Please let me know if any of the problems are unclear, have typos, or have any other mistakes.

Exercise 7.1. Let $T^3 = S^1 \times S^1 \times S^1$ be the three-torus. We give S^1 a CW–complex structure with one zero-cell and one one-cell.

1. Describe the resulting product CW–complex structure on T^3, including a count of the cells in each dimension, their various attaching maps, and the k–skeleta.
2. Use the above to give explicit generators for the cohomology groups $H^k(T^3; \mathbb{Z})$.
3. In terms of these generators, describe the cup product structure on $H^*(T^3; \mathbb{Z})$. [Hint: use the Künneth formula.]
4. Let X be a subcomplex of the two-skeleton obtained by deleting a single (open) two-cell. Give explict generators for the cohomology groups (again over $\mathbb{Z}$) and describe the cup product structure on $H^*(X; \mathbb{Z})$.

Exercise 7.2. For the two given manifolds X and Y, show that they are not homeomorphic.

1. $X = S^2 \times S^4$ and $Y = \mathbb{CP}^3$.
2. $X = S^2 \times S^2$ and $Y = \mathbb{CP}^2 \# \overline{\mathbb{CP}}^2$.

Exercise 7.3. Prove that a direct proof that a surface S is orientable if and only if it does not contain a homeomorphic copy of the Möbius band M^2. [Here "direct" means "without appealing to the classification of surfaces".]

Exercise 7.4. Suppose that M is an orientable n–manifold. Suppose that $N \subset M$ is a closed connected $(n-1)$–dimensional sub-manifold which separates M. Prove that N is orientable.

Exercise 7.5. Suppose that $N \subset S^n$ is a closed connected $(n-1)$–dimensional submanifold. Prove that N is orientable.