AAT Short Talks

- MICHAŁ ADAMASZEK, *"Vietoris-Rips complexes of circular points"*.

  We study the Vietoris-Rips complexes of subsets of the circle with an arbitrary distance parameter, and we show that they are always homotopy equivalent to an odd sphere or a wedge of even spheres. In particular, this gives the homotopy types of all Vietoris-Rips complexes of S^1, the first such result for a manifold. Our technique is robust and allows us to study the stochastic evolution of the complex when points from S^1 are sampled at random.

  Joint work with Henry Adams, Corrine Previte and Christopher Peterson.

- MANUEL AMANN, *"Computational complexity of topological invariants"*. It is an interesting question to determine how complicated it is to actually compute topological invariants of certain spaces from a suitable algebraic model.

  In this talk I shall approach the following concrete problem of that kind: Given a simply-connected space X with both $H_*(X, \mathbb{Q})$ and $\pi_*(X) \otimes \mathbb{Q}$ being finite-dimensional, what is the computational complexity of an algorithm computing the cup-length and the rational Lusternik–Schnirelmann category of X?

  Basically, by a reduction from the decision problem whether a given graph is k-colourable for $k \geq 3$ it will be shown that even stricter versions of the problem above are **NP**-hard.

- IBAI BASABE, *"Minimum Instructions for Robot Motion on Certain State Spaces."*

  A topological approach to the problem of finding the minimum number of instructions for robot motion appeared in the early 2000s in the work of topologist Michael Farber. I will present ideas related to the concept of Topological Complexity of robot motion, outline some techniques used to estimate its value and find the value for some topological state spaces.

- FRANCISCO BELCHÍ, *"New soaps: A_∞ persistence."*

  Persistent Homology can be seen as some soap you can use to clean the glasses you are wearing when looking at your data. In this talk, we will show some of our new soaps – namely, A_∞ persistence, a family of homological invariants that may detect noise beyond what persistent Betti numbers can, through the geometrical interpretation of some A_∞ structures. This allows us to get persistent information related to cup product, Massey products, linking number...

- BARBARA DI FABIO, *"A stable combinatorial distance for Reeb graphs of surfaces."*

For every smooth closed manifold M, the Reeb graph labeled on the vertices is an invariant for the equivalence classes of simple Morse functions $f : M \to \mathbb{R}$ induced by the action of the set of self-diffeomorphisms of M.

Reeb graphs have been used as an effective tool for shape analysis and description tasks since [4]. Their main characteristics are the one-dimensional structure, the high modularity, the low computational cost for their construction, the ability to discriminate shapes indistinguishable by other similar invariants.

The increasing interest in Reeb graphs as topological shape descriptors has led to propose several methodologies for their comparison to estimate the similarity of the associated shapes. Only recently this problem has been tackled taking into account the property of robustness under function perturbations. As an example, in the preprint [1] a functional distortion distance between Reeb graphs has been proposed, with proven stable and discriminative properties.

Generalizing to the case of surfaces the techniques developed in [2], we provide a dissimilarity measure for Reeb graphs, that is combinatorial. It is defined as the infimum cost we have to pay to transform a Reeb graph into another by edit operations.

We prove that this metric is stable under function perturbations, and that, despite its combinatorial nature, coincides with the natural pseudodistance [3]. Our distance is therefore more discriminative than the bottleneck distance of persistent homology and also than the functional distortion distance between Reeb graphs, when applicable.

[1] Ulrich Bauer and Xiaoyin Ge and Yusu Wang. "Measuring Distance between Reeb Graphs", arXiv:1307.2839v1 (2013)

[2] Di Fabio, B. and Landi, C., "Reeb graphs of curves are stable under function perturbations", Mathematical Methods in the Applied Sciences. (2012), **35** 1456–1471.

[3] Donatini, P. and Frosini, P., "Natural pseudodistances between closed manifolds.", Forum Mathematicum. (2004) **16**, no 5 696–715.

[4] Shinagawa, Y. and Kunii, T. L. and Kergosien, Y. L., "Surface coding based on Morse theory", IIEEE Computer Graphics and Applications (1991), **11**, no. 5, 66–78.

- Florian Frick, *"Tverberg plus constraints."*

Tverberg type results, named after Helge Tverberg, are concerned with intersection patterns of faces in a simplicial complex when mapped to some Euclidean space. For example, maps of 1-dimensional complexes to the plane are the well-studied area of graph drawings and in particular graph planarity.

We will give completely elementary proofs of Tverberg type results that were believed to require advanced machinery from algebraic topology. Our simplification builds on a combinatorial reduction to the topological Tverberg Theorem, which we use as a "black box."

This is joint work with Pavle V. M. Blagojević and Günter M. Ziegler.

- ALBERT HAASE, *"Equipartitions of Measures by Hyperplanes."*

  Remember the Ham Sandwich Theorem? It states that every sandwich with measurable layers of bread, cheese, and ham can be cut in half by an affine hyperplane. If the measures are sufficiently nice, this result can be proved using the Borsuk–Ulam Theorem. More generally, given integers m and h, we can ask for the minimal dimension $d = \Delta(m, h)$ such that for any m nice probability measures on $\mathbb{R}^d$ there are h affine hyperplanes defining 2^h cells that capture the same fraction $\frac{1}{2^h}$ of each of the measures.

  Branko Grnbaum raised this question in 1960. While some upper and lower bounds for $\Delta(m, h)$ have been established, only few exact values for the function $\Delta(m, h)$ are known. In my talk I will survey results obtained using configuration and test map schemes and explain some of the problems that arise when trying to directly apply typical Borsuk–Ulam type arguments for general m and h. This is reflected in the fact that some of the results claimed in the literature still lack complete proofs.

- SALMAN PARSA, *"On the complexity of computing Betti numbers and Homology".*

  In this talk I try to present an answer to the following question: How fast one can compute Betti numbers of a simplicial complex? It is known that computing Betti numbers can be done by computing ranks of a constant number of sparse boundary matrices whose dimensions is in the order of the size of the complex. We tried to show that this is the best possible. In other words, given any (sparse) matrix, one can build a 2-complex in linear time such that from its Betti numbers the rank can be computed in linear time. This work was motivated by trying to find a faster algorithm for Betti numbers of complexes which embed in Euclidean 4-space.

- MARÍA JOSÉ PEREIRA, *"LS category of the Quaternionic Projective Space".*

- HUBERT WAGNER, *"Fast persistent homology computation using the BitTree data-structure".*

  In this talk I focus on computing persistent homology using a modification of the standard matrix reduction algorithm. In particular, I present a new data-structure, called BitTree, which is designed to efficiently handle column additions. BitTree is an efficient implementation of a bitset, supporting fast insertions, deletions, iteration, clearing, maximum and minimum queries. It is used to store an intermediate state of a column during the reduction. Experimental evaluation using the PHAT library shows that this technique speeds up the overall computation by up to a factor of 10, compared to the typical implementations.