Please let me know if any of the problems are unclear or have typos. Please turn in a *single* worked exercise. Record your name, the date, and the problem solved. Please also write the names of people and/or references you consult.

Exercise 10.1. [Hatcher page 157, problem 27.] Suppose that (X, A) is a pair. Prove the short exact sequence $0 \to C_n(A) \to C_n(X) \to C_n(X, A) \to 0$ splits. Does this imply $H_n(X) \cong H_n(A) \oplus H_n(X, A)$? Explain.

Exercise 10.2. Recall that S_g is the closed (compact without boundary), connected, orientable surface of genus g. That is, S_g is obtained by attaching g handles to a planar surface with g boundary components. Prove that S_g is homeomorphic to the following CW-complex, having one vertex, $2g$ edges labelled $\{a_i, b_i\}$, and a single 2–cell attached via the path $a_1 b_1 \bar{a}_1 \bar{b}_1 a_2 b_2 \bar{a}_2 \bar{b}_2 \ldots a_g b_g \bar{a}_g \bar{b}_g$.

Exercise 10.3. [Medium. Hatcher page 19, problem 16.] Prove S^∞ is contractible.

Exercise 10.4. Compute the homology groups of P^∞, of $T^n = \times^n S^1$, and of $S^\ell \times S^m$.

Exercise 10.5. [Hatcher page 157, problems 20–23.] Suppose that X and Y are finite CW–complexes. Prove any one of the following.

- $\chi(X \times Y) = \chi(X) \cdot \chi(Y)$.
- If X is the union of subcomplexes A and B then $\chi(X) = \chi(A) + \chi(B) - \chi(A \cap B)$.
- If $p \colon Y \to X$ is an n–fold covering map then $\chi(Y) = n \cdot \chi(X)$.
- If $p \colon S_h \to S_g$ is an n–fold covering map (of surfaces) then $h = n(g - 1) + 1$. Show this is the only restriction.