Please let me know if any of the problems are unclear or have typos. Please let me know if you have suggestions for exercises. Each problem is followed by a number in brackets – this is the number of marks the question is worth. Please turn in at least 10 marks worth of answers by the end of week six. You should feel free to work with others; if you do so, please say whom you collaborate with (and acknowledge other sources as necessary).

Exercise 2.1. Suppose that M is an n–manifold with boundary. Suppose that M is oriented. Let $N = \partial M$ be the boundary of M; we assume that N is non-empty. Define, in the topological category, the *induced orientation* on N. [2]

Exercise 2.2. Suppose that M is a three-manifold. Prove that $M \# S^3 \cong M$. [3]

Exercise 2.3. Suppose that M and N are connected n–manifolds, with $n \geq 3$. Suppose that $D \subset M$ and $E \subset N$ are embedded n–balls. Suppose further that ∂D and ∂E have product regular neighbourhoods in M and N, respectively. Fix a homeomorphism $\phi \colon \partial D \to \partial E$. Compute the fundamental group of $M \#_\phi N$ in terms of the fundamental groups of M and N. [3]

Exercise 2.4. With hypotheses as in the previous problem, compute the homology groups of $M \#_\phi N$. [3]

Exercise 2.5. Set $P^3 = \mathbb{RP}^3$. Find (up to homeomorphism) all covers of $P^3 \# P^3$. [2]

Exercise 2.6. Suppose that $A \subset S^3$ is the Alexander horned sphere (say, as described in lecture). Show that there is a Cantor set $C \subset A$ so that A is locally flat away from C and is not locally flat at the points of C. [3]

Exercise 2.7. Suppose that T and $\overline{T}$ are the right and left trefoil knots in the three-sphere. Let $G = T \# T$ and let $S = T \# \overline{T}$ be the granny and square knots, respectively. Prove that the exteriors X_G and X_S have isomorphic fundamental groups. [3]

Exercise 2.8. With hypotheses as in the previous problem, prove that G and S are not isotopic. [2]

Exercise 2.9. Prove that the figure-eight knot is *achiral*: that is, isotopic to its mirror image. [1]

Exercise 2.10. Let $N = N_3$ be the non-orientable surface of "non-orientable genus three": that is, $N = P^2 \# P^2 \# P^2$. Give (with proof) a finite presentation of the mapping class group of N. [5]

Exercise 2.11. Suppose that $U \subset S^3$ is the unknot. Prove that the U is prime, in the sense that if $U \cong K \# K'$ then both K and K' are unknots. [4]

Exercise 2.12. We consider knots as embeddings of S^1 (oriented) into S^3 (also oriented). An *oriented connect sum* of knots is any connect sum that respects both orientations. Prove that oriented connect sum is well-defined at the level of isotopy classes. Prove further that this makes the collection of isotopy classes of oriented knots into a commutative monoid, having the unknot as the identity. (The previous problem shows that non-trivial knots do not have inverses.) [4]

Exercise 2.13. Prove that the following manifolds are prime.

- P^3 – real projective space.
- $S^2 \times S^1$ – sphere cross circle.
- T^3 – three-torus. [3]