theorem of Félix and Halperin, which accounts for the title of the talk.

- Albert Haase, *"Equipartitions of Measures by Hyperplanes."*

  Remember the Ham Sandwich Theorem? It states that every sandwich with measurable layers of bread, cheese, and ham can be cut in half by an affine hyperplane. If the measures are sufficiently nice, this result can be proved using the Borsuk–Ulam Theorem. More generally, given integers m and h, we can ask for the minimal dimension $d = \Delta(m, h)$ such that for any m nice probability measures on $\mathbb{R}^d$ there are h affine hyperplanes defining 2^h cells that capture the same fraction $\frac{1}{2^h}$ of each of the measures.

  Branko Grnbaum raised this question in 1960. While some upper and lower bounds for $\Delta(m, h)$ have been established, only few exact values for the function $\Delta(m, h)$ are known. In my talk I will survey results obtained using configuration and test map schemes and explain some of the problems that arise when trying to directly apply typical Borsuk–Ulam type arguments for general m and h. This is reflected in the fact that some of the results claimed in the literature still lack complete proofs.

- Matthew Kahle, *"The threshold for integer homology"*.

  Linial and Meshulam introduced the topological study of random simplicial complexes, and found the vanishing threshold for homology with field coefficients. In new work, Hoffman, Paquette, and I show that the vanishing threshold for integer coefficients is the same, at least up to a constant factor. I will discuss these new techniques and mention a few open problems.

- Giorgi Khimshiashvili, *"Geometry and control of polygonal linkages"*.

  We will discuss a natural cell decomposition of the planar configuration space of polygonal linkage constructed by G.Panina, and its applications to the navigation and control problems for planar linkages. Special attention will be given to the vertex-edge graph of this decomposition which is crucial for solving the navigation problem. The resulting navigation algorithm involves not more than 14 steps each of which is a disguised flex of a quadrilateral linkage. It will also be shown that complete control of quadrilateral linkage can be achieved by using the Coulomb potential of controlled point charges placed at its vertices. We will also show that combining the latter result with the navigation algorithm one can obtain a universal control algorithm for planar polygonal linkages with arbitrary number of vertices.

- VITALIY KURLIN, *"A fast and robust algorithm to count topologically persistent holes in noisy clouds"*.

  Preprocessing a 2D image often produces a noisy cloud of interest points. We study the problem of counting holes in noisy clouds in the plane. The holes in a given cloud are quantified by the topological persistence of their boundary contours when the cloud is analyzed at all possible scales. We design the algorithm to count holes that are most persistent in the filtration of offsets (neighborhoods) around given points. The input is a cloud of n points in the plane without any user-defined parameters. The algorithm has a near linear time and a linear space. The output is the array (number of holes, relative persistence in the filtration). We prove theoretical guarantees when the algorithm finds the correct number of holes (components in the complement) of an unknown shape approximated by a cloud.

- ROY MESHULAM, *"Uncertainty principles and sum complexes"*.

  Uncertainty type inequalities reflect quantitative aspects of the general principle that a nonzero function and its Fourier transform cannot both be sharply localized. In this talk we'll describe a link between discrete uncertainty inequalities on the cyclic group $\mathbb{C}_p$ and the topology of certain arithmetically defined simplicial complexes called sum complexes. The main ingredient in the proofs is the determination of the homology of sum complexes with arbitrary field coefficients. The computation depends on some properties of generalized Vandermonde determinants over the modular group algebra $\mathbb{F}_p[C_p^k]$ and involves Schur functions.

- TAHL NOWIK, *"Complexity of curves and knot diagrams"*.

  We will be interested in the number of singular moves required for passing from one planar or spherical curve to another. Similarly, we will be interested in the number of Reidemeister moves required for passing from one knot diagram to another, or for splitting a diagram of a split link. We present invariants of curves, of knot diagrams, and of link diagrams, with which we establish lower bounds for the number of such moves. Upper bounds are established, on the other hand, by presenting explicit sequences of moves.

- PETAR PAVESIC, *"Incremental stability of persistence modules"*.

  We associate to every persistent module M a diagram of increments that provides a group valued measure of the variation of M in the neighborhood of its critical points. We then prove general stability theorems for increments of interleaved persistence modules.

- SALMAN PARSA, *"On the complexity of computing Betti numbers and Homology"*.

  In this talk I try to present an answer to the following question: How fast one can compute Betti numbers of a simplicial complex? It is known that computing Betti numbers can be done by computing ranks of a constant number of sparse boundary matrices whose dimensions is in the order of the size of the complex. We tried to show that this is the best possible. In other words, given any (sparse) matrix, one can build a 2-complex in linear time such that from its Betti numbers the rank can be computed in linear time. This work was motivated by trying to find a faster algorithm for Betti numbers of complexes which embed in Euclidean 4-space.

- MARÍA JOSÉ PEREIRA, *"LS category of the Quaternionic Projective Space"*.

- DIRK SCHÜTZ , *"Intersection homology of linkage spaces in odd dimensional Euclidean space"*.

We consider the moduli spaces $\mathcal{M}_d(\ell)$ of a closed linkage with n links and prescribed lengths $\ell \in \mathbb{R}^n$ in d-dimensional Euclidean space. For $d > 3$ these spaces are no longer manifolds generically, but they have the structure of a pseudomanifold.

We use intersection homology to assign a ring to these spaces that can be used to distinguish the homeomorphism types of $\mathcal{M}_d(\ell)$ for a large class of length vectors. These rings behave rather differently depending on whether d is even or odd. In this talk we will focus on the case where $d \geq 5$ is odd, with the main difference being the existence of an extra generator in the ring which can be thought of as an Euler class of a stratified bundle. We will also highlight the similarities to the case $d = 3$, which has been known for some time.

- Lucile Vandembroucq, *"On the topological complexity of a two-cell complex "*.

  It is well-known that the Farber topological complexity of a space X is less than or equal to the Lusternik-Schnirelmann category of the product $X \times X$ and that the strict inequality can occur. In this talk, I will give a sufficient condition for a space of the form $X = S^p \cup_\alpha e^q$ to satisfy $\mathrm{TC(X)} < \mathrm{cat(X \times X)}$. This condition will be given in terms of a Hopf invariant of the attaching map α.

- Sergey Yuzvinsky, *"Higher topological complexity of hyperplane arrangement complements"*.

  Topological complexity $\mathrm{TC}(X)$ of a topological space X was defined by M. Farber about 10 years ago as a specialization of the Schwarz genus. About 5 years ago, Yu. Rudyak extended Farber's definition to higher (s^{th}) topological complexity $\mathrm{TC}_s(X)$ which coincides with $\mathrm{TC}(X)$ for $s = 2$. One of common features of these invariants is a lower bound (depending on s) determined by $H^*(X)$.

  For X which is the complement of a complex arrangement of hyperplanes there were previous attempts to calculate $\mathrm{TC}(X)$. They turned out to be successful for some particular classes of arrangements such as Coxeter infinite series (Farber and Yu) and general position arrangements (D. Cohen, Yu). These examples prompted Conjecture that for all arrangement complements TC coincides with the cohomological low bound. In a recent paper by J. Gonzalez and M. Grant, TC_s was computed for every s for the Coxeter series of type A.

  In the talk we will give a simple combinatorial condition that allows us to compute TC_s for a wide class of arrangements (including all complex reflection arrangements and general position arrangements). In all arrangements of this class the value of TC_s coincides with the cohomological low bound.

- Hubert Wagner, *"Fast persistent homology computation using the BitTree data-structure"*.

  In this talk I focus on computing persistent homology using a modification of the standard matrix reduction algorithm. In particular, I present a new data-structure, called BitTree, which is designed to efficiently handle column additions. BitTree is an efficient implementation of a bitset, supporting fast insertions, deletions, iteration, clearing, maximum and minimum queries. It is used to store an intermediate state of a column during the reduction. Experimental evaluation using the PHAT library shows that this technique speeds up the overall computation by up to a factor of 10, compared to the typical implementations.

- Rade T. Živaljević, *"Configuration spaces in applied and computational topology"*.

  One of the central paradigms for applying topological methods in discrete geometry and combinatorics is based on the so called Configuration Space/Test Map scheme (see R.T. Živaljević. Topological methods. Chapter 14 in *Handbook of Discrete and Computational Geometry*, J.E. Goodman, J. O'Rourke, eds, Chapman & Hall/CRC 2004). Configuration spaces arising in these applications are typically arrangements of geometric objects (points, lines, flags, convex polytopes), combinatorial objects (trees, graphs, partitions), or even more frequently spaces parameterizing configurations of mixed, geometric and combinatorial type (e.g. partitions convex polytopes with allocation functions, Lovász graph complexes, etc.).

A good illustration is the configuration space of all binary trees of height d, with $2^d - 1$ internal nodes labelled by oriented hyperplanes in $\mathbb{R}^n$, used by Gromov for the proof of his Borsuk-Ulam type theorem, as a topological tool needed for the proof of his celebrated Waist of the Sphere theorem. Another example is the configuration space of 'Voronoi polyhedral partitions', associated to classical Fadell-Neuwirth configuration spaces, introduced by Aronov and Hubard and used (Aronov, Hubard, Soberon, Karasev, Blagojević, Matschke, Ziegler) for proofs of far reaching polyhedral equipartitions of measurable sets (measures).

We review some of the more recent constructions of configurations spaces including the *illumination complexes* and new generalizations of the polyhedral product functor. It is demonstrated how the illumination complexes and their relatives can be used as configuration spaces, leading to new 'fair division theorems'. Among the highlights is the 'polyhedral curtain theorem' (R. Živaljević, arXiv:1307.5138 [math.MG]) which is a relative of both the 'ham sandwich theorem' and the 'splitting necklaces theorem'.