# Implementations: overconvergent modular forms


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18th August 2008

# Description and purpose

These packages are intended for computation of Hecke eigenvalues of overconvergent $p$-adic modular forms, via the two algorithms presented in my talk:

- Direct parametrisation of modular curves
- The $p$-adic Eichler-Hijikata trace formula

# Parametrisations of modular curves

- Implementations in SAGE and PARI/GP
- Works for $p = 2, 3, 5, 7, 13$, tame level 1
- Weight must be a non-negative integer (at present)
- Radius of convergence customisable (but doesn't affect result)
- Functions to calculate matrix of $U_p$ and approximate $q$-expansions of small slope eigenforms

# Explicit parametrisation: SAGE implementation

We compute the 1/2-overconvergent 3-adic modular forms of weight 12 and tame level 1:

```
sage: attach "overconvergent.py"
sage: o = OverconvergentModularFormsSpace(3, 12, 1/2)
sage: f = o.eigenfunctions(4)[1][2]
```

(computes a rough approximation to Delta using a 4x4 truncated matrix)

```
sage: f[8] - 84480
2*3^14 + 2*3^16 + 3^17 + 2*3^18 + 3^19 + [etc]
```

(this should be close to 0, and it is)

```
sage: f = o.eigenfunctions(6)[1][2] // better
sage: f[8] - 84480
3^21 + 2*3^24 + 2*3^25 + 3^26 + 3^28 + 2*3^29 + [etc]
```

(better approximation using a 6x6 truncation)

# $p$-adic Eichler–Hijikata trace formula

- PARI/GP script based on classical trace formula script by William Stein
- Returns trace of $U_m$ for $m \in p\mathbb{N}$, or characteristic power series of $U_p$
- Tame level $\Gamma_0(N)$ for any $N$ coprime to $p$
- Results expressed in terms of parameter $w = (1+p)^k$ on weight space
- (In particular $k$ need not be integral)

# Trace formula: PARI/GP implementation example

Prime $p$ set as parameter in file header (here it is 5). We compute the trace of $U_5$ acting on 5-adic overconvergent forms in the 0 component of weight space, as a power series in the coordinate $w = (1+p)^k - 1$:

```
? trformula(trformula(5, 1, 0)
%1 = [massive mess]
```

Result is a pair (component of weight space, power series in W). We evaluate this at the point of weight space corresponding to $k = 20$:

```
? treval(%, 20)
%2 = 3*5 + 3*5^2 + 5^4 + 4*5^5 + 2*5^6 + 4*5^7 + 3*5^8 +
```

Characteristic power series of $U_5$ in weight 20:

```
? cpeval(cpformula(3, 1, 0), 20)*(1 + O(5^2))
%3 = (1 + O(5^2)) + (2*5 + 5^2 + O(5^3))*z + (2*5^10 + 5
```

(a power series in the auxilliary variable $z$ with $p$-adic coefficients)