

Please let me know if any of the problems are unclear or have typos.

Exercise 7.1. Suppose E is a surface and $h: E \rightarrow E$ is a finite order diffeomorphism; that is, there is a positive power n so that $h^n = \text{Id}$. Show $F = E/\langle h \rangle$ has the structure of an orbifold. Deduce F is good. Can F have a corner reflector?

Exercise 7.2. [Medium] Suppose $p, q > 1$. Give a direct proof (via “unfolding”) showing $F = D^2(p, \bar{q})$ has a finite cover by a surface.

Exercise 7.3. Recall the definition of a fibered solid torus, $T(p, q)$, with underlying space and all fibers oriented. Show $T(p, q)$ is isomorphic to $T(p', q')$ if and only if $p = p'$ and $q \equiv \pm q' \pmod{p}$. If the isomorphism preserves orientations then the sign is removed.

Exercise 7.4. Give necessary and sufficient conditions on p, q, r, s for $T(p, q)$ to cover $T(r, s)$, preserving orientations and fibers.

Exercise 7.5. [Hard] Show \mathbb{R}^3 can be partitioned into circles. [Here a “circle” is a map from S^1 to \mathbb{R}^3 that is a homeomorphism onto its image.] Show \mathbb{R}^3 does not admit a Seifert fibered structure.

Exercise 7.6. [Classwork] Let $h: \mathbb{T}^2 \rightarrow \mathbb{T}^2$ be defined by $h(x, y) = (1 - x, 1 - y)$. Define

$$M_h = I \times \mathbb{T}^2 / (1, p) \sim (0, h(p)).$$

Show M_h is a three-manifold. Show M_h admits at least two non-isomorphic Seifert fibered structures.