Please let me know if any of the problems are unclear or have typos.

Exercise 5.1. Set $\mathbb{H} = \{(x, y) \in \mathbb{R}^2 \mid y > 0\}$. Let $ds_H = ds/y$ and define $\mathbb{H}^2 = (\mathbb{H}, ds_H)$. Following the discussion in class, prove *in detail* that arcs of vertical Euclidean lines are geodesics in this geometry.

Exercise 5.2. [Reading] Following Scott's article, show that $\mathrm{PSL}(2, \mathbb{R}) < \mathrm{Isom}^+(\mathbb{H}^2)$, the group of orientation preserving isometries. Deduce that there is a unique geodesic between any pair of distinct points of $\mathbb{H}^2$. Classify geodesics in this geometry.

Exercise 5.3. Show $\mathrm{PSL}(2, \mathbb{R}) = \mathrm{Isom}^+(\mathbb{H}^2)$. [By Exercise 5.2 the former is contained in the latter.]

Exercise 5.4. Show that $\mathbb{H}^2$ is complete.

Exercise 5.5. Suppose that F is a two-orbifold. Let $|F|$ be the *underlying surface* of F: the surface obtained by forgetting the orbifold structure and retaining only the topological space. If $x \in F$ is singular point, let $\mathrm{ord}(x)$ be the order of x. If $|F|$ is compact then prove directly from the definitions

$$\chi^{\mathrm{orb}}(F) = \chi\left(|F|\right) - \sum\left(1 - \frac{1}{\mathrm{ord}(x)}\right) - \sum\left(\frac{1}{2} - \frac{1}{2\,\mathrm{ord}(y)}\right) - \frac{\mathrm{corners}(F)}{4}.$$

Here the first sum ranges over cone points $x \in F$, the second sum ranges over the corner reflectors $y \in F$, and $\mathrm{corners}(F)$ counts the number of half-mirrored corners. [Hint: First consider the case where $|F|$ is closed.]

Exercise 5.6. We say a two-orbifold F is *closed* if $|F|$ is compact and connected and F has no regular boundary. Classify the closed two-orbifolds F with $\chi^{\mathrm{orb}}(F) > 0$. [Hint: First consider the case where $|F|$ is closed.]

Exercise 5.7. Suppose $\rho\colon E \to F$ is an orbifold covering map (with $|E|$ and $|F|$ compact). Show

$$\chi^{\mathrm{orb}}(E) = \deg(\rho) \cdot \chi^{\mathrm{orb}}(F).$$

Exercise 5.8. We may partially order orbifolds via the "covering relation" $F \leq E$ if and only if there is an orbifold covering map $\rho\colon E \to F$. Prove the covering relation is transitive. Let $\mathcal{F}$ be the set of frieze orbifolds up to *isomorphism*: homeomorphisms that preserve the orbifold structure. Draw the Hasse diagram for the covering relation restricted to $\mathcal{F}$. Give a reasonable justification for the resulting picture. [Challenge: Do the same for the set of isomorphism classes of two-orbifolds with positive χ^{orb}. Exercise 5.7 will be very useful.]