AAT Advanced Courses

- Pavle Blagojevic, *"Topological methods in Geometric Combinatorics"*.

  Lecture 1: *Some problems of geometric combinatorics*

  This is a gentle introduction into the interplay between geometric combinatorics problems on the one side and the equivariant topology problems on the other side. We present a number of well-known problems from geometric combinatorics: the topological Tverberg conjecture, the Bárány–Larman conjecture, the Grünbaum mass partition problem, the k-fan mass partition problem, and the Nandakumar & Ramana-Rao conjecture. Using different schemes these problems are transformed into questions of equivariant topology. Applying classical results like Borsuk–Ulam or Dold theorem we immediately address some of these problems. For the remaining open problems we seek for more advanced topological tools.

  Lecture 2: *The Fadell–Husseini index theory*

  Using the notions of group cohomology, Borel construction and Serre spectral sequences, we develop the Fadell–Husseini ideal valued index theory in order to answer questions about of the existence of equivariant maps between spaces. Applying this method we obtain the topological Tverberg conjecture for prime powers, give upper bounds for the Grünbaum mass partition problem, establish a solution in a particular case of 4-fan mass partitions, and finally prove the Nandakumar & Ramana-Rao conjecture for primes.

  Lecture 3: *An equivariant obstruction theory*

  In this lecture we present an equivariant obstruction theory, as introduced by tom Dieck. This tool, which can give complete answers to questions about the existence of equivariant maps, is used to show a failure of the methods in the case of the topological Tverberg conjecture for non prime powers, the Bárany–Larman conjecture for primes -1, but also to establish the Nandakumar & Ramana-Rao conjecture for prime powers.

- Alexander Dranishnikov, *"Navigational Complexity of Configuration Spaces"*.

  Michael Farber introduced a topological invariant TC(X) of a space X inspired by problems of robotics. It is a numerical invariant that brought to measure the navigational complexity of X viewed as the configuration space of a system. In these lectures we will discuss the connection of TC(X) with the Lusternik-Schnirelman category cat(X). We will explore the similarities and differences in the difficult problem of computation of TC(X) and cat(X). In particular we will show estimates for both invariants for the wedge, product and twisted product of spaces. We finish the presentation by discussion of new directions and open problems in the area. In particular we will consider the Iwase-Sakai conjecture on monoidal topological complexity and Farber's problem about the topological complexity of a discrete group.

- Robert Ghrist, *"Sheaves and the Topology of Networks"*.

- Roy Meshulam, *"Aspects of Random Complexes"*.

  In recent years there is a growing interest in higher dimensional random complexes, both as natural extensions of random graphs, and as potential tools for new applications, e.g. in topological data analysis. In these talks we shall discuss some results and methodologies of this emerging theory.

Lecture 1: *Topology of random complexes.*

In this survey talk we shall describe several models of random complexes including the k-dimensional Erdős-Rényi model, flag complexes of random graphs, models of bounded degrees complexes and random trees. We shall discuss results and conjectures concerning the topology of typical spaces in these models, including their homology, fundamental group and collapsibility.

Lecture 2: *Cohomological Expansion.*

The notion of expansion in graphs as quantified by the Cheeger constant has been extremely useful throughout mathematics and theoretical computer science. A k-dimensional version of the graphical Cheeger constant, called "cohomological expansion", came up independently in the study of homological connectivity of random complexes and in Gromov's remarkable work on the topological overlap property. In this talk we shall discuss this notion and its relevance to random complexes.

Lecture 3: *Higher Laplacians and Garland's method.*

A celebrated result of Garland (1973) asserts that the real cohomology of a lattice in a p-adic group vanishes below the dimension. A major ingredient in the proof is a method of bounding the spectral gap of higher Laplacians of a complex in terms of the spectral gaps of certain lower dimensional links. In this talk we shall discuss Garland's method and some of its applications to random complexes and to combinatorics.

- Dirk Schütz , *"Topology of configuration spaces"*.

  Lecture 1: *Configuration spaces of Linkages*

  A linkage L can be thought of as a graph such that every edge has a given length. A configuration of a linkage is then given by a length preserving immersion of the graph into a given metric space. In this lecture we introduce various types of linkage spaces given through configurations, and meet basic techniques such as Morse theory to study the algebraic topology of these spaces. We will mainly focus on closed linkages, where the graph represents a circle.

  Lecture 2: *Homology and Cohomology of planar Linkage spaces* The Morse-theoretic methods from the previous lecture apply very well to planar linkage spaces, and allow us to calculate the homology groups of these spaces, which turn out to be torsion free. Furthermore, there is a simple combinatorial formula for the Betti numbers. We show how to construct perfect Morse functions for these linkage spaces and obtain result on the cohomology ring structure.

  Lecture 3: *Rigidity results for Linkage spaces* Homology calculations are not enough to distuinguish planar or spatial linkage spaces, but we show that Walker's conjecture, that the cohomology ring suffices, is true. We also discuss various other classes of linkage spaces where a version of Walker's conjecture is true.