Please turn in Exercises 4.1 and 4.3 before 2pm, on 23 November (Friday), in the slot near the front office. Please let me know if any of the problems are unclear, have typos, or have mistakes.

Exercise 4.1. For each polynomial $f(x) \subset K[x]$ given below determine a splitting field L/K and state its degree. Give a short justification for each answer.

1. $K = \mathbb{Q}$ and $f(x) = x^2 - 1$.
2. $K = \mathbb{Q}$ and $f(x) = x^2 - 2$.
3. $K = \mathbb{Q}$ and $f(x) = x^2 + 1$.
4. $K = \mathbb{Q}$ and $f(x) = (x^2 - 2)(x^2 - 3)$.
5. $K = \mathbb{Q}$ and $f(x) = x^3 - 2$.

Exercise 4.2. For each polynomial $f(x) \subset K[x]$ given below determine a splitting field L/K and state its degree. Give a short justification for each answer.

1. $K = \mathbb{R}$ and $f(x) = x^2 - 2$.
2. $K = \mathbb{R}$ and $f(x) = x^2 + 1$.
3. For p prime: $K = \mathbb{F}_p$ and $f(x) = x^{p-1} - 1$.
4. For p prime: $K = \mathbb{F}_p(t)$ and $f(x) = x^p - t$.
5. $K = \mathbb{F}_2$ and $f(x) = x^3 + x + 1$.

Exercise 4.3. For each extension L/K you found in Exercise 4.1 compute $\mathrm{Aut}(L/K)$. Give a short justification for each answer.

Exercise 4.4. For each extension L/K you found in Exercise 4.2 compute $\mathrm{Aut}(L/K)$. Give a short justification for each answer.

Exercise 4.5. Consider the polynomial $f(x) = x^3 - 3x + 1 \in \mathbb{Q}$.

1. Prove that f is irreducible over $\mathbb{Q}$.
2. Prove that f has three real roots $\alpha, \beta, \gamma \in \mathbb{R}$. Give numerical approximations of these.
3. Show that $L = \mathbb{Q}(\alpha)$ is a splitting field for f. [Hint: Consider the element $\alpha^2 - 2 \in L$.]
4. Compute the group $\mathrm{Aut}(L)$ and give a short justification of your answer.